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# MINIMAL CYCLIC CODES OF LENGTH $16p^n$ OVER $GF(q)$, WHERE $q$ IS PRIME OR PRIME POWER OF THE FORM $16k+7$

VISHVAJIT SINGH[1,*], MANJU PRUTHI[1], JAGBIR SINGH[2]

[1]Department of Mathematics, I.G. University, Meerpur, Rewari, India

[2]Department of Mathematics, Maharshi Dayanand University, Rohtak, India



**Abstract.** In this paper, the expressions for primitive idempotents in group algebra of cyclic group G of length $16p^n$, where $p$ is prime and $q$ is some prime or prime power (of type $16k+7$), $n$ is a positive integer, order of $q$ modulo $p^n$ is $\frac{\phi(p^n)}{2}$, are obtained. Associated with this the generating polynomials and minimum distance bounds for the corresponding cyclic codes are obtained.

**Keywords:** cyclotomic cosets; primitive idempotents; generating polynomials; minimum distance.

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## 1. Introduction

The group algebra $FC_{16p^n}$, $F$ is field of order $q$ and $C_{16p^n}$ is cyclic group of order $16p^n$ such that $g.c.d.(q,16p) = 1$, is semi-simple having finite cardinality of collection of primitive idempotents which is equal to the cardinality of collection of $q$-cyclotomic cosets modulo $16p^n$. The primitive idempotents of minimal cyclic codes of length $m$, where order of $q$ modulo $m$ is $\phi(m)$ for $m = 2,4,p^n,2p^n$ were calculated by Pruthy and Arrora. The primitive idempotents of length $p^n$ with order of $q$ modulo $p^n$ is $\frac{\phi(p^n)}{2}$ were computed by Arora et. all and minimal

*Corresponding author

E-mail address: vishvajit73.sheoran@gmail.com

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quadratic residue codes of length $p^n$ by Batra and Arrora. Cyclic codes of length $2p^n$ over $F$, where order of $q$ modulo $2p^n$ is $\frac{\phi(2p^n)}{2}$ were calculated by Batra and Arrora. Minimal cyclic codes of length $p^n q$, where $p$ and $q$ are distinct odd primes were computed by Sahni and Bakshi. Further, when order of $q$ modulo $p^n$ is $\phi(p^n)$, the minimal cyclic codes of length $8p^n$ were obtained by J. Singh and Arrora. Irreducible cyclic codes of length $4p^n$ and $8p^n$, where $q \equiv 3(mod\ 8)$ and $p/(q-1)$ were obtained in [2].

In this paper, we computed cyclic codes of length $16p^n$ over $F$ where $q$ is some prime or prime power of the form $16k+7$ and order of $q$ modulo $p^n$ is $\frac{\phi(p^n)}{2}$. For every odd prime $p$, there exist an integer $g$ such that $1 < g < 16p$ which is primitive root modulo $p$. For $p$ of the type $4k+1$, the order of $g$ modulo 4, modulo 8 is 2 and modulo 16 is 4 and for $p$ of the form $4k+3$ then order of g modulo 4, modulo 8 and modulo 16 is 2. For $q$ any prime or prime power with $(q,p) = 1$, there exist $g \notin \{1,q,q^2,...,q^{\frac{\phi(p)}{2}-1}\}$. For $S = \{1,2,...,16p^n\}$ and for all $a,b \in S$, the equivalence relation defined by $a \equiv bq^i(mod\ 16p^n)$ for some integer $i \geq 0$, partition the set $S$ into $32n+9$ equivalent classes namely $q-$ cyclotomic cosets modulo $16p^n$ given by $\Omega_{ap^n} = \{ap^n\}$ for $a \in \{0,8\}$, $\Omega_{bp^n} = \{bp^n, bp^n q\}$ for $b \in B = \{1,2,3,4,6,9,11\}$ and for $0 \leq i \leq n-1$, $\Omega_{tp^i} = \{tp^i, tp^i q, ..., tp^i q^{\frac{\phi(p^{n-i})}{2}-1}\}$, $\Omega_{tgp^i} = \{tgp^i, tgp^i q, tgp^i q^2, ..., tgp^i q^{\frac{\phi(p^{n-i})}{2}-1}\}$, such that $t \in A = \{1,2,4,8,16,\lambda,2\lambda,4\lambda,\mu,2\mu,\nu,2\nu,\eta,\xi,\rho,\chi\}$, $A' = B \bigcup \{0,8\}$. We are taking the case for $p$ is a prime of the form $8k+1$. However in the other cases whenever p is prime or prime power of the type $8k+3$, $8k+5$ and $8k+7$ the expression for primitive idempotents can be computed by using permutation on the set $A$. Idempotents corresponding to $\Omega_{tp^n}$ for $t \in B$ are computed in the section 2. In section 3, we compute expression for primitive idempotents corresponding to $\Omega_{tp^i}$ fot $t = 8,16,8g,16g$ and for $t = 2,4,2\lambda,4\lambda,2\mu,2\nu,2g,4g,2\lambda g,4\lambda g,2\mu g,2\nu g$ in section 4. And rest of all expressions are obtained in section 5. In section 6, we derived the generating polynomials and dimensions for the corresponding cyclic codes of length $16p^n$. Section 7 consist of minimum distance or the bounds for minimum distance of these codes. At the end, an example is obtained to illustrate the various parameters for these codes in section 8.

## 2. PRIMITIVE IDEMPOTENTS CORRESPONDING TO $\Omega_{tp^n}$, $t \in A'$

Throughout this paper, we consider $\alpha$ to be $16p^n$th root of unity in some extension field of $F$. Let $M_s$ be the minimal ideal in $R_{16p^n} = \frac{F[x]}{<x^{16p^n}-1>} \cong FC_{16p^n}$, generated by $\frac{(x^{16p^n}-1)}{m_s(x)}$, where $m_s(x)$ is the minimal polynomial for $\alpha^s$, $s \in \Omega_s$. We take $P_s(x)$, the primitive idempotent in $R_{16p^n}$, corresponding to the minimal ideal $M_s$, given by $P_s(x) = \frac{1}{16p^n} \sum_{t=0}^{16p^n-1} \varepsilon_i^s x^s$ where $\varepsilon_i^s = \sum_{s \in \Omega_s} \alpha^{-is}$ and $\overline{C}_s = \sum_{s \in \Omega_s} x^s$.

Then,

$$P_s(x) = \frac{1}{16p^n}[\sum_{a \in A'} \varepsilon^s_{ap^n}\overline{C}_{ap^n} + \sum_{i=0}^{n-1}\{\sum_{a \in \mathbb{A}} \varepsilon^s_{ap^i}\overline{C}_{ap^i} + \sum_{a \in \mathbb{A}} \varepsilon^s_{agp^i}\overline{C}_{agp^i}\}] \tag{1}$$

**Lemma 2.1.** *For cyclotomic cosets $\Omega_{ap^n}$, $a \in A'$, $\Omega_{ap^n} = -\Omega_{(8+a)p^n}$ if $a \in \{1,3\}$ and $\Omega_{ap^n} = -\Omega_{ap^n}$ if $a \in \{2,4,6,8\}$.*

**Proof.** *Since $\Omega_{ap^n} = \{ap^n\}$, so $\{(8+a)p^n q + ap^n\} = p^n\{(8+a)q + a\} = p^n\{(8+a)(16k+7)+a\} \equiv 0(mod 16p^n)$. Similarly other result holds.*

**Theorem 2.2.** *The explicit expression for the primitive idempotents $P_{ap^n}$, $a,w \in A'$ in $R_{16p^n}$ are given by*

$$P_{wp^n}(x) = \frac{1}{16p^n}[\{\sum_{a \in A'}(-1)^{wa}\alpha^{wap^{2n}}\overline{C}_{p^n}\} + \sum_{i=0}^{n-1}\{\sum_{a \in \mathbb{A}}(-1)^{wa}\alpha^{wap^{n+i}}\overline{C}_{ap^i} + \sum_{a \in \mathbb{A}}(-1)^{wa}\alpha^{wagp^{n+i}}\overline{C}_{agp^i}\}].$$

**Proof.** *By definition, $\varepsilon_i^s = \sum_{s \varepsilon \Omega_s} \alpha^{-is}$ and $\overline{C}_s = \sum_{s \varepsilon \Omega_s} x^s$ where $\alpha$ is $16p^n$th root of unity. For $s = 0$, since $\varepsilon_k^0 = 1$ for every $0 \le k \le 16p^n - 1$, therefore $P_0(x) = \frac{1}{16p^n}[\sum_{a \in A'} \overline{C}_{ap^n} + \sum_{i=0}^{n-1}\{\sum_{a \in \mathbb{A}} \overline{C}_{ap^i} + \sum_{a \in \mathbb{A}} \overline{C}_{agp^i}\}]$.*

*From lemma 2.1, $\Omega_{p^n} = -\Omega_{9p^n}$, therefore $\varepsilon_k^{p^n} = \sum_{s \varepsilon \Omega_{p^n}} \alpha^{-ks} = \alpha^{-p^n k} = \alpha^{9p^n k}$*

$\varepsilon_0^{p^n} = -\varepsilon_{8p^n}^{p^n} = 1$, $\varepsilon_{p^n}^{p^n} = -\varepsilon_{9p^n}^{p^n} = -\alpha^{p^{2n}}$, $\varepsilon_{2p^n}^{p^n} = \alpha^{2p^{2n}}$,

$\varepsilon_{3p^n}^{p^n} = -\varepsilon_{11p^n}^{p^n} = -\alpha^{3p^{2n}}$, $\varepsilon_{4p^n}^{p^n} = \alpha^{4p^{2n}}$, $\varepsilon_{6p^n}^{p^n} = \alpha^{6p^{2n}}$,

$\varepsilon_{p^i}^{p^n} = -\varepsilon_{\eta p^i}^{p^n} = -\alpha^{p^{n+i}}$, $\varepsilon_{2p^i}^{p^n} = -\varepsilon_{2\mu p^i}^{p^n} = \alpha^2 p^{n+i}$, $\varepsilon_{4p^i}^{p^n} = -\varepsilon_{4\lambda p^i}^{p^n} = \alpha^4 p^{n+i}$,

$\varepsilon^{p^n}_{8p^i} = -\varepsilon^{p^n}_{16p^i} = -1$, $\varepsilon^{p^n}_{\lambda p^i} = -\varepsilon^{p^n}_{\xi p^i} = -\alpha^{\lambda p^{n+i}}$, $\varepsilon^{p^n}_{2\lambda p^i} = -\varepsilon^{p^n}_{2\nu p^i} = \alpha^{2\lambda p^{n+i}}$,
$\varepsilon^{p^n}_{\mu p^i} = -\varepsilon^{p^n}_{\rho p^i} = -\alpha^{\mu p^{n+i}}$, $\varepsilon^{p^n}_{\nu p^i} = -\varepsilon^{p^n}_{\chi p^i} = -\alpha^{\nu p^{n+i}}$, $\varepsilon^{p^n}_{gp^i} = -\varepsilon^{p^n}_{\eta gp^i} = -\alpha^{gp^{n+i}}$,
$\varepsilon^{p^n}_{2gp^i} = -\varepsilon^{p^n}_{2\mu gp^i} = \alpha^{2gp^{n+i}}$, $\varepsilon^{p^n}_{4gp^i} = -\varepsilon^{p^n}_{4\lambda gp^i} = \alpha^{4gp^{n+i}}$, $\varepsilon^{p^n}_{8gp^i} = -\varepsilon^{p^n}_{16gp^i} = -1$,
$\varepsilon^{p^n}_{\lambda gp^i} = -\varepsilon^{p^n}_{\xi gp^i} = -\alpha^{\lambda gp^{n+i}}$, $\varepsilon^{p^n}_{2\lambda gp^i} = -\varepsilon^{p^n}_{2\nu gp^i} = \alpha^{2\lambda gp^{n+i}}$, $\varepsilon^{p^n}_{\mu gp^i} = -\varepsilon^{p^n}_{\rho gp^i} = -\alpha^{\mu gp^{n+i}}$,
$\varepsilon^{p^n}_{\nu gp^i} = -\varepsilon^{p^n}_{\chi gp^i} = -\alpha^{\nu gp^{n+i}}$, .

*Using all these in* $(2.1)$ *the expression for* $P_{p^n}(x)$ *can be computed.*

*Similarly, by lemma* 2.1*,* $P_{wp^n}(x)$*,* $w \in A' - \{0,1\}$ *can be computed.*

## 3. Primitive Idempotents Corresponding to $P_{tp^i}$, $t = 8, 16, 8g, 16g$

We define $H_j = p^j \sum_{s\varepsilon\Omega_{8gp^j}} \alpha^s$ ; $I_j = p^j \sum_{s\varepsilon\Omega_{16gp^j}} \alpha^s$ ; $Q_j = p^j \sum_{s\varepsilon\Omega_{8p^j}} \alpha^s$ ;$R_j = p^j \sum_{s\varepsilon\Omega_{16p^j}} \alpha^s$, for $0 \le j \le n-1$. Since $Q_j^q = (p^j \sum_{s\varepsilon\Omega_{8p^j}} \alpha^s)^q = (p^j)^q(\sum_{s\varepsilon\Omega_{8p^j}} \alpha^s)^q = (p^j)^q \sum_{s\varepsilon\Omega_{8p^j}} \alpha^{qs} = (p^j)^q \sum_{s\varepsilon q\Omega_{8p^j}} \alpha^s$. However, $(p^j)^q = p^j$, so $Q_j^q = p^j \sum_{s\varepsilon q\Omega_{8p^j}} \alpha^s$. Moreover, $\Omega_{8p^j}$ is a cyclotomic coset, so $q\Omega_{8p^j} = \Omega_{8p^j}$. Hence $Q_j^q = p^j \sum_{s\varepsilon\Omega_{8p^j}} \alpha^s = Q_j$ and $Q_j \in GF(q)$. Similarly $H_j, I_j, R_j \in GF(q)$.

**Lemma 3.1.** $I_j + R_j = \begin{cases} -p^{n-1} \text{ if } j = n-1 \\ 0 \text{ otherwise.} \end{cases}$ *for* $0 \le j \le n-1$.

**Proof.** *By definition* $I_j + R_j = p^j \sum_{t=0}^{\frac{\phi(p^{n-j})}{2}-1} (\delta^{gq^t} + \delta^{q^t})$ *where* $\delta = \alpha^{16p^j}$.
*The set* $\{1, q, q^2, ..., q^{\frac{\phi(p^{n-j})}{2}-1}, g, gq, gq^2, ..., gq^{\frac{\phi(p^{n-j})}{2}-1}\}$ *is a reduced residue system mod*$(p^{n-j})$ *so* $I_j + R_j = p^j[\sum_{t=0}^{p^{n-j}} \delta^t - \sum_{t=1,p/t}^{p^{n-j}} \delta^t] = p^j[\sum_{t=0}^{p^{n-j}} \delta^t - \sum_{t=1}^{p^{n-j-1}} \delta^{pt}] = \begin{cases} -p^{n-1} \text{ if } j = n-1 \\ 0 \text{ otherwise.} \end{cases}$

**Lemma 3.2.** *For* $0 \le j \le n-1$, $H_j + Q_j = \begin{cases} p^{n-1} \text{ if } j = n-1 \\ 0 \text{ otherwise.} \end{cases}$

**Proof.** *This result can be computed on similar lines as that of lemma* 3.1 *and using that* $\{1, q, q^2, ..., q^{\frac{\phi(p^{n-j})}{2}-1}, g, gq, gq^2, ..., gq^{\frac{\phi(p^{n-j})}{2}-1}\}$ *is a reduced residue system mod*$(2p^{n-j})$.

**Lemma 3.3.** *For cyclotomic cosets* $\Omega_{p^i}$*,* $0 \le i \le n-1$

*(i)* $\eta^2\Omega_{p^i} = \eta\Omega_{\eta p^i} = \Omega_{p^i} = \nu^2\Omega_{p^i} = \chi^2\Omega_{p^i}$

$(ii)$ $\mu^2\Omega_{p^i} = \Omega_{\eta p^i} = \eta\Omega_{p^i} = \rho^2\Omega_{p^i} = \lambda^2\Omega_{p^i} = \xi^2\Omega_{p^i}$

$(iii)$ $(2\lambda)^2\Omega_{p^i} = 2\Omega_{2p^i} = 4\Omega_{p^i} = \Omega_{4p^i} = (2\mu)^2\Omega_{p^i} = (2\nu)^2\Omega_{p^i}$

$(iv)$ $(4\lambda)^2\Omega_{p^i} = \Omega_{16p^i} = 2\Omega_{8p^i} = 4\Omega_{4p^i} = 16\Omega_{p^i}$.

**Proof.** $(i)$ *Since* $\eta = 1+8p^n$. *Therefore* $\eta^2 = (1+8p^n)^2 = 1+64p^{2n}+16p^n \equiv 1(mod 16p^n)$. *This implies* $\eta^2 \equiv 1(mod 16p^n) \Rightarrow \eta^2\Omega_p i = \Omega p^i$. *Now* $\eta^2\Omega_p i = \{\eta^2 p^i, \eta^2 p^i q, \eta^2 p^i q^2, ..., \eta^2 p^i q^{\frac{\phi(p^{n-i})}{2}-1}\} = \eta\{\eta p^i, \eta p^i q, \eta p^i q^2, ..., \eta p^i q^{\frac{\phi(p^{n-i})}{2}-1}\}$ $= \eta\Omega_{\eta p^i}$. *Now* $\nu^2 = (1+6p^n)^2 = 1+36p^{2n}+12p^n \equiv 1(mod 16p^n)$ *and* $\chi^2 = (1+14p^n)^2 = 1+196p^{2n}+28p^n = 1(mod 16p^n)$. *Hence* $\nu^2\Omega_{p^i} = \chi^2\Omega_{p^i} = \Omega_{p^i}$. *Similarly other result holds.*

**Lemma 3.4.** $(i)$ $\Omega_{(1+tp^n)p^i} = -\Omega_{\{1+(14-t)p^n\}p^i}$ *and hence* $\Omega_{(1+tp^n)gp^i} = -\Omega_{\{1+(14-t)p^n\}gp^i}$, *for* $t = 0,2,4,6$

$(ii)$ $\Omega_{2(1+tp^n)p^i} = -\Omega_{2\{1+(6-t)p^n\}p^i}$ *and hence* $\Omega_{2(1+tp^n)gp^i} = -\Omega_{2\{1+(6-t)p^n\}gp^i}$, *for* $t = 0,2$

$(iii)$ $\Omega_{4p^i} = -\Omega_{4\lambda p^i}$ *and hence* $\Omega_{4gp^i} = -\Omega_{4\lambda gp^i}$,

$(iv)$ $\Omega_{tp^i} = -\Omega_{tp^i}$ *and hence* $\Omega_{tgp^i} = -\Omega_{tgp^i}$, *for* $t = 8,16$. *Where* $1 \le i \le n-1$.

**Proof.** *Since* $\chi = 1+14p^n \equiv -1(mod 16)$ *and* $q^{\frac{\phi(p^n)}{2}} \equiv 1(mod 16)$. *Further,* $q^{\frac{\phi(p^n)}{4}} \equiv 1(mod 16)$, *as* $q \equiv 1(mod 16)$, *therefore* $\chi q^{\frac{\phi(p^n)}{4}} \equiv -1(mod 16)$. *Also* $q^{\frac{\phi(p^n)}{2}} \equiv 1(mod p^n)$ *so* $q^{\frac{\phi(p^n)}{4}} \equiv -1(mod p^n)$ *and* $\chi \equiv 1(mod p^n)$, *thus* $\chi q^{\frac{\phi(p^n)}{4}} \equiv -1(mod p^n)$. *However* $(16, p^n) = 1$ *thus* $\chi q^{\frac{\phi(p^n)}{4}} \equiv -1(mod 16p^n)$ *and so* $-\Omega_{\chi p^i} = \Omega_{p^i}$. *Hence* $-\Omega_{\chi gp^i} = \Omega_{gp^i}$.

Proof of remaining parts can be computed using relations of congruences and similar reasons as for the relation obtained.

**Lemma 3.5.** *For* $0 \le i \le n$; $0 \le j \le n-1$,

$$\sum_{s\varepsilon\Omega_{p^j}} \alpha^{8gp^i s} = \sum_{s\varepsilon\Omega_{2p^j}} \alpha^{4gp^i s} = \sum_{s\varepsilon\Omega_{2p^j}} \alpha^{4\lambda gp^i s} = \sum_{s\varepsilon\Omega_{4p^j}} \alpha^{2\lambda gp^i s} = \sum_{s\varepsilon\Omega_{4p^j}} \alpha^{2\mu gp^i s} =$$
$$\sum_{s\varepsilon\Omega_{4p^j}} \alpha^{2\nu gp^i s} = \sum_{s\varepsilon\Omega_{8p^j}} \alpha^{\lambda gp^i s} = \sum_{s\varepsilon\Omega_{8p^j}} \alpha^{\mu gp^i s} = \sum_{s\varepsilon\Omega_{8p^j}} \alpha^{\nu gp^i s} = \sum_{s\varepsilon\Omega_{8p^j}} \alpha^{\eta gp^i s} = \sum_{s\varepsilon\Omega_{8p^j}} \alpha^{\xi gp^i s} =$$
$$\sum_{s\varepsilon\Omega_{8p^j}} \alpha^{\rho gp^i s} = \sum_{s\varepsilon\Omega_{8p^j}} \alpha^{\chi gp^i s} = \sum_{s\varepsilon\Omega_{2\lambda p^j}} \alpha^{4\lambda gp^i s} = \sum_{s\varepsilon\Omega_{4\lambda p^j}} \alpha^{2\mu gp^i s} = \sum_{s\varepsilon\Omega_{4\lambda p^j}} \alpha^{2\nu gp^i s} =$$

$$\begin{cases} -\frac{\phi(p^{n-j})}{2},\ if\ i+j \geq n, \\ \frac{1}{p^j} H_{i+j},\ if\ i+j \leq n-1,\ g \neq 1, \\ \frac{1}{p^j} Q_{i+j},\ if\ i+j \leq n-1,\ g = 1. \end{cases}$$

**Proof.** *As $\alpha$ is $16p^n$th root of unity in some extension field of $GF(q)$, so*

$$\sum_{s\varepsilon\Omega_{4\lambda p^j}} \alpha^{2\nu g p^i s} = \sum_{t=0}^{\frac{\phi(p^{n-j})}{2}-1} \alpha^{8(1+2p^n)(1+6p^n)gp^{i+j}q^t} = \sum_{t=0}^{\frac{\phi(p^{n-j})}{2}-1} \alpha^{8gp^{i+j}q^t} = \sum_{s\varepsilon\Omega_{p^j}} \alpha^{8gp^i s}.$$

*If* $\beta = \alpha^{8gp^{i+j}}$ *then* $\sum_{s\varepsilon\Omega_{p^j}} \alpha^{8gp^i s} = \sum_{t=0}^{\frac{\phi(p^{n-j})}{2}-1} (\alpha^{8gp^{i+j}q^t}) = \sum_{t=0}^{\frac{\phi(p^{n-j})}{2}-1} (\beta^{q^t})$.

*For* $i+j \geq n$, $\beta$ *is* $16$*th root of unity, and therefore* $\sum_{s\varepsilon\Omega_{p^j}} \alpha^{8gp^i s} = \sum_{t=0}^{\frac{\phi(p^{n-j})}{2}-1} \alpha^{8gp^{i+j}q^t} = -\frac{\phi(p^{n-j})}{2}$.

*If* $i+j \leq n-1$, $\beta$ *is* $16p^{n-i-j}$*th root of unity. Then* $\beta^{q^l} = \beta^{q^r}$ *which is possible when* $l \equiv r (mod \frac{\phi(p^{n-i-j})}{2})$. *So* $\sum_{s\varepsilon\Omega_{p^j}} \alpha^{8gp^i s} = \frac{\phi(p^{n-j})}{\phi(p^{n-i-j})} \sum_{t=0}^{\frac{\phi(p^{n-i-j})}{2}-1} \beta^{q^t} = \frac{p^{i+j}}{p^j} \sum_{s\varepsilon\Omega_{8gp^{i+j}}} \alpha^s = \frac{1}{p^j} H_{i+j}$.

*Similar result hold for* $Q_{i+j}$.

Proof of lemma 3.6, similarly can be obtained using definition of $I_j$ and $R_j$.

**Lemma 3.6.** *For* $0 \leq i \leq n$; $0 \leq j \leq n-1$

$$\sum_{s\varepsilon\Omega_{p^j}} \alpha^{16gp^i s} = \sum_{s\varepsilon\Omega_{2p^j}} \alpha^{8gp^i s} = \sum_{s\varepsilon\Omega_{2p^j}} \alpha^{16gp^i s} = \sum_{s\varepsilon\Omega_{4p^j}} \alpha^{4gp^i s} = \sum_{s\varepsilon\Omega_{4p^j}} \alpha^{8gp^i s} =$$
$$\sum_{s\varepsilon\Omega_{4p^j}} \alpha^{16gp^i s} = \sum_{s\varepsilon\Omega_{4p^j}} \alpha^{4\lambda gp^i s} = \sum_{s\varepsilon\Omega_{8p^j}} \alpha^{16gp^i s} = \sum_{s\varepsilon\Omega_{8p^j}} \alpha^{2\lambda gp^i s} = \sum_{s\varepsilon\Omega_{8p^j}} \alpha^{4\lambda gp^i s} =$$
$$\sum_{s\varepsilon\Omega_{8p^j}} \alpha^{2\mu gp^i s} = \sum_{s\varepsilon\Omega_{8p^j}} \alpha^{2\nu gp^i s} = \sum_{s\varepsilon\Omega_{16p^j}} \alpha^{16gp^i s} = \sum_{s\varepsilon\Omega_{16p^j}} \alpha^{\lambda gp^i s} = \sum_{s\varepsilon\Omega_{16p^j}} \alpha^{2\lambda gp^i s} =$$
$$\sum_{s\varepsilon\Omega_{16p^j}} \alpha^{4\lambda gp^i s} = \sum_{s\varepsilon\Omega_{16p^j}} \alpha^{\mu gp^i s} = \sum_{s\varepsilon\Omega_{16p^j}} \alpha^{2\mu gp^i s} = \sum_{s\varepsilon\Omega_{16p^j}} \alpha^{\nu gp^i s} = \sum_{s\varepsilon\Omega_{16p^j}} \alpha^{2\nu gp^i s} =$$
$$\sum_{s\varepsilon\Omega_{16p^j}} \alpha^{\eta gp^i s} = \sum_{s\varepsilon\Omega_{16p^j}} \alpha^{\xi gp^i s} = \sum_{s\varepsilon\Omega_{16p^j}} \alpha^{\rho gp^i s} = \sum_{s\varepsilon\Omega_{16p^j}} \alpha^{\chi gp^i s} = \sum_{s\varepsilon\Omega_{4\lambda p^j}} \alpha^{4\lambda gp^i s} =$$

$$\begin{cases} \frac{\phi(p^{n-j})}{2}, if\ i+j \geq n, \\ \frac{1}{p^j} I_{i+j},\ if\ i+j \leq n-1,\ g \neq 1, \\ \frac{1}{p^j} R_{i+j},\ if\ i+j \leq n-1,\ g = 1. \end{cases}$$

**Theorem 3.7.** *When $p \equiv 1(mod 8)$, the expressions for primitive idempotents corresponding to $P_{8p^i}$, $P_{8gp^i}$, $P_{16p^i}$ and $P_{16gp^i}$ in $R_{16p^n}$ are given by*

$$P_{8p^j}(x) = \frac{1}{16p^n}\Big[\frac{\phi(p^{n-j})}{2}\{\sum_{t\in A'}(-1)^t\overline{C}_{tp^n}\} + \frac{\phi(p^{n-j})}{2}\sum_{i=n-j}^{n-1}\{\sum_{a\in\mathbb{A}}(-1)^a\{\overline{C}_{ap^i} + \overline{C}_{agp^i}\}\} + \frac{1}{p^j}\sum_{i=0}^{n-j-1}\{Q_{i+j}(\overline{C}_{p^i}+\overline{C}_{\lambda p^i}+\overline{C}_{\mu p^i}+\overline{C}_{\nu p^i}+\overline{C}_{\eta p^i}+\overline{C}_{\xi p^i}+\overline{C}_{\rho p^i}+\overline{C}_{\chi p^i}) + R_{i+j}(\overline{C}_{2p^i}+\overline{C}_{4p^i}+\overline{C}_{8p^i}+\overline{C}_{16p^i}+\overline{C}_{2\lambda p^i}+\overline{C}_{4\lambda p^i}+\overline{C}_{2\mu p^i}+\overline{C}_{2\nu p^i}) + H_{i+j}(\overline{C}_{gp^i}+\overline{C}_{\lambda gp^i}+\overline{C}_{\mu gp^i}+\overline{C}_{\nu gp^i}+\overline{C}_{\eta gp^i}+\overline{C}_{\xi gp^i}+\overline{C}_{\rho gp^i}+\overline{C}_{\chi gp^i}) + I_{i+j}(\overline{C}_{2gp^i}+\overline{C}_{4gp^i}+\overline{C}_{8gp^i}+\overline{C}_{16gp^i}+\overline{C}_{2\lambda gp^i}+\overline{C}_{4\lambda gp^i}+\overline{C}_{2\mu gp^i}+\overline{C}_{2\nu gp^i})\}\Big]$$

$$P_{16p^j}(x) = \frac{1}{16p^n}\Big[\frac{\phi(p^{n-j})}{2}\{\sum_{t\in A'}\overline{C}_{tp^n}\} + \frac{\phi(p^{n-j})}{2}\sum_{i=n-j}^{n-1}\{\sum_{a\in\mathbb{A}}\{\overline{C}_{ap^i} + \overline{C}_{agp^i}\}\} + \frac{1}{p^j}\sum_{i=0}^{n-j-1}\{\sum_{a\in\mathbb{A}}\{R_{i+j}\overline{C}_{ap^i}+I_{i+j}\overline{C}_{agp^i}\}\}\Big]$$

*where* $Q_{n-1} = \frac{1}{2}[p^{n-1} + \frac{\sqrt{p^{n-1}(16p^n+7p-7)}}{4}]$, $H_{n-1} = \frac{1}{2}[p^{n-1} - \frac{\sqrt{p^{n-1}(16p^n+7p-7)}}{4}]$, $I_{n-1} = \frac{1}{2}[-p^{n-1} + \frac{\sqrt{p^{n-1}(16p^n+7p-7)}}{4}]$ *and* $R_{n-1} = -\frac{1}{2}[p^{n-1} + \frac{\sqrt{p^{n-1}(16p^n+7p-7)}}{4}]$ *and for every* $j \le n-2$ $Q_j = H_j = I_j = R_j = 0$.

**Proof.** *Since* $\Omega_{8p^j} = -\Omega_{8p^j}$, *as obtained in lemma* 3.4, *so* $\varepsilon_k^{8p^j} = \sum_{s\varepsilon\Omega_{p^j}}\alpha^{-ks} = \sum_{s\varepsilon\Omega_{p^j}}\alpha^{8ks}$.

*Thus* $\varepsilon_{tp^n}^{8p^j} = (-1)^t\frac{\phi(p^{n-j})}{2}$ *for* $t \in A'$ *and using Lemma* 3.3, $3.5-3.6$, *wherever is required we obtain*

$$\varepsilon_{p^i}^{8p^j} = \varepsilon_{\lambda p^i}^{8p^j} = \varepsilon_{\mu p^i}^{8p^j} = \varepsilon_{\nu p^i}^{8p^j} = \varepsilon_{\eta p^i}^{8p^j} = \varepsilon_{\xi p^i}^{8p^j} = \varepsilon_{\rho p^i}^{8p^j} = \varepsilon_{\chi p^i}^{8p^j} = \begin{cases} -\frac{\phi(p^{n-j})}{2} \text{ if } i+j \ge n \\ \frac{1}{p^j}Q_{i+j} \text{ if } i+j \le n-1 \end{cases}$$

$$\varepsilon_{2p^i}^{8p^j} = \varepsilon_{4p^i}^{8p^j} = \varepsilon_{8p^i}^{8p^j} = \varepsilon_{16p^i}^{8p^j} = \varepsilon_{2\lambda p^i}^{8p^j} = \varepsilon_{4\lambda p^i}^{8p^j} = \varepsilon_{2\mu p^i}^{8p^j} = \varepsilon_{2\nu p^i}^{8p^j} = \begin{cases} \frac{\phi(p^{n-j})}{2} \text{ if } i+j \ge n \\ \frac{1}{p^j}R_{i+j} \text{ if } i+j \le n-1 \end{cases}$$

$$\varepsilon_{gp^i}^{8p^j} = \varepsilon_{\lambda gp^i}^{8p^j} = \varepsilon_{\mu gp^i}^{8p^j} = \varepsilon_{\nu gp^i}^{8p^j} = \varepsilon_{\eta gp^i}^{8p^j} = \varepsilon_{\xi gp^i}^{8p^j} = \varepsilon_{\rho gp^i}^{8p^j} = \varepsilon_{\chi gp^i}^{8p^j} = \begin{cases} -\frac{\phi(p^{n-j})}{2} \text{ if } i+j \ge n \\ \frac{1}{p^j}H_{i+j} \text{ if } i+j \le n-1 \end{cases}$$

$$\varepsilon_{2gp^i}^{8p^j} = \varepsilon_{4gp^i}^{8p^j} = \varepsilon_{8gp^i}^{8p^j} = \varepsilon_{16gp^i}^{8p^j} = \varepsilon_{2\lambda gp^i}^{8p^j} = \varepsilon_{4\lambda gp^i}^{8p^j} = \varepsilon_{2\mu gp^i}^{8p^j} = \varepsilon_{2\nu gp^i}^{8p^j} = \begin{cases} \frac{\phi(p^{n-j})}{2} \text{ if } i+j \ge n \\ \frac{1}{p^j}I_{i+j} \text{ if } i+j \le n-1 \end{cases}$$

*Using all these in* (2.1), *expression for* $P_{8p^j}$ *is obtained.*

*Using Lemma* $3.3-3.6$, *the expression for* $P_{16p^j}$, $P_{8gp^j}$ *and* $P_{16gp^j}$ *can be computed. However,* *the expression for $P_{8gp^j}(x)$, $P_{16gp^j}(x)$ can be written by interchanging $Q$ and $R$ by $H$ and $I$ in the expressions of $P_{8p^j}$, $P_{16p^j}$.*

*Since $\overline{C}_k = \sum_{s\varepsilon\Omega_k} x^s$ and $(\overline{C}_k)_{\alpha^{16p^j}} = \overline{C}_k(\alpha^{16p^j}) = \sum_{s\varepsilon\Omega_k} (\alpha^{16p^j})^s$. Therefore $\overline{C}_{tp^n}(\alpha^{16p^j}) = 1$ for $t \in A'$.*

$$\overline{C}_{ap^i}(\alpha^{16p^j}) = \begin{cases} \frac{\phi(p^{n-j})}{2}, if\ i+j \geq n \\ \frac{1}{p^j}R_{i+j}, if\ i+j \leq n-1. \end{cases} \quad and\ \overline{C}_{agp^i}(\alpha^{16p^j}) = \begin{cases} \frac{\phi(p^{n-j})}{2}, if\ i+j \geq n \\ \frac{1}{p^j}I_{i+j}, if\ i+j \leq n-1. \end{cases}$$

*Using all these in $P_{16p^j}(\alpha^{16p^j}) = 1$, to obtain*

$$16p^n = \frac{\phi(p^{n-j})}{2}[9 + \sum_{i=n-j}^{n-1} \frac{\phi(p^{n-j})}{2} 32] + \frac{1}{p^j} \sum_{i=0}^{n-j-1} \frac{1}{p^i}(16R^2{}_{i+j} + 16I^2{}_{i+j})$$

*which in turn implies* $\frac{1}{p^j} \sum_{i=0}^{n-j-1} \frac{1}{p^i}(R^2{}_{i+j} + I^2{}_{i+j}) = \frac{p^{n-1}}{2}(p+1) + \frac{7p^{n-j-1}}{32}(p-1)$.

*In particular for $j = n-1$, $\frac{1}{p^{n-1}}(R^2{}_{n-1} + I^2{}_{n-1}) = \frac{p^{n-1}}{2}(p+1) + \frac{7p^{n-1}}{32}(p-1)$.*

*Using lemma 3.1 to obtain $I_{n-1} = \frac{1}{2}[-p^{n-1} + \frac{\sqrt{p^{n-1}(16p^n+7p-7)}}{4}]$ and $R_{n-1} = -\frac{1}{2}[p^{n-1} + \frac{\sqrt{p^{n-1}(16p^n+7p-7)}}{4}]$ and so $I_{n-2} = R_{n-2} = I_{n-3} = R_{n-3} = ... = 0$.*

*Relations for $Q_{i+j}$ and $H_{i+j}$ can be derived by using lemma 3.2 and the fact that $P_{8p^j}(\alpha^{8p^j}) = 1$.*

## 4. Primitive Idempotents Corresponding to $P_{tp^i}$, $t = 2, 4, 2\lambda, 4\lambda, 2\mu, 2\nu, 2g, 4g, 2\lambda g, 4\lambda g, 2\mu g, 2\nu g$

For $0 \leq j \leq n-1$, we define

$C_j = p^j \sum_{s\varepsilon\Omega_{2\lambda gp^j}} \alpha^s$, $F_j = p^j \sum_{s\varepsilon\Omega_{2gp^j}} \alpha^s$, $G_j = p^j \sum_{s\varepsilon\Omega_{4gp^j}} \alpha^s$, $K_j = p^j \sum_{s\varepsilon\Omega_{2\lambda p^j}} \alpha^s$, $O_j = p^j \sum_{s\varepsilon\Omega_{2p^j}} \alpha^s$, $P_j = p^j \sum_{s\varepsilon\Omega_{4p^j}} \alpha^s$. Due to similar procedure as in section 3, we obtained $C_j$, $F_j$, $G_j$, $K_j$, $O_j$, $P_j$ $\in GF(q)$.

**Lemma 4.1.** *For $0 \leq j \leq n-1$, $K_j - \beta^{p^j} O_j = 0$; $C_j - \beta^{gp^j} F_j = 0$ where $\beta = \alpha^{4p^n}$.*

**Proof.** *Since $\alpha^{2\lambda p^j} = \alpha^{2(1+2p^n)p^j} = \alpha^{2p^j}\beta^{p^j}$. So $K_j = p^j \sum_{s\varepsilon\Omega_{\lambda p^i}} \alpha^{2s} = p^j[\alpha^{2\lambda p^i} + \alpha^{2q\lambda p^i} + \alpha^{2q^2\lambda p^i} + ... + \alpha^{2q^{\frac{\phi(p^{n-j})}{2}-1}\lambda p^i}] = p^j[\alpha^{2p^i}\beta^{p^i} + \alpha^{2qp^i}\beta^{p^i} + \alpha^{2q^2p^i}\beta^{p^i} + ... + \alpha^{2p^iq^{\frac{\phi(p^{n-j})}{2}-1}}\beta^{p^i}] = \beta^{p^i}[p^j \sum_{s\varepsilon\Omega_{2p^j}} \alpha^s] = \beta^{p^i} O_j$.*

*Remaining can be obtained in similar lines.*

*Proof of lemma* $4.2-4.4$ *can be computed on similar lines as that of Lemma* $3.5$ *and represent* $C_{i+j},\ F_{i+j},\ G_{i+j},\ K_{i+j},\ O_{i+j},\ P_{i+j}$.

**Lemma 4.2.** *For* $0 \le i \le n;\ 0 \le j \le n-1$

$$\sum_{s\varepsilon\Omega_{2p^j}} \alpha^{\lambda gp^i s} = \sum_{s\varepsilon\Omega_{2p^j}} \alpha^{\xi gp^i s} = \sum_{s\varepsilon\Omega_{2\lambda p^j}} \alpha^{\eta gp^i s} = \sum_{s\varepsilon\Omega_{2\mu p^j}} \alpha^{\nu gp^i s} = \sum_{s\varepsilon\Omega_{2\mu p^j}} \alpha^{\chi gp^i s} = \sum_{s\varepsilon\Omega_{2\nu p^j}} \alpha^{\rho gp^i s}$$
$$= -\sum_{s\varepsilon\Omega_{2p^j}} \alpha^{\nu gp^i s} = -\sum_{s\varepsilon\Omega_{2p^j}} \alpha^{\chi gp^i s} = -\sum_{s\varepsilon\Omega_{2\lambda p^j}} \alpha^{\mu gp^i s} = -\sum_{s\varepsilon\Omega_{2\lambda p^j}} \alpha^{\rho gp^i s} = -\sum_{s\varepsilon\Omega_{2\mu p^j}} \alpha^{\xi gp^i s}$$
$$= -\sum_{s\varepsilon\Omega_{2\nu p^j}} \alpha^{\eta gp^i s} = \begin{cases} \frac{\phi(p^{n-j})}{4}\{\alpha^{6gp^{i+j}} + \alpha^{10gp^{i+j}}\},\ if\ i+j \ge n,\ g \ne 1, \\ \frac{1}{p^j} C_{i+j},\ if\ i+j \le n-1,\ g \ne 1, \\ \frac{\phi(p^{n-j})}{4}\{\alpha^{6p^{i+j}} + \alpha^{10p^{i+j}}\},\ if\ i+j \ge n,\ g = 1, \\ \frac{1}{p^j} K_{i+j},\ if\ i+j \le n-1,\ g = 1. \end{cases}$$

**Lemma 4.3.** *For* $0 \le i \le n;\ 0 \le j \le n-1$

$$\sum_{s\varepsilon\Omega_{p^j}} \alpha^{2gp^i s} = \sum_{s\varepsilon\Omega_{2\lambda p^j}} \alpha^{\lambda gp^i s} = \sum_{s\varepsilon\Omega_{2\lambda p^j}} \alpha^{\xi gp^i s} = \sum_{s\varepsilon\Omega_{2\mu p^j}} \alpha^{\rho gp^i s} = \sum_{s\varepsilon\Omega_{2\nu p^j}} \alpha^{\chi gp^i s} =$$
$$-\sum_{s\varepsilon\Omega_{2\lambda p^j}} \alpha^{\nu gp^i s} = -\sum_{s\varepsilon\Omega_{2\lambda p^j}} \alpha^{\chi gp^i s} = -\sum_{s\varepsilon\Omega_{2\mu p^j}} \alpha^{\eta gp^i s} = -\sum_{s\varepsilon\Omega_{2\nu p^j}} \alpha^{\xi gp^i s} =$$
$$\begin{cases} \frac{\phi(p^{n-j})}{4}\{\alpha^{2gp^{i+j}} + \alpha^{14gp^{i+j}}\},\ if\ i+j \ge n,\ g \ne 1, \\ \frac{1}{p^j} F_{i+j},\ if\ i+j \le n-1, g \ne 1, \\ \frac{\phi(p^{n-j})}{4}\{\alpha^{2p^{i+j}} + \alpha^{14p^{i+j}}\},\ if\ i+j \ge n,\ g = 1, \\ \frac{1}{p^j} O_{i+j},\ if\ i+j \le n-1, g = 1. \end{cases}$$

**Lemma 4.4.** *For* $0 \le i \le n;\ 0 \le j \le n-1$

$$\sum_{s\varepsilon\Omega_{p^j}} \alpha^{4gp^i s} = \sum_{s\varepsilon\Omega_{4\lambda p^j}} \alpha^{\lambda gp^i s} = \sum_{s\varepsilon\Omega_{4\lambda p^j}} \alpha^{\nu gp^i s} = \sum_{s\varepsilon\Omega_{4\lambda p^j}} \alpha^{\xi gp^i s} = \sum_{s\varepsilon\Omega_{4\lambda p^j}} \alpha^{\chi gp^i s} = \sum_{s\varepsilon\Omega_{2\lambda p^j}} \alpha^{2\nu gp^i s}$$
$$= \sum_{s\varepsilon\Omega_{2\mu p^j}} \alpha^{2\mu gp^i s} = \sum_{s\varepsilon\Omega_{2\nu p^j}} \alpha^{2\nu gp^i s} = -\sum_{s\varepsilon\Omega_{4\lambda p^j}} \alpha^{\mu gp^i s} = -\sum_{s\varepsilon\Omega_{4\lambda p^j}} \alpha^{\eta gp^i s} = -\sum_{s\varepsilon\Omega_{4\lambda p^j}} \alpha^{\rho gp^i s}$$
$$= -\sum_{s\varepsilon\Omega_{2\lambda p^j}} \alpha^{2\mu gp^i s} = \begin{cases} \frac{\phi(p^{n-j})}{4}\{\alpha^{4gp^{i+j}} + \alpha^{12gp^{i+j}}\},\ if\ i+j \ge n,\ g \ne 1, \\ \frac{1}{p^j} G_{i+j},\ if\ i+j \le n-1,\ g \ne 1, \\ \frac{\phi(p^{n-j})}{4}\{\alpha^{4p^{i+j}} + \alpha^{12p^{i+j}}\},\ if\ i+j \ge n,\ g = 1, \\ \frac{1}{p^j} P_{i+j},\ if\ i+j \le n-1,\ g = 1. \end{cases}$$

**Theorem 4.5.** *When $p\equiv 1(mod 8)$, the expressions for the primitive idempotents corresponding to $P_{2p^i}$, $P_{4p^i}$, $P_{2\lambda p^i}$, $P_{4\lambda p^i}$, $P_{2\mu p^i}$, $P_{2\nu p^i}$, $P_{2gp^i}$, $P_{4gp^i}$, $P_{2\lambda gp^i}$, $P_{4\lambda gp^i}$, $P_{2\mu gp^i}$ and $P_{2\nu gp^i}$ are given by*

$$P_{2p^j}(x) = \tfrac{1}{16p^n}[\tfrac{\phi(p^{n-j})}{2}\{\sum_{t\in A'}(-1)^t\alpha^{2\lambda tp^{n+j}}\overline{C}_{tp^n}\} + \frac{\phi(p^{n-j})}{2}\sum_{i=n-j}^{n-1}\{\sum_{a\in\mathbb{A}}(-1)^a\{\alpha^{2\lambda ap^{i+j}}\overline{C}_{ap^i} + \alpha^{2\lambda agp^{i+j}}\overline{C}_{agp^i}\}\} + \frac{1}{p^j}\sum_{i=0}^{n-j-1}\{-K_{i+j}\overline{C}_{p^i} - P_{i+j}\overline{C}_{2p^i} + Q_{i+j}\overline{C}_{4p^i} + R_{i+j}\overline{C}_{8p^i} + R_{i+j}\overline{C}_{16p^i} - O_{i+j}\overline{C}_{\lambda p^i} + P_{i+j}\overline{C}_{2\lambda p^i} + Q_{i+j}\overline{C}_{4\lambda p^i} + K_{i+j}\overline{C}_{\mu p^i} - P_{i+j}\overline{C}_{2\mu p^i} + O_{i+j}\overline{C}_{\nu p^i} + P_{i+j}\overline{C}_{2\nu p^i} - K_{i+j}\overline{C}_{\eta p^i} - O_{i+j}\overline{C}_{\xi p^i} + K_{i+j}\overline{C}_{\rho p^i} + O_{i+j}\overline{C}_{\chi p^i} - C_{i+j}\overline{C}_{gp^i} - G_{i+j}\overline{C}_{2gp^i} + H_{i+j}\overline{C}_{4gp^i} + I_{i+j}\overline{C}_{8gp^i} + I_{i+j}\overline{C}_{16gp^i} - F_{i+j}\overline{C}_{\lambda gp^i} + G_{i+j}\overline{C}_{2\lambda gp^i} + H_{i+j}\overline{C}_{4\lambda gp^i} + C_{i+j}\overline{C}_{\mu gp^i} - G_{i+j}\overline{C}_{2\mu gp^i} + F_{i+j}\overline{C}_{\nu gp^i} + G_{i+j}\overline{C}_{2\nu gp^i} - C_{i+j}\overline{C}_{\eta gp^i} - F_{i+j}\overline{C}_{\xi gp^i} + C_{i+j}\overline{C}_{\rho gp^i} + F_{i+j}\overline{C}_{\chi gp^i}\}]$$

$$P_{4p^j}(x) = \tfrac{1}{16p^n}[\tfrac{\phi(p^{n-j})}{2}\{\sum_{t\in A'}(-1)^t\alpha^{4tp^{n+j}}\overline{C}_{tp^n}\} + \frac{\phi(p^{n-j})}{2}\sum_{i=n-j}^{n-1}\{\sum_{a\in\mathbb{A}}(-1)^a\{\alpha^{4ap^{i+j}}\overline{C}_{ap^i} + \alpha^{4agp^{i+j}}\overline{C}_{agp^i}\}\} + \frac{1}{p^j}\sum_{i=0}^{n-j-1}\{-P_{i+j}\overline{C}_{p^i} + Q_{i+j}\overline{C}_{2p^i} + R_{i+j}\overline{C}_{4p^i} + R_{i+j}\overline{C}_{8p^i} + R_{i+j}\overline{C}_{16p^i} + P_{i+j}\overline{C}_{\lambda p^i} + Q_{i+j}\overline{C}_{2\lambda p^i} + R_{i+j}\overline{C}_{4\lambda p^i} - P_{i+j}\overline{C}_{\mu p^i} + Q_{i+j}\overline{C}_{2\mu p^i} + P_{i+j}\overline{C}_{\nu p^i} + Q_{i+j}\overline{C}_{2\nu p^i} - P_{i+j}\overline{C}_{\eta p^i} + P_{i+j}\overline{C}_{\xi p^i} - P_{i+j}\overline{C}_{\rho p^i} + P_{i+j}\overline{C}_{\chi p^i} - G_{i+j}\overline{C}_{gp^i} + H_{i+j}\overline{C}_{2gp^i} + I_{i+j}\overline{C}_{4gp^i} + I_{i+j}\overline{C}_{8gp^i} + I_{i+j}\overline{C}_{16gp^i} + G_{i+j}\overline{C}_{\lambda gp^i} + H_{i+j}\overline{C}_{2\lambda gp^i} + I_{i+j}\overline{C}_{4\lambda gp^i} - G_{i+j}\overline{C}_{\mu gp^i} + H_{i+j}\overline{C}_{2\mu gp^i} + G_{i+j}\overline{C}_{\nu gp^i} + H_{i+j}\overline{C}_{2\nu gp^i} - G_{i+j}\overline{C}_{\eta gp^i} + G_{i+j}\overline{C}_{\xi gp^i} - G_{i+j}\overline{C}_{\rho gp^i} + G_{i+j}\overline{C}_{\chi gp^i}\}]$$

$$P_{2\lambda p^j}(x) = \tfrac{1}{16p^n}[\tfrac{\phi(p^{n-j})}{2}\{\sum_{t\in A'}(-1)^t\alpha^{2tp^{n+j}}\overline{C}_{tp^n}\} + \frac{\phi(p^{n-j})}{2}\sum_{i=n-j}^{n-1}\{\sum_{a\in\mathbb{A}}(-1)^a\{\alpha^{2ap^{i+j}}\overline{C}_{ap^i} + \alpha^{2agp^{i+j}}\overline{C}_{agp^i}\}\} + \frac{1}{p^j}\sum_{i=0}^{n-j-1}\{-O_{i+j}\overline{C}_{p^i} + P_{i+j}\overline{C}_{2p^i} + Q_{i+j}\overline{C}_{4p^i} + R_{i+j}\overline{C}_{8p^i} + R_{i+j}\overline{C}_{16p^i} - K_{i+j}\overline{C}_{\lambda p^i} - P_{i+j}\overline{C}_{2\lambda p^i} + Q_{i+j}\overline{C}_{4\lambda p^i} + O_{i+j}\overline{C}_{\mu p^i} + P_{i+j}\overline{C}_{2\mu p^i} + K_{i+j}\overline{C}_{\nu p^i} - P_{i+j}\overline{C}_{2\nu p^i} - O_{i+j}\overline{C}_{\eta p^i} - K_{i+j}\overline{C}_{\xi p^i} + O_{i+j}\overline{C}_{\rho p^i} + K_{i+j}\overline{C}_{\chi p^i} - F_{i+j}\overline{C}_{gp^i} + G_{i+j}\overline{C}_{2gp^i} + H_{i+j}\overline{C}_{4gp^i} + I_{i+j}\overline{C}_{8gp^i} + I_{i+j}\overline{C}_{16gp^i} - C_{i+j}\overline{C}_{\lambda gp^i} - G_{i+j}\overline{C}_{2\lambda gp^i} + H_{i+j}\overline{C}_{4\lambda gp^i} + C_{i+j}\overline{C}_{\mu gp^i} + G_{i+j}\overline{C}_{2\mu gp^i} + C_{i+j}\overline{C}_{\nu gp^i} - G_{i+j}\overline{C}_{2\nu gp^i} - F_{i+j}\overline{C}_{\eta gp^i} - C_{i+j}\overline{C}_{\xi gp^i} + F_{i+j}\overline{C}_{\rho gp^i} + C_{i+j}\overline{C}_{\chi gp^i}\}]$$

$$P_{4\lambda p^j}(x) = \tfrac{1}{16p^n}[\tfrac{\phi(p^{n-j})}{2}\{\sum_{t\in A'}\alpha^{4tp^{n+j}}\overline{C}_{tp^n}\} + \frac{\phi(p^{n-j})}{2}\sum_{i=n-j}^{n-1}\{\sum_{a\in\mathbb{A}}(-1)^a\{\alpha^{4ap^{i+j}}\overline{C}_{ap^i} + \alpha^{4agp^{i+j}}\overline{C}_{agp^i}\}\} + \frac{1}{p^j}\sum_{i=0}^{n-j-1}\{P_{i+j}\overline{C}_{p^i} + Q_{i+j}\overline{C}_{2p^i} + R_{i+j}\overline{C}_{4p^i} + R_{i+j}\overline{C}_{8p^i} + R_{i+j}\overline{C}_{16p^i} - P_{i+j}\overline{C}_{\lambda p^i} + Q_{i+j}\overline{C}_{2\lambda p^i} +$$

$R_{i+j}\overline{C}_{4\lambda p^i}+P_{i+j}\overline{C}_{\mu p^i}+Q_{i+j}\overline{C}_{2\mu p^i}-P_{i+j}\overline{C}_{\nu p^i}+Q_{i+j}\overline{C}_{2\nu p^i}+P_{i+j}\overline{C}_{\eta p^i}-P_{i+j}\overline{C}_{\xi p^i}+P_{i+j}\overline{C}_{\rho p^i}-P_{i+j}\overline{C}_{\chi p^i}+G_{i+j}\overline{C}_{gp^i}+H_{i+j}\overline{C}_{2gp^i}+I_{i+j}\overline{C}_{4gp^i}+I_{i+j}\overline{C}_{8gp^i}+I_{i+j}\overline{C}_{16gp^i}-G_{i+j}\overline{C}_{\lambda gp^i}+H_{i+j}\overline{C}_{2\lambda gp^i}+I_{i+j}\overline{C}_{4\lambda gp^i}+G_{i+j}\overline{C}_{\mu gp^i}+H_{i+j}\overline{C}_{2\mu gp^i}-G_{i+j}\overline{C}_{\nu gp^i}+H_{i+j}\overline{C}_{2\nu gp^i}+G_{i+j}\overline{C}_{\eta gp^i}-G_{i+j}\overline{C}_{\xi gp^i}+G_{i+j}\overline{C}_{\rho gp^i}-G_{i+j}\overline{C}_{\chi gp^i}\}]$

$$P_{2\mu p^j}(x) = \tfrac{1}{16p^n}[\tfrac{\phi(p^{n-j})}{2}\{\sum_{t\in A'}\alpha^{2\lambda tp^{n+j}}\overline{C}_{tp^n}\} + \frac{\phi(p^{n-j})}{2}\sum_{i=n-j}^{n-1}\{\sum_{a\in\mathbb{A}}(-1)^a\{\alpha^{2\lambda ap^{i+j}}\overline{C}_{ap^i} + \alpha^{2\lambda agp^{i+j}}\overline{C}_{agp^i}\}\} + \frac{1}{p^j}\sum_{i=0}^{n-j-1}\{K_{i+j}\overline{C}_{p^i} - P_{i+j}\overline{C}_{2p^i} + Q_{i+j}\overline{C}_{4p^i} + R_{i+j}\overline{C}_{8p^i} + R_{i+j}\overline{C}_{16p^i} + O_{i+j}\overline{C}_{\lambda p^i}+P_{i+j}\overline{C}_{2\lambda p^i}+Q_{i+j}\overline{C}_{4\lambda p^i}-K_{i+j}\overline{C}_{\mu p^i}-P_{i+j}\overline{C}_{2\mu p^i}-O_{i+j}\overline{C}_{\nu p^i}+P_{i+j}\overline{C}_{2\nu p^i}+K_{i+j}\overline{C}_{\eta p^i}+O_{i+j}\overline{C}_{\xi p^i}-K_{i+j}\overline{C}_{\rho p^i}-O_{i+j}\overline{C}_{\chi p^i}+C_{i+j}\overline{C}_{gp^i}-G_{i+j}\overline{C}_{2gp^i}+H_{i+j}\overline{C}_{4gp^i}+I_{i+j}\overline{C}_{8gp^i}+I_{i+j}\overline{C}_{16gp^i}+F_{i+j}\overline{C}_{\lambda gp^i}+G_{i+j}\overline{C}_{2\lambda gp^i}+H_{i+j}\overline{C}_{4\lambda gp^i}-C_{i+j}\overline{C}_{\mu gp^i}-G_{i+j}\overline{C}_{2\mu gp^i}-F_{i+j}\overline{C}_{\nu gp^i}+G_{i+j}\overline{C}_{2\nu gp^i}+C_{i+j}\overline{C}_{\eta gp^i}+F_{i+j}\overline{C}_{\xi gp^i}-C_{i+j}\overline{C}_{\rho gp^i}-F_{i+j}\overline{C}_{\chi gp^i}\}]$$

$$P_{2\nu p^j}(x) = \tfrac{1}{16p^n}[\tfrac{\phi(p^{n-j})}{2}\{\sum_{t\in A'}\alpha^{2tp^{n+j}}\overline{C}_{tp^n}\} + \frac{\phi(p^{n-j})}{2}\sum_{i=n-j}^{n-1}\{\sum_{a\in\mathbb{A}}(-1)^a\{\alpha^{2ap^{i+j}}\overline{C}_{ap^i} + \alpha^{2agp^{i+j}}\overline{C}_{agp^i}\}\}+\frac{1}{p^j}\sum_{i=0}^{n-j-1}\{O_{i+j}\overline{C}_{p^i}+P_{i+j}\overline{C}_{2p^i}+Q_{i+j}\overline{C}_{4p^i}+R_{i+j}\overline{C}_{8p^i}+R_{i+j}\overline{C}_{16p^i}+K_{i+j}\overline{C}_{\lambda p^i}-P_{i+j}\overline{C}_{2\lambda p^i}+Q_{i+j}\overline{C}_{4\lambda p^i}-O_{i+j}\overline{C}_{\mu p^i}+P_{i+j}\overline{C}_{2\mu p^i}-K_{i+j}\overline{C}_{\nu p^i}-P_{i+j}\overline{C}_{2\nu p^i}+O_{i+j}\overline{C}_{\eta p^i}+K_{i+j}\overline{C}_{\xi p^i}-O_{i+j}\overline{C}_{\rho p^i}-K_{i+j}\overline{C}_{\chi p^i}+F_{i+j}\overline{C}_{gp^i}+G_{i+j}\overline{C}_{2gp^i}+H_{i+j}\overline{C}_{4gp^i}+I_{i+j}\overline{C}_{8gp^i}+I_{i+j}\overline{C}_{16gp^i}+C_{i+j}\overline{C}_{\lambda gp^i}-G_{i+j}\overline{C}_{2\lambda gp^i}+H_{i+j}\overline{C}_{4\lambda gp^i}-F_{i+j}\overline{C}_{\mu gp^i}+G_{i+j}\overline{C}_{2\mu gp^i}-C_{i+j}\overline{C}_{\nu gp^i}-G_{i+j}\overline{C}_{2\nu gp^i}+F_{i+j}\overline{C}_{\eta gp^i}+C_{i+j}\overline{C}_{\xi gp^i}-F_{i+j}\overline{C}_{\rho gp^i}-C_{i+j}\overline{C}_{\chi gp^i}\}],$$ *where*

$P_{n-1} = \frac{1}{8}[4\sqrt{-p^{2(n-1)}} + \sqrt{p^{n-1}(-16p^{n-1}-7p+7)}]$ , $G_{n-1} = -\frac{1}{8}[4\sqrt{-p^{2(n-1)}} - \sqrt{p^{n-1}(-16p^{n-1}-7p+7)}]$ *and* $C_{i+j}$, $F_{i+j}$, $K_{i+j}$, $O_{i+j}$ *can be obtained, from the following relations,*

$$\tfrac{1}{p^j}\sum_{i=0}^{n-j-1}\{\frac{K_{i+j}F_{i+j}+O_{i+j}C_{i+j}}{p^i}\} = -\frac{p^{n-1}(p-1)}{2}+\frac{9p^{n-j-1}(p-1)}{32},$$

$$\tfrac{1}{p^j}\sum_{i=0}^{n-j-1}\{\frac{K_{i+j}O_{i+j}+C_{i+j}F_{i+j}}{p^i}\} = -\frac{p^{n-1}}{4}(p-1)-\frac{7p^{n-j-1}(p-1)}{32},$$

$$\tfrac{1}{p^j}\sum_{i=0}^{n-j-1}\{\frac{K_{i+j}C_{i+j}+O_{i+j}F_{i+j}}{p^i}\} = -\frac{p^{n-j-1}(p-1)}{16},$$

$$\tfrac{1}{p^j}\sum_{i=0}^{n-j-1}\{\frac{K^2{}_{i+j}+C^2{}_{i+j}+F^2{}_{i+j}+O^2{}_{i+j}}{p^i}\} = -p^{(n-1)}+\frac{p^{n-j-1}(p-1)}{8}.$$

*The expressions for* $P_{2gp^j}$, $P_{4gp^j}$, $P_{2\lambda gp^j}$, $P_{4\lambda gp^j}$, $P_{2\mu gp^j}$ *and* $P_{2\nu gp^j}$ *can be obtained by replacing*

*P, Q, R, K, O by G, H, I, C, F and* $\alpha^{up^{i+j}}$ *by* $\alpha^{ugp^{i+j}}$ *respectively in the expression of* $P_{2p^j}$, $P_{4p^j}$, $P_{2\lambda p^j}$, $P_{4\lambda p^j}$, $P_{2\mu p^j}$ *and* $P_{2\nu p^j}$.

**Proof.** *These expressions can be obtained using Lemmas* $3.3-3.6$, $4.2-4.4$ *and similar procedure as in theorem* 3.7. *Also the relations can be derived using* $P_{2p^j}(\alpha^{2p^i})=1$, $P_{2p^j}(\alpha^{2gp^i})=0$, $P_{2p^j}(\alpha^{2\lambda p^j})=0$ *and* $P_{2p^j}(\alpha^{2\nu gp^j})=0$.

## 5. Primitive Idempotents Corresponding to $P_{tp^i}, t=1,\lambda,\mu,\nu,\eta,\xi,\rho,\chi,g,\lambda g,\mu g,\nu g,\eta g,\xi g,\rho g,\chi g$

For $0\leq j\leq n-1$, define $A_j=p^j\sum\limits_{s\varepsilon\Omega_{gp^j}}\alpha^s$; $B_j=p^j\sum\limits_{s\varepsilon\Omega_{\lambda gp^j}}\alpha^s$ ; $D_j=p^j\sum\limits_{s\varepsilon\Omega_{\mu gp^j}}\alpha^s$ ; $E_j=p^j\sum\limits_{s\varepsilon\Omega_{\nu gp^j}}\alpha^s$; $J_j=p^j\sum\limits_{s\varepsilon\Omega_{\lambda p^j}}\alpha^s$; $L_j=p^j\sum\limits_{s\varepsilon\Omega_{\mu p^j}}\alpha^s$ ; $M_j=p^j\sum\limits_{s\varepsilon\Omega_{\nu p^j}}\alpha^s$; $N_j=p^j\sum\limits_{s\varepsilon\Omega_{p^j}}\alpha^s$.
Using similar procedure as in section 3 we obtain $A_j, B_j, D_j, E_j, J_j, L_j, M_j, N_j\in GF(q)$.
Proof of lemma 5.1 is similar to that of lemma 4.1.

**Lemma 5.1.** *For* $0\leq j\leq n-1$, $L_j-\beta^{p^j}N_j=0$; $M_j-\beta^{p^j}J_j=0$; $D_j-\beta^{gp^j}A_j=0$; $E_j-\beta^{gp^j}B_j=0$.

Proof of Lemma $5.2-5.5$ is similarly as that of lemma 3.5.

**Lemma 5.2.** *For* $0\leq i\leq n$; $0\leq j\leq n-1$

$$\sum_{s\varepsilon\Omega_{p^j}}\alpha^{gp^is}=\sum_{s\varepsilon\Omega_{\lambda p^j}}\alpha^{\xi gp^is}=\sum_{s\varepsilon\Omega_{\mu p^j}}\alpha^{\rho gp^is}=\sum_{s\varepsilon\Omega_{\nu p^j}}\alpha^{\nu gp^is}=\sum_{s\varepsilon\Omega_{\eta p^j}}\alpha^{\eta gp^is}=\sum_{s\varepsilon\Omega_{\chi p^j}}\alpha^{\chi gp^is}$$
$$=-\sum_{s\varepsilon\Omega_{\nu p^j}}\alpha^{\chi gp^is}=-\sum_{s\varepsilon\Omega_{\lambda p^j}}\alpha^{\lambda gp^is}=-\sum_{s\varepsilon\Omega_{\mu p^j}}\alpha^{\mu gp^is}=-\sum_{s\varepsilon\Omega_{\xi p^j}}\alpha^{\xi gp^is}=-\sum_{s\varepsilon\Omega_{\rho p^j}}\alpha^{\rho gp^is}$$
$$=\begin{cases}\frac{\phi(p^{n-j})}{4}\{\alpha^{gp^{i+j}}+\alpha^{7gp^{i+j}}\},\ if\ i+j\geq n,\ g\neq 1,\\ \frac{1}{p^j}A_{i+j},\ if\ i+j\leq n-1,\ g\neq 1,\\ \frac{\phi(p^{n-j})}{4}\{\alpha^{p^{i+j}}+\alpha^{7p^{i+j}}\},\ if\ i+j\geq n,\ g=1,\\ \frac{1}{p^j}N_{i+j},\ if\ i+j\leq n-1,\ g=1.\end{cases}$$

**Lemma 5.3.** *For* $0\leq i\leq n$; $0\leq j\leq n-1$

$$\sum_{s\varepsilon\Omega_{p^j}}\alpha^{\lambda gp^is}=\sum_{s\varepsilon\Omega_{\eta p^j}}\alpha^{\xi gp^is}=\sum_{s\varepsilon\Omega_{\mu p^j}}\alpha^{\nu gp^is}=\sum_{s\varepsilon\Omega_{\rho p^j}}\alpha^{\chi gp^is}=-\sum_{s\varepsilon\Omega_{p^j}}\alpha^{\xi p^is}=-\sum_{s\varepsilon\Omega_{\nu p^j}}\alpha^{\rho gp^is}$$

$$= -\sum_{s\varepsilon\Omega_{\lambda p^j}} \alpha^{\eta g p^i s} = -\sum_{s\varepsilon\Omega_{\mu p^j}} \alpha^{\chi g p^i s} = \begin{cases} \frac{\phi(p^{n-j})}{4}\{\alpha^{3gp^{i+j}} + \alpha^{5gp^{i+j}}\},\ if\ i+j \geq n,\ g \neq 1, \\ \frac{1}{p^j} B_{i+j},\ if\ i+j \leq n-1,\ g \neq 1, \\ \frac{\phi(p^{n-j})}{4}\{\alpha^{3p^{i+j}} + \alpha^{5p^{i+j}}\},\ if\ i+j \geq n,\ g = 1, \\ \frac{1}{p^j} J_{i+j},\ if\ i+j \leq n-1,\ g = 1. \end{cases}$$

**Lemma 5.4.** *For* $0 \leq i \leq n$; $0 \leq j \leq n-1$

$$\sum_{s\varepsilon\Omega_{p^j}} \alpha^{\mu g p^i s} = \sum_{s\varepsilon\Omega_{\lambda p^j}} \alpha^{\nu g p^i s} = \sum_{s\varepsilon\Omega_{\eta p^j}} \alpha^{\rho g p^i s} = \sum_{s\varepsilon\Omega_{\xi p^j}} \alpha^{\chi g p^i s} = -\sum_{s\varepsilon\Omega_{p^j}} \alpha^{\rho g p^i s} = -\sum_{s\varepsilon\Omega_{\lambda p^j}} \alpha^{\chi g p^i s}$$

$$= -\sum_{s\varepsilon\Omega_{\mu p^j}} \alpha^{\eta g p^i s} = -\sum_{s\varepsilon\Omega_{\nu p^j}} \alpha^{\xi g p^i s} = \begin{cases} \frac{\phi(p^{n-j})}{4}\{\alpha^{3gp^{i+j}} + \alpha^{5gp^{i+j}}\},\ if\ i+j \geq n,\ g \neq 1, \\ \frac{1}{p^j} D_{i+j},\ if\ i+j \leq n-1,\ g \neq 1, \\ \frac{\phi(p^{n-j})}{4}\{\alpha^{3p^{i+j}} + \alpha^{5p^{i+j}}\},\ if\ i+j \geq n,\ g = 1, \\ \frac{1}{p^j} L_{i+j},\ if\ i+j \leq n-1,\ g = 1. \end{cases}$$

**Lemma 5.5.** *For* $0 \leq i \leq n$; $0 \leq j \leq n-1$

$$\sum_{s\varepsilon\Omega_{p^j}} \alpha^{\nu g p^i s} = \sum_{s\varepsilon\Omega_{\lambda p^j}} \alpha^{\rho g p^i s} = \sum_{s\varepsilon\Omega_{\mu p^j}} \alpha^{\xi g p^i s} = \sum_{s\varepsilon\Omega_{\eta p^j}} \alpha^{\chi g p^i s} = -\sum_{s\varepsilon\Omega_{p^j}} \alpha^{\chi g p^i s} = -\sum_{s\varepsilon\Omega_{\lambda p^j}} \alpha^{\mu g p^i s}$$

$$= -\sum_{s\varepsilon\Omega_{\nu p^j}} \alpha^{\eta g p^i s} = -\sum_{s\varepsilon\Omega_{\xi p^j}} \alpha^{\rho g p^i s} = \begin{cases} \frac{\phi(p^{n-j})}{4}\{\alpha^{gp^{i+j}} + \alpha^{7gp^{i+j}}\},\ if\ i+j \geq n,\ g \neq 1, \\ \frac{1}{p^j} E_{i+j},\ if\ i+j \leq n-1,\ g \neq 1, \\ \frac{\phi(p^{n-j})}{4}\{\alpha^{p^{i+j}} + \alpha^{7p^{i+j}}\},\ if\ i+j \geq n,\ g = 1, \\ \frac{1}{p^j} M_{i+j},\ if\ i+j \leq n-1,\ g = 1. \end{cases}$$

**Theorem 5.6.** *For $p \equiv 1(mod 8)$, the expressions for the primitive idempotents corresponding to $P_{p^i}$, $P_{\lambda p^i}$,$P_{\mu p^i}$,$P_{\nu p^i}$,$P_{\eta p^i}$,$P_{\xi p^i}$,$P_{\rho p^i}$,$P_{\chi p^i}$,$P_{gp^i}$,$P_{\lambda gp^i}$,$P_{\mu gp^i}$,$P_{\nu gp^i}$,$P_{\eta gp^i}$,$P_{\xi gp^i}$,$P_{\rho gp^i}$ and $P_{\chi gp^i}$ are given by*

$$P_{p^j}(x) = \tfrac{1}{16p^n}[\tfrac{\phi(p^{n-j})}{2}\{\sum_{t\in A'}(-1)^t\alpha^{t\nu p^{n+j}}\overline{C}_{tp^n}\} + \frac{\phi(p^{n-j})}{2}\sum_{i=n-j}^{n-1}\{\sum_{a\in\mathbb{A}}(-1)^a\{\alpha^{a\nu p^{i+j}}\overline{C}_{ap^i} + \alpha^{a\nu g p^{i+j}}\overline{C}_{agp^i}\}\} + \frac{1}{p^j}\sum_{i=0}^{n-j-1}\{-M_{i+j}\overline{C}_{p^i} - K_{i+j}\overline{C}_{2p^i} - P_{i+j}\overline{C}_{4p^i} + Q_{i+j}\overline{C}_{8p^i} + R_{i+j}\overline{C}_{16p^i} - L_{i+j}\overline{C}_{\lambda p^i} - O_{i+j}\overline{C}_{2\lambda p^i} + P_{i+j}\overline{C}_{4\lambda p^i} - J_{i+j}\overline{C}_{\mu p^i} + K_{i+j}\overline{C}_{2\mu p^i} - N_{i+j}\overline{C}_{\nu p^i} + O_{i+j}\overline{C}_{2\nu p^i} + M_{i+j}\overline{C}_{\eta p^i} + L_{i+j}\overline{C}_{\xi p^i} + J_{i+j}\overline{C}_{\rho p^i} + N_{i+j}\overline{C}_{\chi p^i} - E_{i+j}\overline{C}_{gp^i} - C_{i+j}\overline{C}_{2gp^i} - G_{i+j}\overline{C}_{4gp^i} + H_{i+j}\overline{C}_{8gp^i} + I_{i+j}\overline{C}_{16gp^i} - D_{i+j}\overline{C}_{\lambda gp^i} - F_{i+j}\overline{C}_{2\lambda gp^i} + G_{i+j}\overline{C}_{4\lambda gp^i} - B_{i+j}\overline{C}_{\mu gp^i} + C_{i+j}\overline{C}_{2\mu gp^i} - A_{i+j}\overline{C}_{\nu gp^i} + F_{i+j}\overline{C}_{2\nu gp^i} + E_{i+j}\overline{C}_{\eta gp^i} + D_{i+j}\overline{C}_{\xi gp^i} + B_{i+j}\overline{C}_{\rho gp^i} + A_{i+j}\overline{C}_{\chi gp^i}\}]$$

$P_{\lambda p^j}(x) = \frac{1}{16p^n}[\frac{\phi(p^{n-j})}{2}\{\sum_{t\in A'}(-1)^t\alpha^{t\mu p^{n+j}}\overline{C}_{tp^n}\} + \frac{\phi(p^{n-j})}{2}\sum_{i=n-j}^{n-1}\{\sum_{a\in\mathbb{A}}(-1)^a\{\alpha^{a\mu p^{i+j}}\overline{C}_{ap^i} + \alpha^{a\mu gp^{i+j}}\overline{C}_{agp^i}\}\} + \frac{1}{p^j}\sum_{i=0}^{n-j-1}\{-L_{i+j}\overline{C}_{p^i} - O_{i+j}\overline{C}_{2p^i} + P_{i+j}\overline{C}_{4p^i} + Q_{i+j}\overline{C}_{8p^i} + R_{i+j}\overline{C}_{16p^i} + M_{i+j}\overline{C}_{\lambda p^i} - K_{i+j}\overline{C}_{2\lambda p^i} - P_{i+j}\overline{C}_{4\lambda p^i} + N_{i+j}\overline{C}_{\mu p^i} + O_{i+j}\overline{C}_{2\mu p^i} - J_{i+j}\overline{C}_{\nu p^i} + K_{i+j}\overline{C}_{2\nu p^i} + L_{i+j}\overline{C}_{\eta p^i} - M_{i+j}\overline{C}_{\xi p^i} - N_{i+j}\overline{C}_{\rho p^i} + J_{i+j}\overline{C}_{\chi p^i} - D_{i+j}\overline{C}_{gp^i} - F_{i+j}\overline{C}_{2gp^i} + G_{i+j}\overline{C}_{4gp^i} + H_{i+j}\overline{C}_{8gp^i} + I_{i+j}\overline{C}_{16gp^i} + E_{i+j}\overline{C}_{\lambda gp^i} - C_{i+j}\overline{C}_{2\lambda gp^i} - G_{i+j}\overline{C}_{4\lambda gp^i} + A_{i+j}\overline{C}_{\mu gp^i} + F_{i+j}\overline{C}_{2\mu gp^i} - B_{i+j}\overline{C}_{\nu gp^i} + C_{i+j}\overline{C}_{2\nu gp^i} + D_{i+j}\overline{C}_{\eta gp^i} - E_{i+j}\overline{C}_{\xi gp^i} - A_{i+j}\overline{C}_{\rho gp^i} + B_{i+j}\overline{C}_{\chi gp^i}\}]$

$P_{\mu p^j}(x) = \frac{1}{16p^n}[\frac{\phi(p^{n-j})}{2}\{\sum_{t\in A'}(-1)^t\alpha^{\lambda tp^{n+j}}\overline{C}_{tp^n}\} + \frac{\phi(p^{n-j})}{2}\sum_{i=n-j}^{n-1}\{\sum_{a\in\mathbb{A}}(-1)^a\{\alpha^{a\lambda p^{i+j}}\overline{C}_{ap^i} + \alpha^{a\lambda gp^{i+j}}\overline{C}_{agp^i}\}\} + \frac{1}{p^j}\sum_{i=0}^{n-j-1}\{-J_{i+j}\overline{C}_{p^i} + K_{i+j}\overline{C}_{2p^i} - P_{i+j}\overline{C}_{4p^i} + Q_{i+j}\overline{C}_{8p^i} + R_{i+j}\overline{C}_{16p^i} + N_{i+j}\overline{C}_{\lambda p^i} + O_{i+j}\overline{C}_{2\lambda p^i} + P_{i+j}\overline{C}_{4\lambda p^i} + M_{i+j}\overline{C}_{\mu p^i} - K_{i+j}\overline{C}_{2\mu p^i} - L_{i+j}\overline{C}_{\nu p^i} - O_{i+j}\overline{C}_{2\nu p^i} + J_{i+j}\overline{C}_{\eta p^i} - N_{i+j}\overline{C}_{\xi p^i} - M_{i+j}\overline{C}_{\rho p^i} + L_{i+j}\overline{C}_{\chi p^i} - B_{i+j}\overline{C}_{gp^i} + C_{i+j}\overline{C}_{2gp^i} - G_{i+j}\overline{C}_{4gp^i} + H_{i+j}\overline{C}_{8gp^i} + I_{i+j}\overline{C}_{16gp^i} + A_{i+j}\overline{C}_{\lambda gp^i} + F_{i+j}\overline{C}_{2\lambda gp^i} + G_{i+j}\overline{C}_{4\lambda gp^i} + E_{i+j}\overline{C}_{\mu gp^i} - C_{i+j}\overline{C}_{2\mu gp^i} - D_{i+j}\overline{C}_{\nu gp^i} - F_{i+j}\overline{C}_{2\nu gp^i} + B_{i+j}\overline{C}_{\eta gp^i} - A_{i+j}\overline{C}_{\xi gp^i} - E_{i+j}\overline{C}_{\rho gp^i} + D_{i+j}\overline{C}_{\chi gp^i}\}]$

$P_{\nu p^j}(x) = \frac{1}{16p^n}[\frac{\phi(p^{n-j})}{2}\{\sum_{t\in A'}(-1)^t\alpha^{tp^{n+j}}\overline{C}_{tp^n}\} + \frac{\phi(p^{n-j})}{2}\sum_{i=n-j}^{n-1}\{\sum_{a\in\mathbb{A}}(-1)^a\{\alpha^{ap^{i+j}}\overline{C}_{ap^i} + \alpha^{agp^{i+j}}\overline{C}_{agp^i}\}\} + \frac{1}{p^j}\sum_{i=0}^{n-j-1}\{-N_{i+j}\overline{C}_{p^i} + O_{i+j}\overline{C}_{2p^i} + P_{i+j}\overline{C}_{4p^i} + Q_{i+j}\overline{C}_{8p^i} + R_{i+j}\overline{C}_{16p^i} - J_{i+j}\overline{C}_{\lambda p^i} + K_{i+j}\overline{C}_{2\lambda p^i} - P_{i+j}\overline{C}_{4\lambda p^i} - L_{i+j}\overline{C}_{\mu p^i} - O_{i+j}\overline{C}_{2\mu p^i} - M_{i+j}\overline{C}_{\nu p^i} - K_{i+j}\overline{C}_{2\nu p^i} + N_{i+j}\overline{C}_{\eta p^i} + J_{i+j}\overline{C}_{\xi p^i} + L_{i+j}\overline{C}_{\rho p^i} + M_{i+j}\overline{C}_{\chi p^i} - A_{i+j}\overline{C}_{gp^i} + F_{i+j}\overline{C}_{2gp^i} + G_{i+j}\overline{C}_{4gp^i} + H_{i+j}\overline{C}_{8gp^i} + I_{i+j}\overline{C}_{16gp^i} - B_{i+j}\overline{C}_{\lambda gp^i} + C_{i+j}\overline{C}_{2\lambda gp^i} - G_{i+j}\overline{C}_{4\lambda gp^i} - D_{i+j}\overline{C}_{\mu gp^i} - F_{i+j}\overline{C}_{2\mu gp^i} - E_{i+j}\overline{C}_{\nu gp^i} - C_{i+j}\overline{C}_{2\nu gp^i} + A_{i+j}\overline{C}_{\eta gp^i} + B_{i+j}\overline{C}_{\xi gp^i} + D_{i+j}\overline{C}_{\rho gp^i} + E_{i+j}\overline{C}_{\chi gp^i}\}]$

$P_{\eta p^j}(x) = \frac{1}{16p^n}{}_{i+j}[\frac{\phi(p^{n-j})}{2}\{\sum_{t\in A'}\alpha^{t\nu p^{n+j}}\overline{C}_{tp^n}\} + \frac{\phi(p^{n-j})}{2}\sum_{i=n-j}^{n-1}\{\sum_{a\in\mathbb{A}}\alpha^{a\nu p^{i+j}}\overline{C}_{ap^i} + \sum_{a\in\mathbb{A}}\alpha^{a\nu gp^{i+j}}\overline{C}_{agp^i}\} + \frac{1}{p^j}\sum_{i=0}^{n-j-1}\{M_{i+j}\overline{C}_{p^i} - K_{i+j}\overline{C}_{2p^i} - P_{i+j}\overline{C}_{4p^i} + Q_{i+j}\overline{C}_{8p^i} + R_{i+j}\overline{C}_{16p^i} + L_{i+j}\overline{C}_{\lambda p^i} - O_{i+j}\overline{C}_{2\lambda p^i} + P_{i+j}\overline{C}_{4\lambda p^i} + J_{i+j}\overline{C}_{\mu p^i} + K_{i+j}\overline{C}_{2\mu p^i} + N_{i+j}\overline{C}_{\nu p^i} + O_{i+j}\overline{C}_{2\nu p^i} - M_{i+j}\overline{C}_{\eta p^i} - L_{i+j}\overline{C}_{\xi p^i} - J_{i+j}\overline{C}_{\rho p^i} - N_{i+j}\overline{C}_{\chi p^i} + E_{i+j}\overline{C}_{gp^i} - C_{i+j}\overline{C}_{2gp^i} - G_{i+j}\overline{C}_{4gp^i} + H_{i+j}\overline{C}_{8gp^i} + I_{i+j}\overline{C}_{16gp^i} + D_{i+j}\overline{C}_{\lambda gp^i} - F_{i+j}\overline{C}_{2\lambda gp^i} + G_{i+j}\overline{C}_{4\lambda gp^i} + B_{i+j}\overline{C}_{\mu gp^i} + C_{i+j}\overline{C}_{2\mu gp^i} + A_{i+j}\overline{C}_{\nu gp^i} + F_{i+j}\overline{C}_{2\nu gp^i} - E_{i+j}\overline{C}_{\eta gp^i} - D_{i+j}\overline{C}_{\xi gp^i} - B_{i+j}\overline{C}_{\rho gp^i} - A_{i+j}\overline{C}_{\chi gp^i}\}]$

$$P_{\xi p^j}(x) = \tfrac{1}{16p^n}[\tfrac{\phi(p^{n-j})}{2}\{\sum_{t\in A'}\alpha^{t\mu p^{n+j}}\overline{C}_{tp^n}\} + \frac{\phi(p^{n-j})}{2}\sum_{i=n-j}^{n-1}\{\sum_{a\in\mathbb{A}}\{\alpha^{a\mu p^{i+j}}\overline{C}_{ap^i} + \alpha^{a\mu gp^{i+j}}\overline{C}_{agp^i}\}\} + \frac{1}{p^j}\sum_{i=0}^{n-j-1}\{L_{i+j}\overline{C}_{p^i} - O_{i+j}\overline{C}_{2p^i} + P_{i+j}\overline{C}_{4p^i} + Q_{i+j}\overline{C}_{8p^i} + R_{i+j}\overline{C}_{16p^i} - M_{i+j}\overline{C}_{\lambda p^i} - K_{i+j}\overline{C}_{2\lambda p^i} - P_{i+j}\overline{C}_{4\lambda p^i} - N_{i+j}\overline{C}_{\mu p^i} + O_{i+j}\overline{C}_{2\mu p^i} + J_{i+j}\overline{C}_{\nu p^i} + K_{i+j}\overline{C}_{2\nu p^i} - L_{i+j}\overline{C}_{\eta p^i} + M_{i+j}\overline{C}_{\xi p^i} + N_{i+j}\overline{C}_{\rho p^i} - J_{i+j}\overline{C}_{\chi p^i} + D_{i+j}\overline{C}_{gp^i} - F_{i+j}\overline{C}_{2gp^i} + G_{i+j}\overline{C}_{4gp^i} + H_{i+j}\overline{C}_{8gp^i} + I_{i+j}\overline{C}_{16gp^i} - E_{i+j}\overline{C}_{\lambda gp^i} - C_{i+j}\overline{C}_{2\lambda gp^i} - G_{i+j}\overline{C}_{4\lambda gp^i} - A_{i+j}\overline{C}_{\mu gp^i} + F_{i+j}\overline{C}_{2\mu gp^i} + B_{i+j}\overline{C}_{\nu gp^i} + C_{i+j}\overline{C}_{2\nu gp^i} - D_{i+j}\overline{C}_{\eta gp^i} + E_{i+j}\overline{C}_{\xi gp^i} + A_{i+j}\overline{C}_{\rho gp^i} - B_{i+j}\overline{C}_{\chi gp^i}\}]$$

$$P_{\rho p^j}(x) = \tfrac{1}{16p^n}[\tfrac{\phi(p^{n-j})}{2}\{\sum_{t\in A'}\alpha^{t\lambda p^{n+j}}\overline{C}_{tp^n}\} + \frac{\phi(p^{n-j})}{2}\sum_{i=n-j}^{n-1}\{\sum_{a\in\mathbb{A}}\alpha^{a\lambda p^{i+j}}\overline{C}_{ap^i} + \sum_{a\in\mathbb{A}}\alpha^{a\lambda gp^{i+j}}\overline{C}_{agp^i}\} + \frac{1}{p^j}\sum_{i=0}^{n-j-1}\{J_{i+j}\overline{C}_{p^i} + K_{i+j}\overline{C}_{2p^i} - P_{i+j}\overline{C}_{4p^i} + Q_{i+j}\overline{C}_{8p^i} + R_{i+j}\overline{C}_{16p^i} - N_{i+j}\overline{C}_{\lambda p^i} + O_{i+j}\overline{C}_{2\lambda p^i} + P_{i+j}\overline{C}_{4\lambda p^i} - M_{i+j}\overline{C}_{\mu p^i} - K_{i+j}\overline{C}_{2\mu p^i} + L_{i+j}\overline{C}_{\nu p^i} - O_{i+j}\overline{C}_{2\nu p^i} - J_{i+j}\overline{C}_{\eta p^i} + N_{i+j}\overline{C}_{\xi p^i} + M_{i+j}\overline{C}_{\rho p^i} - L_{i+j}\overline{C}_{\chi p^i} + B_{i+j}\overline{C}_{gp^i} + C_{i+j}\overline{C}_{2gp^i} - G_{i+j}\overline{C}_{4gp^i} + H_{i+j}\overline{C}_{8gp^i} + I_{i+j}\overline{C}_{16gp^i} - A_{i+j}\overline{C}_{\lambda gp^i} + F_{i+j}\overline{C}_{2\lambda gp^i} + G_{i+j}\overline{C}_{4\lambda gp^i} - E_{i+j}\overline{C}_{\mu gp^i} - C_{i+j}\overline{C}_{2\mu gp^i} + D_{i+j}\overline{C}_{\nu gp^i} - F_{i+j}\overline{C}_{2\nu gp^i} - B_{i+j}\overline{C}_{\eta gp^i} + A_{i+j}\overline{C}_{\xi gp^i} + E_{i+j}\overline{C}_{\rho gp^i} - D_{i+j}\overline{C}_{\chi gp^i}\}]$$

$$P_{\chi p^j}(x) = \tfrac{1}{16p^n}[\tfrac{\phi(p^{n-j})}{2}\{\sum_{t\in A'}\alpha^{tp^{n+j}}\overline{C}_{tp^n}\} + \frac{\phi(p^{n-j})}{2}\sum_{i=n-j}^{n-1}\{\sum_{a\in\mathbb{A}}\alpha^{ap^{i+j}}\overline{C}_{ap^i} + \sum_{a\in\mathbb{A}}\alpha^{agp^{i+j}}\overline{C}_{agp^i}\} + \frac{1}{p^j}\sum_{i=0}^{n-j-1}\{N_{i+j}\overline{C}_{p^i} + O_{i+j}\overline{C}_{2p^i} + P_{i+j}\overline{C}_{4p^i} + Q_{i+j}\overline{C}_{8p^i} + R_{i+j}\overline{C}_{16p^i} + J_{i+j}\overline{C}_{\lambda p^i} + K_{i+j}\overline{C}_{2\lambda p^i} - P_{i+j}\overline{C}_{4\lambda p^i} + L_{i+j}\overline{C}_{\mu p^i} - O_{i+j}\overline{C}_{2\mu p^i} + M_{i+j}\overline{C}_{\nu p^i} - K_{i+j}\overline{C}_{2\nu p^i} - N_{i+j}\overline{C}_{\eta p^i} - J_{i+j}\overline{C}_{\xi p^i} - L_{i+j}\overline{C}_{\rho p^i} - M_{i+j}\overline{C}_{\chi p^i} + A_{i+j}\overline{C}_{gp^i} + F_{i+j}\overline{C}_{2gp^i} + G_{i+j}\overline{C}_{4gp^i} + H_{i+j}\overline{C}_{8gp^i} + I_{i+j}\overline{C}_{16gp^i} + B_{i+j}\overline{C}_{\lambda gp^i} + C_{i+j}\overline{C}_{2\lambda gp^i} - G_{i+j}\overline{C}_{4\lambda gp^i} + D_{i+j}\overline{C}_{\mu gp^i} - F_{i+j}\overline{C}_{2\mu gp^i} + E_{i+j}\overline{C}_{\nu gp^i} - C_{i+j}\overline{C}_{2\nu gp^i} - A_{i+j}\overline{C}_{\eta gp^i} - B_{i+j}\overline{C}_{\xi gp^i} - D_{i+j}\overline{C}_{\rho gp^i} - E_{i+j}\overline{C}_{\chi gp^i}\}]$$

*The expressions for $P_{gp^j}$, $P_{\lambda gp^j}$, $P_{\mu gp^j}$, $P_{\nu gp^j}$, $P_{\eta gp^j}$, $P_{\xi gp^j}$, $P_{\rho gp^j}$ and $P_{\chi gp^j}$ can be computed by interchanging P, Q, R, K, O, L, M, N, J by G, H, I, C, F, D, E, A, B and $\alpha^{up^{i+j}}$ by $\alpha^{ugp^{i+j}}$ respectively, Where $A_{i+j}$, $B_{i+j}$, $D_{i+j}$, $E_{i+j}$, $J_{i+j}$, $L_{i+j}$, $M_{i+j}$, $N_{i+j}$ can be computed, using lemma* 5.1 *and the following relations,*

$$\tfrac{1}{p^j}\sum_{i=0}^{n-j-1}\{\frac{M_{i+j}N_{i+j}+A_{i+j}E_{i+j}+J_{i+j}L_{i+j}+B_{i+j}D_{i+j}}{p^i}\} = -\frac{5p^{n-1}(p+1)}{4} - \frac{7p^{n-j-1}(p-1)}{16},$$

$$\tfrac{1}{p^j}\sum_{i=0}^{n-j-1}\{\frac{M_{i+j}B_{i+j}+N_{i+j}D_{i+j}-J_{i+j}E_{i+j}-A_{i+j}L_{i+j}}{p^i}\} = \frac{3p^{n-j-1}(p-1)}{16},$$

$\frac{1}{p^j}\sum_{i=0}^{n-j-1}\{\frac{{M^2}_{i+j}+{E^2}_{i+j}+{L^2}_{i+j}+{D^2}_{i+j}+{J^2}_{i+j}+{B^2}_{i+j}+{N^2}_{i+j}+{A^2}_{i+j}}{p^i}\}$
$=-2p^(n-1)+\frac{p^{n-j-1}(p-1)(3+4\alpha^{6p^{n+j}}+4\alpha^{2p^{n+j}})}{8},$
$\frac{1}{p^j}\sum_{i=0}^{n-j-1}\{\frac{E_{i+j}L_{i+j}+J_{i+j}A_{i+j}-M_{i+j}D_{i+j}-N_{i+j}B_{i+j}}{p^i}\}$
$=p^{n-1}(p-1)+\frac{5p^{n-j-1}(p-1)}{8}+\frac{p^{n-j-1}(p-1)(\alpha^{3(\chi+\mu g)p^{n+j}}-\alpha^{(\nu+\mu g)p^{n+j}})}{4}.$

**Proof.** *The above expressions can be obtained using Lemmas* $3.3-3.6, 4.2-4.4$ *and* $5.2-5.5$ *and similar procedure as in theorem* 3.7*. Also the relations can be derived using* $P_{p^j}(\alpha^{p^i})=1$*,* $P_{p^j}(\alpha^{\lambda g p^i})=0$*,* $P_{p^j}(\alpha^{\nu p^j})=0$ *and* $P_{\lambda p^j}(\alpha^{\mu g p^j})=0$*.*

## 6. Dimension and Generating Polynomials

The polynomial $m_s(x)=\prod_{s\varepsilon\Omega_s}(x-\alpha^s)$ denote the minimal polynomial for $\alpha^s$ and the generating polynomial for cyclic code $M_s$ of length $16p^n$ corresponding to the cyclotomic coset $\Omega_s$ is $\frac{x^{16p^n}-1}{m_s(x)}$ and the dimension of $M_s$ is equal to the cardinality of the class $\Omega_s$ by J. Singh and Arrora.

**Theorem 6.1.** $(i)$ *The generating polynomial for the codes* $M_{tp^n}$*, for* $t\in A'$ *are* $(1+x+x^2+...+x^{(16p^n-1)})$, $(x^8-1)(x^2+\beta^2)(x^2-\beta^6)(x+\beta)(x+\beta^7)(1+x^{16}+...+x^{16(p^n-1)})$, $(x+\beta^2)(x-\beta^6)(x^4-1)(x^8+1)(1+x^{16}+...+x^{16(p^n-1)})$, $(x+\beta^3)(x+\beta^5)(x^2-\beta^2)(x^8-1)(1+x^{16}+...+x^{16(p^n-1)})$, $(x^2-1)(x^4+1)(x^8+1)(1+x^{16}+...+x^{16(p^n-1)})$, $(x^8+1)(x^4-1)(x-\beta^2)(x+\beta^6)(1+x^{16}+...+x^{16(p^n-1)})$, $(x^8+1)(x^4+1)(x^2+1)(x-1)(1+x^{16}+...+x^{16(p^n-1)})$, $(x^8-1)(x^2-\beta^6)(x^2+\beta^2)(x-\beta^7)(x-\beta)(1+x^{16}+...+x^{16(p^n-1)})$ *and* $(x^8-1)(x^2+\beta^6)(x^2-\beta^2)(x-\beta^3)(x-\beta^5)(1+x^{16}+...+x^{16(p^n-1)})$ *respectively, where* $\beta$ *is* $16th$ *root of unity.*

$(ii)$ *The generating polynomial for* $M_{8p^i}\oplus M_{8gp^i}$*,* $M_{16p^i}\oplus M_{16gp^i}$ *and* $M_{p^i}\oplus M_{2p^i}\oplus M_{4p^i}\oplus M_{\lambda p^i}\oplus M_{2\lambda p^i}\oplus M_{4\lambda p^i}\oplus M_{\mu p^i}\oplus M_{2\mu p^i}\oplus M_{\nu p^i}\oplus M_{2\nu p^i}\oplus M_{\eta p^i}\oplus M_{\xi p^i}\oplus M_{\rho p^i}\oplus M_{\chi p^i}\oplus M_{gp^i}\oplus M_{2gp^i}\oplus M_{4gp^i}\oplus M_{\lambda gp^i}\oplus M_{2\lambda gp^i}\oplus M_{4\lambda gp^i}\oplus M_{\mu gp^i}\oplus M_{2\mu gp^i}\oplus M_{\nu gp^i}\oplus M_{2\nu gp^i}\oplus M_{\eta gp^i}\oplus M_{\xi gp^i}\oplus M_{\rho gp^i}\oplus M_{\chi gp^i}$ *are* $(x^{p^{n-i-1}}+1)(x^{p^{n-i}}-1)(x^{2p^{n-i}}+1)(x^{4p^{n-i}}+1)(x^{8p^{n-i}}+1)(1+x^{16p^{n-i}}+...+x^{16p^{n-i}(p^i-1)})$, $(x^{p^{n-i-1}}-1)(x^{p^{n-i}}+1)(x^{2p^{n-i}}+1)(x^{4p^{n-i}}+1)(x^{8p^{n-i}}+1)(x^{8p^{n-i}}+1)(1+x^{16p^{n-i}}+...+x^{16p^{n-i}(p^i-1)})$ *and* $(x^{2p^{n-i-1}}+1)(x^{4p^{n-i-1}}+1)(x^{8p^{n-i-1}}+1)(x^{2p^{n-i}}-1)(1+x^{16p^{n-i}}+...+x^{16p^{n-i}(p^i-1)})$ *respectively.*

*Proof.* $(i)$ The minimal polynomial for $\alpha^{tp^n}$, for $t \in A'$ are $(x-1)$, $(x-\beta)(x-\beta^7)$, $(x-\beta^2)(x+\beta^6)$, $(x-\beta^3)(x-\beta^5)$, $(x^2+1)$, $(x-\beta^6)(x+\beta^2)$, $(x+1)$, $(x+\beta)(x+\beta^7)$ and $(x+\beta^3)(x+\beta^5)$, respectively. The corresponding generating polynomials are $(1+x+x^2+...+x^(16p^n-1))$, $(x^8-1)(x^2+\beta^2)(x^2-\beta^6)(x+\beta)(x+\beta^7)(1+x^{16}+...+x^{16(p^n-1)})$, $(x^8+1)(x^4-1)(x+\beta^2)(x-\beta^6)(1+x^{16}+...+x^{16(p^n-1)})$, $(x^8-1)(x^2+\beta^6)(x+\beta^3)(x^2-\beta^2)(x+\beta^5)(1+x^{16}+...+x^{16(p^n-1)})$, $(x^2-1)(x^4+1)(x^8+1)(1+x^{16}+...+x^{16(p^n-1)})$, $(x^8+1)(x^4-1)(x-\beta^2)(x+\beta^6)(1+x^{16}+...+x^{16(p^n-1)})$, $(x^8+1)(x^4+1)(x^2+1)(x-1)(1+x^{16}+...+x^{16(p^n-1)})$, $(x^8-1)(x^2-\beta^6)(x-\beta^7)(x-\beta)(x^2+\beta^2)(1+x^{16}+...+x^{16(p^n-1)})$, $(x^8-1)(x-\beta^3)(x-\beta^5)(x^2+\beta^6)(x^2-\beta^2)(1+x^{16}+...+x^{16(p^n-1)})$.

$(ii)$ The product of minimal polynomial satisfied by $\alpha^{8p^i}$ and $\alpha^{8gp^i}$ is $(\frac{x^{p^{n-i}}+1}{x^{p^{n-i-1}}+1})$. Therefore, the generating polynomial for $M_{8p^i} \oplus M_{8gp^i}$ is $(x^{p^{n-i-1}}+1)(x^{p^{n-i}}-1)(x^{2p^{n-i}}+1)(x^{4p^{n-i}}+1)(x^{8p^{n-i}}+1)(1+x^{16p^{n-i}}+...+x^{16p^{n-i}(p^i-1)})$. The product of minimal polynomial satisfied by $\alpha^{16p^i}$ and $\alpha^{16gp^i}$ is $(\frac{x^{p^{n-i}}-1}{x^{p^{n-i-1}}-1})$. Therefore, the generating polynomial for $M_{16p^i} \oplus M_{16gp^i}$ is $(x^{p^{n-i-1}}-1)(x^{p^{n-i}}+1)(x^{2p^{n-i}}+1)(x^{4p^{n-i}}+1)(x^{8p^{n-i}}+1)(x^{8p^{n-i}}+1)(1+x^{16p^{n-i}}+...+x^{16p^{n-i}(p^i-1)})$. Also the product of minimal polynomial satisfied by $\alpha^{p^i}$, $\alpha^{2p^i}$, $\alpha^{4p^i}$,..., $\alpha^{\rho g p^i}$, $\alpha^{\chi g p^i}$ is $\frac{(x^{2p^{n-i}}+1)(x^{4p^{n-i}}+1)(x^{8p^{n-i}}+1)}{(x^{2p^{n-i-1}}+1)(x^{4p^{n-i-1}}+1)(x^{8p^{n-i-1}}+1)}$. Therefore, the generating polynomial for $M_{p^i} \oplus M_{2p^i} \oplus M_{4p^i} \oplus M_{\lambda p^i} \oplus M_{2\lambda p^i} \oplus M_{4\lambda p^i} \oplus M_{\mu p^i} \oplus M_{2\mu p^i} \oplus M_{\nu p^i} \oplus M_{2\nu p^i} \oplus M_{\eta p^i} \oplus M_{\xi p^i} \oplus M_{\rho p^i} \oplus M_{\chi p^i} \oplus M_{gp^i} \oplus M_{2gp^i} \oplus M_{4gp^i} \oplus M_{\lambda g p^i} \oplus M_{2\lambda g p^i} \oplus M_{4\lambda g p^i} \oplus M_{\mu g p^i} \oplus M_{2\mu g p^i} \oplus M_{\nu g p^i} \oplus M_{2\nu g p^i} \oplus M_{\eta g p^i} \oplus M_{\xi g p^i} \oplus M_{\rho g p^i} \oplus M_{\chi g p^i}$ is $(x^{2p^{n-i-1}}+1)(x^{4p^{n-i-1}}+1)(x^{8p^{n-i-1}}+1)(x^{2p^{n-i}}-1)(1+x^{16p^{n-i}}+...+x^{16p^{n-i}(p^i-1)})$. □

## 7. Minimum Distance

If $l$ is a cyclic code of length $m$ generated by $g(x)$ and its minimum distance is $d$, then the code $\bar{l}$ of length $mk$ generated by $g(x)(1+x^m+x^{2m}+...+x^{(k-1)m})$ is a repetition code of $l$ repeated $k$ times and its minimum distance is $dk$ by Bakshi. Here, we find the minimum distance of the minimal cyclic code $M_s$ of length $16p^n$, generated by the primitive idempotent $P_s$.

**Theorem 7.1.** *Each the codes $M_{tp^n}$, for $t \in A'$ are of minimum distance $16p^n$. For $0 \leq i \leq n-1$, the minimum distance of the cyclic codes $M_{cp^i}$, $M_{cgp^i}$, $c = \{8, 16\}$ are greater than or equal*

*$32p^i$ and minimum distance for the codes $M_{ap^i}$, $M_{agp^i}$, $a \in A - \{8, 16\}$ are greater than or equal to $16p^i$.*

*Proof.* As the generating polynomial for the code $M_0$ is a polynomial of length $16p^n$, hence its minimum distance is $16p^n$. Also, the generating polynomial for the cyclic code $M_{p^n}$ is $(x^8 - 1)(x^2 + \beta^2)(x+\beta)(x^2 - \beta^6)(x+\beta^7)(1 + x^{16} + ... + x^{16(p^n-1)})$. Which the code repeat for the cyclic code of length 16 with generating polynomial $(x^8 - 1)(x^2+\beta^2)(x+\beta)(x^2-\beta^6)(x+\beta^7)$, repeated $p^n$ times. So its minimum distance is $16p^n$. Similarly, the minimum distance for each of the cyclic codes $M_{tp^n}$, where $2 \leq t \leq 15$ is $16p^n$.

The cyclic codes $M_{8p^i}$ and $M_{8gp^i}$, with genrating polynomial $(x^{p^{n-i-1}} + 1)(x^{p^{n-i}} - 1)(x^{2p^{n-i}} + 1)(x^{4p^{n-i}} + 1)(x^{8p^{n-i}} + 1)(1 + x^{16p^{n-i}} + ... + x^{16p^{n-i}(p^i-1)})$ is a repetition code of the code generated by $(x^{p^{n-i-1}} + 1)(x^{p^{n-i}} - 1)(x^{2p^{n-i}} + 1)(x^{4p^{n-i}} + 1)(x^{8p^{n-i}} + 1)$ of length $16p^{n-i}$ and minimum distance 32 repeated $p^i$ times. Therefore its minimum distance is $32p^i$. The codes corresponding to $M_{8p^i}$ and $M_{8gp^i}$ are the sub codes of the above codes, so their minimum distances are greater than or equal to $32p^i$.

Similarly, the minimum distance for each of the cyclic codes $M_{16p^i}$ and $M_{16gp^i}$ of length $16p^n$ are also greater than or equal to $32p^i$.

The product of generating polynomials for the cyclic codes $M_{ap^i}$, $M_{agp^i}$, $a \in A - \{8, 16\}$ is $(x^{2p^{n-i-1}} + 1)(x^{4p^{n-i-1}} + 1)(x^{8p^{n-i-1}} + 1)(x^{2p^{n-i}} + 1)(1 + x^{16p^{n-i}} + ... + x^{16p^{n-i}(p^i-1)})$. If we consider a code $C$ of length $16p^{n-i}$ generated by the polynomial $(x^{2p^{n-i-1}} + 1)(x^{4p^{n-i-1}} + 1)(x^{8p^{n-i-1}} + 1)(x^{2p^{n-i}} + 1)$, then the minimum distance for this code is 16. Since the cyclic code of length $16p^n$ generated by the said polynomial is a repetition of the code $C$ repeated $p^i$ times. Hence its minimum distance is $16p^i$.

Since the codes corresponding to $\Omega_{ap^i}$, $\Omega_{agp^i}$, $a \in A$ are the sub codes of the said codes, so their minimum distance is greater than or equal to $16p^i$. □

## 8. EXAMPLE

**Example 6.1.** Cyclic Codes of length 48.

Take $p = 3$, $n = 1$, $q = 103$. Then the q-cyclotomic cosets are

$\Omega_s = \{s, r\}$ where $7s \equiv r(mod 48)$ and $0 \leq s \leq 41$.

Minimal polynomials for $\alpha^s, \alpha^{7s}$ where $0 \le s \le 41$ are $x-1$, $x^2-2x+47$, $x^2+90x+46$, $x^2+68x-1$, $x^2-77x+56$, $x^2-54x+57$, $x^2-94x+1$, $x-47$, $x^2-94x-1$, $x^2+7x+56$, $x^2+3x+57$, $x^2-14x+1$, $x^2-66x+47$, $x-46$, $x^2-92x+57$, $x^2+20x+1$, $x^2+38x+47$, $x^2-40x+46$, $x+1$, $x^2+2x+47$, $x^2-90x+46$, $x^2-68x-1$, $x-56$, $x^2-9x-1$, $x^2+96x+56$, $x-57$ and $x^2-11x+56$ respectively.

The minimal codes $M_0$, $M_1$, $M_2$, $M_3$, $M_4$, $M_5$, $M_6$, $M_7$, $M_8$, $M_9$, $M_{10}$, $M_{11}$, $M_{12}$, $M_{13}$, $M_{16}$, $M_{17}$, $M_{18}$, $M_{19}$, $M_{20}$, $M_{24}$, $M_{25}$, $M_{26}$, $M_{27}$, $M_{32}$, $M_{33}$, $M_{34}$, $M_{40}$ and $M_{41}$ of length 48 are as follows:

| Code | Dim. | Min. Distance Bound | Generating Polynomial |
|---|---|---|---|
| $M_0$ | 1 | 48 | $\sum_{t=0}^{47}\{x^t\}$ |
| $M_1$ | 1 | $16 \le d \le 48$ | $6+46x+61x^2+6x^3+91x^4+41x^5+2x^6+54x^7+68x^8+39x^9+55x^{10}+30x^{11}+79x^{12}+86x^{13}+48x^{14}+55x^{15}+32x^{16}+101x^{17}+8x^{18}+99x^{19}+15x^{20}+84x^{21}+69x^{22}+91x^{23}+55x^{24}+53x^{25}+23x^{26}+13x^{27}+68x^{28}+7x^{29}+12x^{30}+42x^{31}+87x^{32}+5x^{33}+40x^{34}+52x^{35}+43x^{36}+84x^{37}+18x^{38}+45x^{39}+87x^{40}+40x^{41}+13x^{42}+26x^{43}+60x^{44}+2x^{45}+x^{46}$ |
| $M_2$ | 1 | $16 \le d \le 48$ | $42+14x+12x^2+21x^3+93x^4+93x^5+78x^6+76x^7+78x^8+101x^9+47x^{10}+29x^{11}+43x^{12}+16x^{13}+73x^{14}+27x^{15}+15x^{16}+35x^{17}+23x^{18}+55x^{19}+24x^{20}+75x^{21}+5x^{22}+58x^{23}+23x^{24}+3x^{25}+72x^{26}+27x^{27}+71x^{28}+15x^{29}+43x^{30}+79x^{31}+102x^{32}+101x^{33}+42x^{34}+97x^{35}+78x^{36}+58x^{37}+55x^{38}+21x^{39}+54x^{40}+26x^{41}+42x^{42}+74x^{43}+20x^{44}+13x^{45}+x^{46}$ |

| Code | Dim. | Min. Distance Bound | Generating Polynomial |
|---|---|---|---|
| $M_3$ | 1 | 48 | $86+9x+80x^2+93x^3+18x^4+81x^5+67x^6+2x^7+100x^8+4x^9+63x^{10}+65x^{11}+54x^{12}+29x^{13}+69x^{14}+86x^{15}+46x^{16}+21x^{17}+32x^{18}+34x^{19}+78x^{20}+85x^{21}+90x^{22}+25x^{23}+39x^{24}+102x^{25}+74x^{26}+87x^{27}+16x^{28}+42x^{29}+91x^{30}+50x^{31}+92x^{32}+23x^{33}+8x^{34}+52x^{35}+42x^{36}+24x^{37}+26x^{38}+41x^{39}+33x^{40}+19x^{41}+89x^{42}+97x^{43}+93x^{44}+35x^{45}+x^{46}$ |
| $M_4$ | 1 | $16 \leq d \leq 48$ | $51+45x+72x^2+89x^3+42x^4+58x^5+79x^6+80x^7+81x^8+75x^9+98x^{10}+58x^{11}+78x^{12}+40x^{13}+72x^{14}+10x^{15}+75x^{16}+68x^{17}+94x^{18}+14x^{19}+47x^{20}+x^{22}+40x^{23}+9x^{24}+65x^{25}+23x^{26}+93x^{27}+87x^{28}+26x^{29}+25x^{30}+10x^{31}+74x^{32}+28x^{33}+85x^{34}+52x^{35}+24x^{36}+10x^{37}+17x^{38}+14x^{39}+87x^{40}+55x^{41}+52x^{42}+65x^{43}+2x^{44}+77x^{45}+x^{46}$ |
| $M_5$ | 1 | $16 \leq d \leq 48$ | $61+37x+90x^2+81x^3+86x^4+15x^5+38x^6+52x^7+101x^8+102x^9+28x^{10}+41x^{11}+51x^{12}+100x^{13}+83x^{14}+57x^{15}+67x^{16}+95x^{17}+31x^{18}+53x^{19}+84x^{20}+66x^{21}+99x^{22}+33x^{23}+100x^{24}+2x^{25}+67x^{26}+2x^{27}+73x^{28}+89x^{29}+74x^{30}+83x^{31}+43x^{32}+70x^{33}+24x^{34}+45x^{35}+91x^{36}+80x^{37}+76x^{38}+20x^{39}+14x^{40}+87x^{41}+37x^{42}+x^{43}+78x^{44}+54x^{45}+x^{46}$ |

| Code | Dim. | Min. Distance Bound | Generating Polynomial |
|---|---|---|---|
| $M_6$ | 1 | 48 | $36 + 78x + 86x^2 + 75x^3 + 63x^4 + 79x^5 + 50x^6 + 89x^7 + 76x^8 + 51x^9 + 83x^{10} + 26x^{11} + 95x^{12} + 46x^{13} + 6x^{14} + 3x^{15} + 70x^{16} + 88x^{17} + 65x^{18} + 48x^{19} + 18x^{20} + 99x^{21} + 18x^{22} + 48x^{23} + 65x^{24} + 88x^{25} + 70x^{26} + 3x^{27} + 6x^{28} + 46x^{29} + 95x^{30} + 26x^{31} + 83x^{32} + 51x^{33} + 76x^{34} + 89x^{35} + 50x^{36} + 79x^{37} + 63x^{38} + 75x^{39} + 86x^{40} + 78x^{41} + 36x^{42} + 10x^{43} + 80x^{44} + 94x^{45} + x^{46}$ |
| $M_9$ | 1 | 48 | $27 + 57x + 25x^2 + 76x^3 + 91x^4 + 71x^5 + 9x^6 + 49x^7 + 38x^8 + 82x^9 + 55x^{10} + 62x^{11} + 98x^{12} + 17x^{13} + 45x^{14} + 10x^{15} + 32x^{16} + 92x^{17} + 36x^{18} + 4x^{19} + 72x^{20} + 34x^{21} + 69x^{22} + 37x^{23} + 93x^{24} + 50x^{25} + 28x^{26} + 96x^{27} + 68x^{28} + 90x^{29} + 54x^{30} + 61x^{31} + 88x^{32} + 29x^{33} + 40x^{34} + 80x^{35} + 39x^{36} + 19x^{37} + 4x^{38} + 55x^{39} + 87x^{40} + 14x^{41} + 7x^{42} + 77x^{43} + 82x^{44} + 94x^{45} + x^{46}$ |
| $M_{10}$ | 1 | $16 \leq d \leq 48$ | $86 + 89x + 37x^2 + 60x^3 + 93x^4 + 48x^5 + 42x^6 + 27x^7 + 86x^8 + 9x^9 + 47x^{10} + 5x^{1}1 + 39x^{1}2 + 87x^{1}3 + 62x^{1}4 + 33x^{1}5 + 15x^{1}6 + 38x^{1}7 + 52x^{1}8 + 48x^{1}9 + 74x^{2}0 + 23x^{2}1 + 5x^{2}2 + 10x^{2}3 + 52x^{2}4 + 100x^{2}5 + 16x^{2}6 + 33x^{2}7 + 71x^{2}8 + 31x^{2}9 + 39x^{3}0 + 24x^{3}1 + 57x^{3}2 + 9x^{3}3 + 42x^{3}4 + 70x^{3}5 + 42x^{3}6 + 45x^{3}7 + 58x^{3}8 + 60x^{3}9 + 54x^{4}0 + 40x^{4}1 + 86x^{4}2 + 29x^{4}3 + 96x^{4}4 + 96x^{4}5 + x^{4}6$ |

| Code | Dim. | Min. Distance Bound | Generating Polynomial |
|---|---|---|---|
| $M_{11}$ | 1 | $16 \leq d \leq 48$ | $78+94x+75x^2+45x^3+18x^4+85x^5+44x^6+101x^7+68x^8+85x^9+63x^{10}+100x^{11}+37x^{12}+74x^{13}+84x^{14}+25x^{15}+46x^{16}+64x^{17}+41x^{18}+69x^{19}+86x^{20}+81x^{21}+90x^{22}+86x^{23}+21x^{24}+x^{25}+5x^{26}+72x^{27}+16x^{28}+25x^{29}+49x^{30}+53x^{31}+9x^{32}+51x^{33}+8x^{34}+80x^{35}+86x^{36}+79x^{37}+63x^{38}+73x^{39}+33x^{40}+53x^{41}+40x^{42}+6x^{43}+55x^{44}+100x^{45}+x^{46}$ |
| $M_{12}$ | 1 | 48 | $75+58x+16x^2+63x^3+42x^4+10x^5+98x^6+23x^7+18x^8+23x^9+98x^{10}+10x^{11}+42x^{12}+63x^{13}+16x^{14}+58x^{15}+75x^{16}+65x^{17}+11x^{18}+89x^{19}+102x^{20}+x^{22}+14x^{23}+92x^{24}+38x^{25}+28x^{26}+45x^{27}+87x^{28}+40x^{29}+61x^{30}+93x^{31}+5x^{32}+80x^{33}+85x^{34}+80x^{35}+5x^{36}+93x^{37}+61x^{38}+40x^{39}+87x^{40}+45x^{41}+28x^{42}+38x^{43}+92x^{44}+14x^{45}+x^{46}$ |
| $M_{13}$ | 1 | $16 \leq d \leq 48$ | $17+66x+20x^2+99x^3+86x^4+31x^5+68x^6+51x^7+11x^8+56x^9+28x^{10}+71x^{11}+75x^{12}+3x^{13}+7x^{14}+x^{15}+67x^{16}+59x^{17}+88x^{18}+50x^{19}+53x^{20}+12x^{21}+99x^{22}+27x^{23}+38x^{24}+101x^{25}+95x^{26}+94x^{27}+73x^{28}+26x^{29}+24x^{30}+20x^{31}+21x^{32}+97x^{33}+24x^{34}+93x^{35}+49x^{36}+23x^{37}+97x^{38}+13x^{39}+14x^{40}+15x^{41}+12x^{42}+102x^{43}+86x^{44}+66x^{45}+x^{46}$ |

| Code | Dim. | Min. Distance Bound | Generating Polynomial |
|---|---|---|---|
| $M_{17}$ | 1 | $16 \leq d \leq 48$ | $70 + 46x + 17x^2 + 70x^3 + 91x^4 + 30x^5 + 92x^6 + 54x^7 + 100x^8 + 43x^9 + 55x^{10} + 32x^{11} + 29x^{12} + 86x^{13} + 10x^{14} + 58x^{15} + 32x^{16} + 94x^{17} + 59x^{18} + 99x^{19} + 16x^{20} + 53x^{21} + 69x^{22} + 49x^{23} + 58x^{24} + 53x^{25} + 52x^{26} + 83x^{27} + 68x^{28} + 83x^{29} + 37x^{30} + 42x^{31} + 31x^{32} + 24x^{33} + 40x^{34} + 28x^{35} + 21x^{36} + 84x^{37} + 81x^{38} + 10x^{39} + 87x^{40} + 77x^{41} + 83x^{42} + 26x^{43} + 64x^{44} + 92x^{45} + x^{46}$ |
| $M_{18}$ | 1 | 48 | $78 + 14x + 54x^2 + 39x^3 + 93x^4 + 58x^5 + 86x^6 + 76x^7 + 42x^8 + 11x^9 + 47x^{10} + 79x^{11} + 21x^{12} + 16x^{13} + 71x^{14} + 6x^{15} + 15x^{16} + 3x^{17} + 28x^{18} + 55x^{19} + 5x^{20} + 51x^{21} + 5x^{22} + 55x^{23} + 28x^{24} + 3x^{25} + 15x^{26} + 6x^{27} + 71x^{28} + 16x^{29} + 21x^{30} + 79x^{31} + 47x^{32} + 11x^{33} + 42x^{34} + 76x^{35} + 86x^{36} + 58x^{37} + 93x^{38} + 39x^{39} + 54x^{40} + 14x^{41} + 78x^{42} + 74x^{43} + 90x^{44} + 83x^{45} + x^{46}$ |
| $M_{19}$ | 1 | $16 \leq d \leq 48$ | $42 + 9x + 51x^2 + 55x^3 + 18x^4 + 4x^5 + 95x^6 + 2x^7 + 38x^8 + 81x^9 + 63x^{10} + 35x^{11} + 12x^{12} + 29x^{13} + 53x^{14} + 42x^{15} + 46x^{16} + 43x^{17} + 30x^{18} + 34x^{19} + 42x^{20} + 99x^{21} + 90x^{22} + 61x^{23} + 43x^{24} + 102x^{25} + 24x^{26} + 88x^{27} + 16x^{28} + 86x^{29} + 66x^{30} + 50x^{31} + 2x^{32} + 28x^{33} + 8x^{34} + 28x^{35} + 78x^{36} + 24x^{37} + 14x^{38} + 32x^{39} + 33x^{40} + 34x^{41} + 77x^{42} + 97x^{43} + 58x^{44} + 65x^{45} + x^{46}$ |

| Code | Dim. | Min. Distance Bound | Generating Polynomial |
|---|---|---|---|
| $M_{20}$ | 1 | $16 \leq d \leq 48$ | $80+45x+15x^2+77x^3+42x^4+55x^5+29x^6+80x^7+4x^8+51x^9+98x^{10}+55x^{11}+86x^{12}+40x^{13}+15x^{14}+48x^{15}+75x^{16}+100x^{17}+101x^{18}+14x^{19}+57x^{20}+x^{22}+77x^{23}+2x^{24}+65x^{25}+52x^{26}+55x^{27}+87x^{28}+14x^{29}+17x^{30}+10x^{31}+24x^{32}+52x^{33}+85x^{34}+28x^{35}+74x^{36}+10x^{37}+25x^{38}+26x^{39}+87x^{40}+93x^{41}+23x^{42}+65x^{43}+9x^{44}+40x^{45}+x^{46}$ |
| $M_{24}$ | 1 | 48 | $\sum_{n=1}^{48}\{102^n x^{n-1}\}(mod 103)$ |
| $M_{25}$ | 1 | $16 \leq d \leq 48$ | $6+57x+61x^2+97x^3+91x^4+62x^5+2x^6+49x^7+68x^8+64x^9+55x^{10}+73x^{11}+79x^{12}+17x^{13}+48x^{14}+48x^{15}+32x^{16}+2x^{17}+8x^{18}+4x^{19}+15x^{20}+19x^{21}+69x^{22}+12x^{23}+55x^{24}+50x^{25}+23x^{26}+90x^{27}+68x^{28}+96x^{29}+12x^{30}+61x^{31}+87x^{32}+98x^{33}+40x^{34}+51x^{35}+43x^{36}+19x^{37}+18x^{38}+58x^{39}+87x^{40}+63x^{41}+13x^{42}+77x^{43}+60x^{44}+101x^{45}+x^{46}$ |
| $M_{26}$ | 1 | $16 \leq d \leq 48$ | $42+89x+12x^2+82x^3+93x^4+10x^5+78x^6+27x^7+78x^8+2x^9+47x^{10}+74x^{11}+43x^{12}+87x^{13}+73x^{14}+76x^{15}+15x^{16}+68x^{17}+23x^{18}+48x^{19}+24x^{20}+28x^{21}+5x^{22}+45x^{23}+23x^{24}+100x^{25}+72x^{26}+76x^{27}+71x^{28}+88x^{29}+43x^{30}+24x^{31}+102x^{32}+2x^{33}+42x^{34}+6x^{35}+78x^{36}+45x^{37}+55x^{38}+82x^{39}+54x^{40}+77x^{41}+42x^{42}+29x^{43}+20x^{44}+90x^{45}+x^{46}$ |

| Code | Dim. | Min. Distance Bound | Generating Polynomial |
|---|---|---|---|
| $M_{27}$ | 1 | 48 | $86+94x+80x^2+10x^3+18x^4+22x^5+67x^6+101x^7+100x^8+99x^9+63x^{10}+38x^{11}+54x^{12}+74x^{13}+69x^{14}+17x^{15}+46x^{16}+82x^{17}+32x^{18}+69x^{19}+78x^{20}+18x^{21}+90x^{22}+78x^{23}+39x^{24}+x^{25}+74x^{26}+16x^{27}+16x^{28}+61x^{29}+91x^{30}+53x^{31}+92x^{32}+80x^{33}+8x^{34}+51x^{35}+42x^{36}+79x^{37}+26x^{38}+62x^{39}+33x^{40}+84x^{41}+89x^{42}+6x^{43}+93x^{44}+68x^{45}+x^{46}$ |
| $M_{33}$ | 1 | 48 | $27+46x+25x^2+27x^3+91x^4+32x^5+9x^6+54x^7+38x^8+21x^9+55x^{10}+41x^{11}+98x^{12}+86x^{13}+45x^{14}+93x^{15}+32x^{16}+11x^{17}+36x^{18}+99x^{19}+72x^{20}+69x^{21}+69x^{22}+66x^{23}+93x^{24}+53x^{25}+28x^{26}+7x^{27}+68x^{28}+13x^{29}+54x^{30}+42x^{31}+88x^{32}+74x^{33}+40x^{34}+23x^{35}+39x^{36}+84x^{37}+4x^{38}+48x^{39}+87x^{40}+89x^{41}+7x^{42}+26x^{43}+82x^{44}+9x^{45}+x^{46}$ |
| $M_{34}$ | 1 | $16\le d\le 48$ | $86+14x+37x^2+43x^3+93x^4+55x^5+42x^6+76x^7+86x^8+94x^9+47x^{10}+98x^{11}+39x^{12}+16x^{13}+62x^{14}+70x^{15}+15x^{16}+65x^{17}+52x^{18}+55x^{19}+74x^{20}+80x^{21}+5x^{22}+93x^{23}+52x^{24}+3x^{25}+16x^{26}+70x^{27}+71x^{28}+72x^{29}+39x^{30}+79x^{31}+57x^{32}+94x^{33}+42x^{34}+33x^{35}+42x^{36}+58x^{37}+58x^{38}+43x^{39}+54x^{40}+63x^{41}+86x^{42}+74x^{43}+96x^{44}+7x^{45}+x^{46}$ |

| Code | Dim. | Min. Distance Bound | Generating Polynomial |
| --- | --- | --- | --- |
| $M_{41}$ | 1 | $16 \leq d \leq 48$ | $102+57x+48x^2+36x^3+43x^4+17x^5+32x^6+63x^7+21x^8+3x^9+37x^{10}+44x^{11}+65x^{12}+69x^{13}+97x^{14}+73x^{15}+31x^{16}+71x^{17}+98x^{18}+75x^{19}+70x^{20}+40x^{21}+25x^{22}+98x^{23}+28x^{24}+81x^{25}+43x^{26}+7x^{27}+19x^{28}+22x^{29}+61x^{30}+87x^{31}+16x^{32}+77x^{33}+13x^{34}+49x^{35}+94x^{36}+93x^{37}+92x^{38}+44x^{39}+7x^{40}+76x^{41}+24x^{42}+99x^{43}+65x^{44}+11x^{45}+x^{46}$ |
| $M_a$ | 1 | $32 \leq d \leq 48$ | $\sum_{n=1}^{48}\{57^{rn}x^{n-1}\}(mod 103)$ where $r=\frac{a}{8}$ and $a \in \{8,16,32,40\}$ |

## Conflict of Interests

The author(s) declare that there is no conflict of interests.

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