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ON PRE CONTINUOUS AND PRE IRRESOLUTE SOFT MULTIFUNCTIONS

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Abstract. In this paper, we define the pre upper and lower continuous multi functions between soft topological spaces and study several properties of these multi functions. Then we define the pre upper and lower irresolute multi functions between soft topological spaces and study several properties of these multifunctions.

Keywords: soft sets; soft topology; soft multifunction; soft upper (lower) pre continuous; soft upper (lower) pre irresolute.

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1. Introduction

The concept of soft set theory is fundamentally important in almost every scientific field. Soft set theory is a new mathematical tool for dealing with uncertainties. Also, this theory is a set associated with parameters and has been applied in several directions. Many complex problems in economics, engineering, environmental science, social science, medical science etc. can not be dealt with by classical methods, because classical methods have inherent difficulties. In 1999, Molodtsov [1] introduced the concept of soft set theory as a new approach for coping with uncertainties and these problems. Also he and et al. [2] presented the basic results of the soft set

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theory. He successfully applied the soft set theory into several directions such as smoothness of functions, theory of probability, game theory, Riemann Integration, Perron Integration, theory of optimization, theory of measurement etc. Maji and Roy [3] applied soft sets theory in a multicriteria decision making problems. Then, Shabir and Naz [4] introduced the notions of soft topological spaces. Cagman et al [5] defined a soft topological space. After that, Kharal and Ahmad,[6] defined a mapping on soft classes and studied properties of these mappings. Then Akdag and Erol studied on soft multifunctions in [9, 14, 15, 16].

In this paper, firstly we define the pre upper and lower continuous multifunction from a soft topological spaces (X, τ, E) to a soft topological spaces (Y, σ, K) and study several properties of these continuous multifunctions. Finally we define the pre upper and lower irresolute multifunctions between soft topological spaces and study several properties of these multifunctions.

2. Preliminaries

Definition 2.1. [1] Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E . A pair (F, A) is called a soft set over X , where F is a mapping given by $F : A \rightarrow P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X . For $e \in A$, $F(e)$ may be considered as the set of e -approximate elements of the soft set (F, A) .

Definition 2.2. [7] A soft set (F, A) over X is called a null soft set, denoted by Φ , if $e \in A$, $F(e) = \emptyset$. If $A = E$, then the null soft set is called universal null soft set, denoted by Φ .

Definition 2.3. [7] A soft set (F, A) over X is called an absolute soft set, denoted by \tilde{A} , if $e \in A$, $F(e) = X$. If $A = E$, then the absolute soft set is called universal soft set, denoted by \tilde{X} .

Definition 2.4. [4] Let Y be a non-empty subset of X , then \tilde{Y} denotes the soft set (Y, E) over X for which $Y(e) = Y$, for all $e \in E$.

Definition 2.5. [7] The union of two soft sets of (F, A) and (G, B) over the common universe X is the soft set (H, C) , where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A, \\ F(e) \cup G(e), & \text{if } e \in A \cap B. \end{cases}$$

We write $(F,A)\tilde{\cup}(G,B) = (H,C)$.

Definition 2.6. [7] The intersection (H,C) of two soft sets (F,A) and (G,B) over a common universe X , denoted $(F,A)\tilde{\cap}(G,B)$, is defined as $C = A \cap B$, and $H(e) = F(e) \cap G(e)$ for all $e \in C$.

Definition 2.7. [7] Let (F,A) and (G,B) be two soft sets over a common universe X . $(F,A)\tilde{\subset}(G,B)$, if $A \subset B$ and $F(e) \subset G(e)$ for all $e \in A$.

Definition 2.8. [8] For a soft set (F,A) over X the relative complement of (F,A) is denoted by $(F,A)^c$ and is defined by $(F,A)^c = (F^c,A)$, where $F^c : A \rightarrow P(X)$ is a mapping given by $F^c(\alpha) = X - F(\alpha)$ for all $\alpha \in A$.

Proposition 2.1. [10] Let $(G,A), (H,A), (S,A), (T,A)$ be soft sets in X . Then the following are true;

- (a) If $(G,A)\tilde{\cap}(H,A) = \Phi$, then $(G,A)\tilde{\subset}(H,A)^c$.
- (b) $(G,A)\tilde{\cup}(G,A)^c = \tilde{X}$.
- (c) If $(G,A)\tilde{\subset}(H,A)$ and $(H,A)\tilde{\subset}(S,A)$, then $(G,A)\tilde{\subset}(S,A)$.
- (d) If $(G,A)\tilde{\subset}(H,A)$ and $(S,A)\tilde{\subset}(T,A)$ then $(G,A)\tilde{\cap}(S,A)\tilde{\subset}(H,A)\tilde{\cap}(T,A)$.
- (e) $(G,A)\tilde{\subset}(H,A)$ if and only if $(H,A)^c\tilde{\subset}(G,A)^c$.

Definition 2.9. [13] The soft set (G,E) over X is called a soft point in \tilde{X} , denoted by E_e^x , if for $e \in E$ there exist $x \in X$ such that $G(e) = \{x\}$ and $G(e') = \emptyset$ for all $e' \in E - \{e\}$.

Definition 2.10. [9] The soft point E_e^x is said to be in the soft set (H,E) , denoted by $E_e^x\tilde{\in}(H,E)$, if $x \in H(e)$.

Proposition 2.2. [10] Let E_e^x be a soft point and (H,E) be a soft set in \tilde{X} . If $E_e^x\tilde{\in}(H,E)$, then $E_e^x\tilde{\notin}(H,E)^c$.

Definition 2.11. [4] Let τ be the collection of soft sets over X , then τ is said to be a soft topology on X if satisfies the following axioms.

- (a) Φ, \tilde{X} belong to τ ,
- (b) the union of any number of soft sets in τ belongs to τ ,
- (c) the intersection of any two soft sets in τ belongs to τ .

The triplet (X, τ, E) is called a soft topological space over X . Let (X, τ, E) be a soft topological space over X , then the members of τ are said to be soft open sets in X . A soft set (F, A) over X is said to be a soft closed set in X , if its relative complement $(F, A)^c$ belongs to τ .

Definition 2.12. [17] Let (F, A) be any soft set of a soft topological space (X, τ, E) . Then (F, A) is called

- (a) soft pre-open set of X if $(F, A) \widetilde{\subset} \text{int}(cl((F, A)))$ and
- (b) soft pre-closed set of X if $cl(\text{int}(F, A)) \widetilde{\subset} (F, A)$.

Proposition 2.3. [11] (a) Arbitrary union of soft pre-open sets is a soft pre-open sets.

- (b) Arbitrary intersection of soft pre-closed sets is a soft pre-closed set.

Definition 2.13. [11] Let (X, τ, E) be a soft topological space over X and (F, A) be a soft set over X . Then:

(a) $spint(F, A) = \widetilde{\cup} \{(O, E) : (O, E) \text{ is a soft pre-open set and } (O, E) \widetilde{\subset} (F, A)\}$ is called soft preinterior.

(b) $spcl(F, A) = \widetilde{\cap} \{(H, E) : (H, E) \text{ is a soft pre-closed set and } (H, E) \widetilde{\supset} (F, A)\}$ is called soft preclosure.

Clearly that $spint(F, A)$ is the largest soft pre-open set which is contained in (F, A) and $spcl(F, A)$ is the smallest soft pre-closed set over which contains (F, A) .

Proposition 2.4. [11, 12] Let (F, A) be any soft set of a soft topological space (X, τ, E) . Then,

- (a) $spcl(\widetilde{X} - (F, A)) = \widetilde{X} - spint(F, A)$.
- (b) $spint(\widetilde{X} - (F, A)) = \widetilde{X} - spcl(F, A)$.
- (c) $spcl(F, A) = (F, A) \widetilde{\cup} cl(\text{int}(F, A))$.
- (d) $spint(F, A) = (F, A) \widetilde{\cap} \text{int}(cl(F, A))$.

Proposition 2.5. [11] Let (F, A) be any soft set of a soft topological space (X, τ, E) . Then, (F, A) is soft pre-closed (soft pre-open) iff $(F, A) = spcl(F, A)$ ($(F, A) = spint(F, A)$).

Proposition 2.6. [11] Let (F, A) and (G, B) be two soft sets in a soft topological space (X, τ, E) . Then the following are hold;

- (a) $spcl(\Phi) = \Phi$.
- (b) $spcl(F, A)$ is soft pre-closed in (X, τ, E) for each soft subset (F, A) of X .
- (c) $spcl(F, A) \widetilde{\subset} spcl(G, B)$ if $(F, A) \widetilde{\subset} (G, B)$.

$$(d) \text{spcl}(F, A) \widetilde{\cup} \text{spcl}(G, B) \widetilde{\subset} \text{spcl}((F, A) \widetilde{\cup} (G, B)).$$

$$(e) \text{spcl}((F, A) \widetilde{\cap} (G, B)) \widetilde{\subset} \text{spcl}(F, A) \widetilde{\cap} \text{spcl}(G, B).$$

Remark 2.1. [11] Similar results hold for soft preinteriors.

Definition 2.14. [9] Let $S(X, E)$ and $S(Y, K)$ be two soft classes. Let $u : X \rightarrow Y$ be multifunction and $p : E \rightarrow K$ be mapping. Then a soft multifunction $F : S(X, E) \rightarrow S(Y, K)$ is defined as follows: For a soft set (G, A) in (X, E) , $(F(G, A), K)$ is a soft set in $S(Y, K)$ given by

$$F(G, A)(k) = \begin{cases} \bigcup_{e \in p^{-1}(k)} u(G(e)), & \text{if } p^{-1}(k) \cap A \neq \emptyset \\ \emptyset, & \text{otherwise} \end{cases} \quad \text{for } k \in K. (F(G, A), K) \text{ is called a soft}$$

image of a soft set (G, A) . Moreover, $F(G, A) = \widetilde{\cup}\{F(E_e^x) : E_e^x \widetilde{\in}(G, A)\}$ for a soft subset (G, A) of X .

Definition 2.15. [9] Let $F : S(X, E) \rightarrow S(Y, K)$ be a soft multifunction.

The soft upper inverse image of (H, K) denoted by $F^+(H, K)$ and defined as

$$F^+(H, K) = \left\{ E_e^x \widetilde{\in} \widetilde{X} : F(E_e^x) \widetilde{\subset} (H, K), e \in E \right\}.$$

The soft lower inverse image of (H, K) denoted by $F^-(H, K)$ and defined as

$$F^-(H, K) = \left\{ E_e^x \widetilde{\in} \widetilde{X} : F(E_e^x) \widetilde{\cap} (H, K) \neq \Phi \right\}.$$

Definition 2.16. [9] Let $F : S(X, E) \rightarrow S(Y, K)$ and $G : S(X, E) \rightarrow S(Y, K)$ be two soft multifunctions. Then, F equal to G if $F(E_e^x) = G(E_e^x)$, for each $E_e^x \widetilde{\in} \widetilde{X}$.

Definition 2.17. [9] The soft multifunction $F : S(X, E) \rightarrow S(Y, K)$ is called surjective if p and u are surjective.

Theorem 2.1. [9] Let $F : S(X, E) \rightarrow S(Y, K)$ be a soft multifunction Then, for soft sets (F, E) , (G, E) and for a family of soft sets $(G_i, E)_{i \in I}$ in the soft class $S(X, E)$ the following are hold;

$$(a) F(\Phi) = \Phi.$$

$$(b) F(\widetilde{X}) \widetilde{\subset} \widetilde{Y}.$$

$$(c) F((G, A) \widetilde{\cup} (H, B)) = F(G, A) \widetilde{\cup} F(H, B) \text{ in general } F(\widetilde{\cup}_i (G_i, E)) = \widetilde{\cup}_i F(G_i, E).$$

$$(d) F((G, A) \widetilde{\cap} (H, B)) \widetilde{\subset} F(G, A) \widetilde{\cap} F(H, B) \text{ in general } F(\widetilde{\cap}_i (G_i, E)) \widetilde{\subset} \widetilde{\cap}_i F(G_i, E).$$

$$(e) \text{ If } (G, E) \widetilde{\subset} (H, E), \text{ then } F(G, E) \widetilde{\subset} F(H, E).$$

Theorem 2.2. [9] Let $F : S(X, E) \rightarrow S(Y, K)$ be a soft multifunction Then, for soft sets (G, K) , (H, K) in the soft class $S(Y, K)$ the following are hold;

$$(a) F^-(\Phi) = \Phi \text{ and } F^+(\Phi) = \Phi.$$

- (b) $F^-(\tilde{Y}) = \tilde{X}$ and $F^+(\tilde{Y}) = \tilde{X}$.
- (c) $F^-((G,K)\tilde{\cup}(H,K)) = F^-(G,K)\tilde{\cup}F^-(H,K)$.
- (d) $F^+(G,K)\tilde{\cup}F^+(H,K) \tilde{\subset} F^+((G,K)\tilde{\cup}(H,K))$.
- (e) $F^-((G,K)\tilde{\cap}(H,K)) \tilde{\subset} F^-(G,K)\tilde{\cap}F^-(H,K)$.
- (f) $F^+(G,K)\tilde{\cap}F^+(H,K) = F^+((G,K)\tilde{\cap}(H,K))$.
- (g) If $(G,K)\tilde{\subset}(H,K)$, then $F^-(G,K)\tilde{\subset}F^-(H,K)$ and $F^+(G,K)\tilde{\subset}F^+(H,K)$.

Proposition 2.7. [9] Let (G_i, K) be soft sets over Y for each $i \in I$. The follows are true for a soft multifunction $F : (X, \tau, E) \rightarrow (Y, \sigma, K)$:

- (a) $F^-(\tilde{\cup}_{i \in I}(G_i, K)) = \tilde{\cup}_{i \in I}(F^-(G_i, K))$
- (b) $\tilde{\cap}_{i \in I}(F^+(G_i, K)) = F^+(\tilde{\cap}_{i \in I}(G_i, K))$
- (c) $\tilde{\cup}_{i \in I}F^+(G_i, K) \tilde{\subset} F^+(\tilde{\cup}_{i \in I}(G_i, K))$
- (d) $F^-(\tilde{\cap}_{i \in I}(G_i, K)) \tilde{\subset} \tilde{\cap}_{i \in I}(F^-(G_i, K))$.

Proposition 2.8. [9] Let $F : S(X, E) \rightarrow S(Y, K)$ be a soft multifunction. Then the follows are true:

- (a) $(G, A) \tilde{\subset} F^+(F(G, A)) \tilde{\subset} F^-(F(G, A))$ for a soft subset (G, A) in X . If F is surjective then $(G, A) = F^+(F(G, A)) = F^-(F(G, A))$.
- (b) $F(F^+(H, B)) \tilde{\subset}(H, B) \tilde{\subset} F(F^-(H, B))$ for a soft subset (H, B) in Y .
- (c) For two soft subsets (H, B) and (U, C) in Y if $(H, B)\tilde{\cap}(U, C) = \Phi$, then $F^+(H, B)\tilde{\cap}F^-(U, C) = \Phi$.

Proposition 2.9. [9] Let $F : S(X, E) \rightarrow S(Y, K)$ and $G : S(Y, K) \rightarrow S(Z, L)$ be two soft multifunction. Then the follows are true:

- (a) $(F^-)^- = F$.
- (b) For a soft subset (T, C) in Z , $(GoF)^-(T, C) = F^-(G^-(T, C))$ and $(GoF)^+(T, C) = F^+(G^+(T, C))$.

Proposition 2.10. [9] Let (G, K) be a soft set over Y . Then the followings are true for a soft multifunction $F : S(X, E) \rightarrow S(Y, K)$

- (a) $F^+(\tilde{Y} - (G, K)) = \tilde{X} - F^-(G, K)$
- (b) $F^-(\tilde{Y} - (G, K)) = \tilde{X} - F^+(G, K)$.

3. Upper and Lower Pre Continuity of Soft Multifunctions

Definition 3.1. Let (X, τ, E) and (Y, σ, K) be two soft topological spaces. Then a soft multifunction $F : (X, \tau, E) \rightarrow (Y, \sigma, K)$ is said to be;

(a) soft upper pre continuous at a soft point $E_e^x \in \tilde{X}$ if for every soft open set (G, K) such that $F(E_e^x) \tilde{\subset} (G, K)$, there exists a soft pre open neighborhood (P, E) of E_e^x such that $F(E_e^z) \tilde{\subset} (G, K)$ for all $E_e^z \in (P, E)$.

(b) soft lower pre continuous at a soft point $E_e^x \in \tilde{X}$ if for every soft open set (G, K) such that $F(E_e^x) \tilde{\cap} (G, K) \neq \Phi$, there exists a soft pre open neighborhood (P, E) of E_e^x such that $F(E_e^z) \tilde{\cap} (G, K) \neq \Phi$ for all $E_e^z \in (P, E)$.

(c) soft upper(lower) pre continuous if F has this property at every soft point of X .

Theorem 3.1. For a soft multifunction $F : (X, \tau, E) \rightarrow (Y, \sigma, K)$ the following statements are equivalent;

- (a) F is soft upper pre continuous.
- (b) $F^+(V, K)$ is a soft pre open set in X for each soft open set (V, K) of Y .
- (c) $F^-(H, K)$ is a soft pre closed set in X for each soft closed set (H, K) of Y .
- (d) $spcl(F^-(V, K)) \tilde{\subset} F^-(cl(V, K))$ for each soft subset (V, K) of Y .
- (e) $F^+(H, K)$ is a soft pre neighborhood of E_e^x , for each soft point E_e^x in X and for each soft neighborhood (H, K) of $F(E_e^x)$.
- (f) for each soft point E_e^x in X and for each soft neighborhood (V, K) of $F(E_e^x)$, there exists a soft pre neighborhood (U, E) of E_e^x such that $F(U, E) \tilde{\subset} (V, K)$.
- (g) $F^+(int(H, K)) \tilde{\subset} spint(F^+(H, K))$ for each soft subset (H, K) of Y .
- (h) $F^+(G, K) \tilde{\subset} int(cl(F^+(G, K)))$ for each soft open set (G, K) of Y .
- (i) $cl(F^+(V, K))$ is a soft neighborhood of E_e^x , for each soft point E_e^x in X and for each soft neighborhood (V, K) of $F(E_e^x)$.

Proof. (a) \Rightarrow (b) Let (V, K) be any soft open set of Y and $E_e^x \in F^+(V, K)$.

Then $F(E_e^x) \tilde{\subset} V$ and there exists soft preopen set (U, E) containing E_e^x such that $F(U, E) \tilde{\subset} (V, K)$.

Since $E_e^x \in (U, E) \tilde{\subset} int(cl(U, E)) \tilde{\subset} int(cl(F^+(V, K)))$, we have $F^+(V, K) \tilde{\subset} int(cl(F^+(V, K)))$.

This shows that $F^+(V, K)$ is soft pre open set in X .

(b) \Leftrightarrow (c) Let (H, K) be any soft closed set of Y . Then, $\tilde{Y} - (H, K)$ is a soft open set in Y . By (b), $F^+ (\tilde{Y} - (H, K)) = \tilde{X} - F^- (H, K)$ is a soft pre open set in X . Then $X - F^- (H, K)$ is a soft pre open set in X . Thus, $F^- (H, K)$ is a soft pre closed set in X .

(c) \Rightarrow (d) Let (V, K) be any soft subset of Y . Since $cl(V, K)$ is a soft closed set in Y , then $F^- (cl(V, K))$ is soft pre closed set in X . Thus $spcl(F^- (V, K)) \subseteq F^- (cl(V, K))$.

(d) \Rightarrow (c) Let (H, K) be any soft closed set of Y . Then $spcl(F^- (H, K)) \tilde{\subset} F^- (cl(H, K)) = F^- (H, K)$ and hence $F^- (H, K)$ is soft pre closed set in X .

(b) \Rightarrow (e) Let $E_e^x \tilde{\in} \tilde{X}$ and (H, K) be any soft neighborhood of $F(E_e^x)$. Then there exists a soft open set (G, K) of Y such that $F(E_e^x) \tilde{\subset} (G, K) \tilde{\subset} (H, K)$. Then we have $E_e^x \tilde{\in} F^+ (G, K) \tilde{\subset} F^+ (H, K)$ and since $F^+ (G, K)$ is soft pre open set in X then $F^+ (H, K)$ is a soft pre neighborhood of E_e^x .

(e) \Rightarrow (f) Let $E_e^x \tilde{\in} \tilde{X}$ and (V, K) be any soft neighborhood of $F(E_e^x)$. If we take $(U, E) = F^+ (V, K)$, then (U, E) is soft pre neighborhood of E_e^x and $F(U, E) \tilde{\subset} (V, K)$.

(f) \Rightarrow (a) Let $E_e^x \tilde{\in} \tilde{X}$ and (V, K) be any soft open set of Y such that $F(E_e^x) \tilde{\subset} (V, K)$. Then (V, K) is a soft neighborhood of $F(E_e^x)$ and there exists (U, E) soft pre neighborhood of E_e^x such that $F(U, E) \tilde{\subset} (V, K)$. Therefore, there exists soft pre open set (H, E) in X such that $E_e^x \tilde{\in} (H, E) \tilde{\subset} (U, E)$ and hence $F(H, E) \tilde{\subset} (V, K)$. This implies that F is soft upper pre continuous.

(b) \Rightarrow (g) For any subset (H, K) of Y , $int(H, K)$ is soft open set in Y . Then $F^+ (int(H, K))$ is soft pre open set in X .

Hence $F^+ (int(H, K)) = spint(F^+ (int(H, K))) \tilde{\subset} spint(F^+ (H, K))$.

(g) \Rightarrow (b) Let (V, K) be any soft open set of Y . Then

$F^+ (V, K) = F^+ (int(V, K)) \tilde{\subset} spint(F^+ (V, K))$ and hence $F^+ (V, K)$ is soft pre open set in X .

(b) \Leftrightarrow (h) Obvious.

(h) \Rightarrow (i) Let $E_e^x \tilde{\in} \tilde{X}$ and (V, K) be any soft open neighborhood of $F(E_e^x)$. Then $E_e^x \tilde{\in} F^+ (V, K) = int(F^+ (V, K)) \tilde{\subset} int(cl(F^+ (V, K))) \tilde{\subset} cl(F^+ (V, K))$ and hence $cl(F^+ (V, K))$ is a neighborhood of E_e^x .

(i) \Rightarrow (h) Let (G, K) be any soft open set of Y and $E_e^x \tilde{\in} F^+ (G, K)$. Then $cl(F^+ (G, K))$ is a soft neighborhood of E_e^x and thus $E_e^x \tilde{\in} int(cl(F^+ (G, K)))$. Hence $F^+ (G, K) \tilde{\subset} int(cl(F^+ (G, K)))$.

Theorem 3.2. For a multifunction $F : (X, \tau, E) \longrightarrow (Y, \sigma, K)$, the following are equivalent;

- (a) F is soft lower pre continuous.
- (b) $F^-(V, K)$ is a soft pre open set in X for each soft open set (V, K) of Y .
- (c) $F^+(H, K)$ is a soft pre closed set in X for each soft closed set (H, K) of Y .
- (d) $spcl(F^+(V, K)) \widetilde{\subset} F^+(cl(V, K))$ for each subset (V, K) of Y .
- (e) for each $E_e^x \widetilde{\in} \widetilde{X}$ and for each soft neighborhood (H, K) which intersects $F(E_e^x)$, $F^-(H, K)$ is a soft pre neighborhood of E_e^x .
- (f) for each $E_e^x \widetilde{\in} \widetilde{X}$ and for each soft neighborhood (V, K) which intersects $F(E_e^x)$, there exists a soft pre neighborhood (U, E) of E_e^x such that a $F(E_e^z) \widetilde{\cap} (V, K) \neq \Phi$ for every $E_e^z \widetilde{\in} (U, E)$.
- (g) $F^-(int(G, K)) \widetilde{\subset} spint(F^-(G, K))$ for each soft subset (G, K) of Y .
- (h) $F^-(H, K) \widetilde{\subset} int(cl(F^-(H, K)))$ for each soft open set (H, K) of Y .
- (i) for each $E_e^x \widetilde{\in} \widetilde{X}$ and for each soft neighborhood (V, K) which intersects $F(E_e^x)$, $cl(F^-(V, K))$ is a soft neighborhood of E_e^x .

Proof. The proof is similar to previous theorem.

Definition 3.2. [9] Let (X, τ, E) and (Y, σ, K) be two soft topological spaces. Then a soft multifunction $F : (X, \tau, E) \rightarrow (Y, \sigma, K)$ is said to be

- (a) soft upper continuous at a soft point $E_e^x \widetilde{\in} \widetilde{X}$ if for each soft open (G, K) such that $F(E_e^x) \widetilde{\subset} (G, K)$, there exists a soft open neighborhood $P(E_e^x)$ of E_e^x such that $F(E_e^z) \widetilde{\subset} (G, K)$ for all $E_e^z \widetilde{\in} P(E_e^x)$.
- (b) soft lower continuous at a point $E_e^x \widetilde{\in} \widetilde{X}$ if for each soft open (G, K) such that $F(E_e^x) \widetilde{\cap} (G, K) \neq \Phi$, there exists a soft open neighborhood $P(E_e^x)$ of E_e^x such that $F(E_e^z) \widetilde{\cap} (G, K) \neq \Phi$ for all $E_e^z \widetilde{\in} P(E_e^x)$.
- (c) soft upper(lower) continuous if F has this property at every soft point of X .

Proposition 3.1. [9] A soft multifunction $F : (X, \tau, E) \rightarrow (Y, \sigma, K)$ is;

- (a) soft upper continuous if and only if $F^+(G, K)$ is soft open in X , for every soft open set (G, K) .
- (b) soft lower continuous if and only if $F^-(G, K)$ is soft open set in X , for every soft set (G, K) .

Remark 3.1. Every soft upper (lower) continuous multifunctions is soft upper (lower) pre continuous. But the converse is not true as shown by the following examples.

Example 3.1. Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$, $E = \{e_1, e_2\}$, $K = \{k_1, k_2\}$.

$$\tau = \left\{ \Phi, \tilde{X}, (G, E) \right\}, \text{ where } (G, E) = \{(e_1, \{x_1\}), (e_2, \{x_2\})\} \text{ and}$$

$$\sigma = \left\{ \Phi, \tilde{Y}, (H, K) \right\}, \text{ where } (H, K) = \{(k_1, \{y_1\}), (k_2, \{y_2\})\}.$$

Let $u : X \rightarrow Y$ be multifunction defined as: $u(x_1) = \{y_1, y_2\}$, $u(x_2) = \{y_2\}$
and $p : E \rightarrow K$ be mapping defined as: $p(e_1) = k_1$, $p(e_2) = k_2$.

Then the soft multifunction $F : (X, \tau, E) \rightarrow (Y, \sigma, K)$ is soft upper pre continuous but is not soft upper continuous .Because for a soft open set (H, K) in Y , $F^+(H, K) = \{(e_2, \{x_2\})\}$ is soft pre open set but is not soft open set in X .

Example 3.2. Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$, $E = \{e_1, e_2\}$, $K = \{k_1, k_2\}$.

$$\tau = \left\{ \Phi, \tilde{X}, (G, E) \right\}, \text{ where } (G, E) = \{(e_1, \{x_1\}), (e_2, \{x_2\})\} \text{ and}$$

$$\sigma = \left\{ \Phi, \tilde{Y}, (H, K) \right\}, \text{ where } (H, K) = \{(k_1, \{y_1\}), (k_2, \{y_2\})\}.$$

Let $u : X \rightarrow Y$ be multifunction defined as: $u(x_1) = \{y_1, y_2\}$, $u(x_2) = \{y_2\}$
and $p : E \rightarrow K$ be mapping defined as: $p(e_1) = k_1$, $p(e_2) = k_2$.

Then the soft multifunction $F : (X, \tau, E) \rightarrow (Y, \sigma, K)$ is soft lower pre continuous but is not soft lower continuous .Because for a soft open set (H, K) in Y , $F^-(H, K) = \{(e_1, \{x_2\}), (e_2, X)\}$ is soft pre open set but is not soft open set in X .

4. Upper and Lower Irresolute Soft Multifunctions

Definition 4.1. Let (X, τ, E) and (Y, σ, K) be two soft topological spaces. Then a soft multifunction $F : (X, \tau, E) \rightarrow (Y, \sigma, K)$ is said to be;

(a) soft upper pre irresolute at a soft point $E_\rho^x \tilde{\in} \tilde{X}$ if for every soft pre open set (G, K) such that $F(E_\rho^x) \tilde{\subset} (G, K)$, there exists a soft pre open neighborhood (P, E) of E_ρ^x such that $F(E_\rho^z) \tilde{\subset} (G, K)$ for all $E_\rho^z \tilde{\in} (P, E)$.

(b) soft lower pre irresolute at a soft point $E_\rho^x \tilde{\in} \tilde{X}$ if for every soft pre open set (G, K) such that $F(E_\rho^x) \tilde{\cap} (G, K) \neq \Phi$, there exists a soft pre open neighborhood (P, E) of E_ρ^x such that $F(E_\rho^z) \tilde{\cap} (G, K) \neq \Phi$ for all $E_\rho^z \tilde{\in} (P, E)$.

(c) soft upper (lower) pre irresolute if F has this property at every soft point of X .

Example 4.1. Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$,

$$E = \{e_1, e_2\}, K = \{k_1, k_2\}. \tau = \left\{ \Phi, \tilde{X}, (F_1, E), (F_2, E), (F_3, E) \right\}, \text{ where}$$

$$(F_1, E) = \{(e_2, \{x_2\})\}, (F_2, E) = \{(e_2, X)\}, (F_3, E) = \{(e_1, X), (e_2, \{x_2\})\} \text{ and}$$

$\sigma = \left\{ \Phi, \tilde{Y}, (H_1, K), (H_2, K), (H_3, K) \right\}$, where

$$(H_1, K) = \{(k_1, \{y_1\})\}, (H_2, K) = \{(k_2, \{y_2\})\}, (H_3, K) = \{(k_1, \{y_1\}), (k_2, \{y_2\})\}.$$

Let $u : X \rightarrow Y$ be multifunction defined as: $u(x_1) = \{y_1, y_2\}$, $u(x_2) = \{y_2\}$ and $p : E \rightarrow K$ be mapping defined as: $p(e_1) = k_1$, $p(e_2) = k_2$. Then the soft multifunction $F : (X, \tau, E) \rightarrow (Y, \sigma, K)$ is soft upper pre irresolute. Because for the soft pre open sets $\{(k_1, \{y_1\})\}$, $\{(k_2, \{y_2\})\}$, $\{(k_1, \{y_1\}), (k_2, \{y_2\})\}$, $\{(k_1, \{y_1\}), (k_2, Y)\}$, $\{(k_1, Y), (k_2, \{y_2\})\}$ in Y ,

$$F^+(\{(k_1, \{y_1\})\}) = \Phi,$$

$$F^+(\{(k_2, \{y_2\})\}) = \{(e_2, \{x_2\})\} = F^+(\{(k_1, \{y_1\}), (k_2, \{y_2\})\}),$$

$$F^+(\{(k_1, \{y_1\}), (k_2, Y)\}) = \{(e_2, X)\},$$

$$F^+(\{(k_1, Y), (k_2, \{y_2\})\}) = \{(e_1, X), (e_2, \{x_2\})\} \text{ is soft pre open set in } X.$$

Example 4.2. Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$,

$$E = \{e_1, e_2\}, K = \{k_1, k_2\}.$$

$\tau = \left\{ \Phi, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E) \right\}$, where

$$(F_1, E) = \{(e_1, \{x_1\})\}, (F_2, E) = \{(e_1, \{x_1\}), (e_2, \{x_1\})\}, (F_3, E) = \{(e_1, X)\},$$

$$(F_4, E) = \{(e_1, \{x_1\}), (e_2, X)\}, (F_5, E) = \{(e_1, X), (e_2, \{x_1\})\}, (F_6, E) = \{(e_2, \{x_1\})\},$$

$$(F_7, E) = \{(e_2, X)\} \text{ and } \sigma = \left\{ \Phi, \tilde{Y}, (H_1, K), (H_2, K), (H_3, K) \right\}, \text{ where } (H_1, K) = \{(k_1, \{y_1\})\},$$

$$(H_2, K) = \{(k_2, \{y_2\})\}, (H_3, K) = \{(k_1, \{y_1\}), (k_2, \{y_2\})\}.$$

Let $u : X \rightarrow Y$ be multifunction defined as: $u(x_1) = \{y_1, y_2\}$, $u(x_2) = \{y_2\}$ and $p : E \rightarrow K$ be mapping defined as: $p(e_1) = k_1$, $p(e_2) = k_2$. Then the soft multifunction $F : (X, \tau, E) \rightarrow (Y, \sigma, K)$ is soft lower pre irresolute. Because for the soft pre open sets $\{(k_1, \{y_1\})\}$, $\{(k_2, \{y_2\})\}$, $\{(k_1, \{y_1\}), (k_2, \{y_2\})\}$, $\{(k_1, \{y_1\}), (k_2, Y)\}$, $\{(k_1, Y), (k_2, \{y_2\})\}$ in Y ,

$$F^-(\{(k_1, \{y_1\})\}) = \{(e_1, \{x_1\})\}, F^-(\{(k_2, \{y_2\})\}) = \{(e_2, X)\},$$

$$F^-(\{(k_1, \{y_1\}), (k_2, \{y_2\})\}) = \{(e_1, \{x_1\}), (e_2, X)\} = F^-(\{(k_1, \{y_1\}), (k_2, Y)\}),$$

$$F^-(\{(k_1, Y), (k_2, \{y_2\})\}) = \tilde{Y} \text{ is soft pre open set in } X.$$

Theorem 4.1. For a soft multifunction $F : (X, \tau, E) \rightarrow (Y, \sigma, K)$ the following properties are equivalent;

(a) F is soft upper pre irresolute.

(b) For each soft point $E_e^x \in \tilde{X}$ and each soft pre open (G, K) containing $F(E_e^x)$, $E_e^x \in \text{int}(cl(F^+(G, K)))$.

(c) For each soft point $E_\epsilon^x \in \tilde{X}$ and each soft pre open (G, K) containing $F(E_\epsilon^x)$, there exists a soft open set (U, E) such that $E_\epsilon^x \in (U, E) \tilde{\subset} cl(F^+(G, K))$.

(d) For each soft point $E_\epsilon^x \in \tilde{X}$ and each soft pre neighborhood (G, K) of $F(E_\epsilon^x)$, $F^+(G, K)$ is soft pre neighborhood of E_ϵ^x .

(e) For each soft point $E_\epsilon^x \in \tilde{X}$ and each soft pre neighborhood (G, K) of $F(E_\epsilon^x)$, there exists (U, E) is soft pre neighborhood of E_ϵ^x such that $F(U, E) \tilde{\subset} (G, K)$.

(f) $F^+(G, K)$ is soft pre open set, for every soft pre open set (G, K) in Y .

(g) $F^-(G, K)$ is soft pre closed set, for every soft pre closed set (G, K) in Y .

(h) $spcl(F^-(G, K)) \tilde{\subset} (F^-(spcl(G, K)))$, for every soft set (G, K) in Y .

Proof. (a) \Rightarrow (b) Let (G, K) be any soft pre open set and $F(E_\epsilon^x) \tilde{\subset} (G, K)$. By (a), there exists (P, E) soft pre open set containing E_ϵ^x such that $F(E_\epsilon^z) \tilde{\subset} (G, K)$ for all $E_\epsilon^z \in (P, E)$. Then $F(P, E) \tilde{\subset} (G, K)$. Since (P, E) soft pre open set, then we have

$$E_\epsilon^x \in (P, E) \tilde{\subset} int(cl(P, E)) \tilde{\subset} int(cl(F^+(G, K))).$$

(b) \Rightarrow (c) If we take $int(cl(F^+(G, K))) = (U, E)$ in (b), then

$$E_\epsilon^x \in (U, E) \tilde{\subset} cl(F^+(G, K)).$$

(c) \Rightarrow (d) Let (G, K) be soft pre neighborhood of $F(E_\epsilon^x)$, then $E_\epsilon^x \in F^+(G, K)$ and by (c), there exists soft open set $E_\epsilon^x \in (U, E) \tilde{\subset} cl(F^+(G, K))$. Then $E_\epsilon^x \in (U, E) = int(U, E) \tilde{\subset} int(cl(F^+(G, K)))$. Thus we have $F^+(G, K) \tilde{\subset} int(cl(F^+(G, K)))$ and $F^+(G, K)$ is soft pre neighborhood of E_ϵ^x .

(d) \Rightarrow (e) Let (G, K) be soft pre neighborhood of $F(E_\epsilon^x)$. If we take $(U, E) = F^+(G, K)$, then by (d), (U, E) is soft pre neighborhood of E_ϵ^x and $F(U, E) \tilde{\subset} (G, K)$.

(e) \Rightarrow (f) Let (G, K) be soft pre open set and $E_\epsilon^x \in F^+(G, K)$. By (e), there exists (U, E) a soft pre neighborhood of E_ϵ^x such that $F(U, E) \tilde{\subset} (G, K)$. Then $E_\epsilon^x \in (U, E) \tilde{\subset} F^+(G, K)$ and thus $E_\epsilon^x \in (U, E) \tilde{\subset} int(cl(U, E)) \tilde{\subset} int(cl(F^+(G, K)))$. Therefore $F^+(G, K) \tilde{\subset} int(cl(F^+(G, K)))$. Then $F^+(G, K)$ is soft pre open set.

(f) \Rightarrow (g) Let (H, K) be soft pre closed set. Then $\tilde{Y} - (H, K)$ is soft pre open set and by (f), $F^+(\tilde{Y} - (H, K)) = \tilde{Y} - (F^-(H, K))$ is soft pre open set. Therefore $F^-(H, K)$ is soft pre closed set.

(g) \Rightarrow (h) Let (G, K) be any soft subset. Then $spcl(G, K)$ is soft pre closed set and by (g), $F^-(spcl(G, K))$ is soft pre closed set. Therefore $spcl(F^-(G, K)) \widetilde{\subset} spcl(F^-(spcl(G, K))) = F^-(spcl(G, K))$.

(h) \Rightarrow (a) Let (G, K) be soft pre open set and $E_e^x \widetilde{\in} F^+(G, K)$. Then $F(E_e^x) \widetilde{\subset} (G, K)$ and $F(E_e^x) \widetilde{\cap} (\widetilde{Y} - (G, K)) = \Phi$. Thus $E_e^x \notin F^-(\widetilde{Y} - (G, K)) = F^-(\widetilde{Y} - spint(G, K)) = F^-(spcl(\widetilde{Y} - (G, K)))$ and by (h), $E_e^x \notin spcl(F^-(\widetilde{Y} - (G, K)))$. Then there exists soft pre open set (U, E) containing E_e^x such that

$(U, E) \widetilde{\cap} F^-(\widetilde{Y} - (G, K)) = \Phi$. Then, $F(U, E) \widetilde{\subset} (G, K)$. Therefore F is upper pre irresolute.

Theorem 4.2. For a soft multifunction $F : (X, \tau, E) \rightarrow (Y, \sigma, K)$ the following properties are equivalent;

- (a) F is soft lower pre irresolute.
- (b) $E_e^x \widetilde{\in} int(cl(F^-(G, K)))$ for each soft point $E_e^x \widetilde{\in} \widetilde{X}$ and each soft pre open (G, K) such that $F(E_e^x) \widetilde{\cap} (G, K) \neq \Phi$.
- (c) For each soft point $E_e^x \widetilde{\in} \widetilde{X}$ and each soft pre open (G, K) containing $F(E_e^x)$, there exists a soft open set (U, E) such that $E_e^x \widetilde{\in} (U, E) \widetilde{\subset} cl(F^-(G, K))$.
- (d) For each soft pre open (G, K) and each soft point $E_e^x \widetilde{\in} F^-(G, K)$, there exists a soft open set (U, E) such that $E_e^x \widetilde{\in} (U, E) \widetilde{\subset} (F^-(G, K))$.
- (e) $F^-(G, K)$ is soft pre open set, for every soft pre open set (G, K) in Y .
- (f) $F^+(G, K)$ is soft pre closed set, for every soft pre closed set (G, K) in Y .
- (g) $cl(int(F^+(G, K))) \widetilde{\subset} (F^+(spcl(G, K)))$, for every soft subset (G, K) in Y .
- (h) $F(cl(int(H, E))) \widetilde{\subset} spcl(F(H, E))$, for every soft subset (H, E) in X .
- (k) $F(spcl(H, E)) \widetilde{\subset} spcl(F(H, E))$, for every soft subset (H, E) in X .
- (l) $spcl(F^+(G, K)) \widetilde{\subset} (F^+(spcl(G, K)))$, for every soft set (G, K) in Y .

Proof. (a) \Rightarrow (b), (b) \Rightarrow (c) and (c) \Rightarrow (d) is similar to previous theorem.

(d) \Rightarrow (e) Let (G, K) be soft pre open set and $E_e^x \widetilde{\in} F^-(G, K)$, then by (d), there exists pre soft open set (U, E) such that $E_e^x \widetilde{\in} (U, E) \widetilde{\subset} (F^-(G, K))$. Then

$E_e^x \widetilde{\in} (U, E) \widetilde{\subset} int(cl(U, E)) \widetilde{\subset} int(cl(F^-(G, K)))$. Therefore we have

$F^-(G, K) \widetilde{cl}(cl(F^-(G, K)))$ and $F^-(G, K)$ is soft pre open set.

(e) \Rightarrow (f) Let (H, K) be soft pre closed set. Then $\widetilde{Y} - (H, K)$ is soft pre open set and by (e), $F^-(\widetilde{Y} - (H, K)) = \widetilde{Y} - (F^+(H, K))$ is soft pre open set. Therefore $F^+(H, K)$ is soft pre closed set.

(f) \Rightarrow (g) Let (G, K) be any soft subset. Then $spcl(G, K)$ is soft pre closed set and by (f), $F^+(spcl(G, K))$ is soft pre closed set. Therefore

$$cl(int(F^+(G, K))) \widetilde{cl}(int(F^+(spcl(G, K)))) \widetilde{cl}(F^+(spcl(G, K))) = F^+(spcl(G, K)).$$

(g) \Rightarrow (h) Let (H, E) be any soft subset. We know that $(H, E) \widetilde{cl}F^+(F(H, E))$ and by (g), $cl(int(H, E)) \widetilde{cl}(int(F^+(F(H, E)))) \widetilde{cl}F^+(spcl(F(H, E)))$.

$$\text{Thus } F(cl(int(H, E))) \widetilde{cl}spcl(F(H, E)).$$

(h) \Rightarrow (k) Let (H, E) be any soft subset. We know that

$$spcl(H, E) = (H, E) \widetilde{cl}(int(H, E)). \text{ Then}$$

$$F(spcl(H, E)) = F((H, E) \widetilde{cl}(int(H, E))) \widetilde{cl}F(H, E) \widetilde{cl}F(cl(int(H, E))) \widetilde{cl}spcl(F(H, E)).$$

(k) \Rightarrow (l) Let (G, K) be any soft subset. For $F^+(G, K)$ by (k),

$$F(spcl(F^+(G, K))) \widetilde{cl}spcl(F(F^+(G, K))) \text{ and thus we have}$$

$$spcl(F^+(G, K)) \widetilde{cl}F^+(spcl(F(F^+(G, K)))) \widetilde{cl}F^+(spcl(G, K)).$$

(l) \Rightarrow (a) Let (G, K) be any soft subset with $E_e^x \widetilde{cl}F^-(G, K)$. Then $F(E_e^x) \widetilde{cl}(G, K) \neq \Phi$ and $F(E_e^x) \widetilde{cl}\widetilde{Y} - (G, K)$. Then $E_e^x \widetilde{cl}F^+(\widetilde{Y} - (G, K))$. Since (G, K) soft pre open set then $\widetilde{Y} - (G, K)$ is soft pre closed set and $E_e^x \widetilde{cl}F^+(spcl(\widetilde{Y} - (G, K)))$. By (l), $E_e^x \widetilde{cl}spcl(F^+(\widetilde{Y} - (G, K)))$ and there exists soft pre open set (U, E) containing E_e^x such that $\Phi = (U, E) \widetilde{cl}F^+(\widetilde{Y} - (G, K)) = (U, E) \widetilde{cl}\widetilde{Y} - (F^-(G, K))$ and $(U, E) \widetilde{cl}F^-(G, K)$. Therefore, F is soft lower pre irresolute.

5. Conclusion

Since the soft sets are very useful in information systems, we hoped that these multifunctions will be useful for the many sciences, such as medical, engineering, economy, social science, etc. In this paper, we define the upper and lower pre continuity of soft multifunctions and several properties of these multifunction have been given. Finally we define the upper and lower pre irresolute soft multifunctions and by examples and counter examples several properties of these multifunction have been given.

Conflict of Interests

The authors declare that there is no conflict of interests.

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