Available online at http://scik.org

J. Math. Comput. Sci. 2 (2012), No. 6, 1700-1711

ISSN: 1927-5307

SOFT MULTISETS

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Abstract: In this paper the notion of soft multisets are redefined using soft count functions and some operations on them have been introduced. The notion of distance and similarity between two soft multisets has also been defined. An application of soft multisets has also been shown.

Keywords: soft set; soft multisets; distance; similarity measure.

2000 AMS Subject Classification: 03E70; 03E75; 03E99

1. Introduction

Multisets are an important generalization of classical set theory which has emerged by violating a basic property of classical sets that an element can belong to a set only once. The term "multiset" has been coined by N.G. Bruijn [5]. It is a 'set' where an element can occur more than once. Multisets are very useful structures arising in many areas of mathematics and computer science such as database queries. Several generalizations of multisets like fuzzy multisets [11, 12, 17] are also popular. One of the important applications of fuzzy multisets is in information retrieval on the web, since an information item may appear more than once with possibly different degrees of relevance to a query. Again the theory of soft sets was initiated by D. Molodtsov [13] in 1999 for modeling uncertainty present in real life. Roughly speaking, a soft set is a parametrized classification of the objects of the universe. He has shown several applications of soft sets in different areas like integration, game theory, decision making

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Received June 14, 2012

etc. Later Maji et al. [6] defined several operations on soft sets. Perhaps this is the only theory available with parameterization tool for modeling uncertainty. H. Aktas & N. Cagman [1] has shown that fuzzy sets and rough sets are special cases of soft sets. Many authors [3, 7, 10, 16] have combined soft sets with other sets to generate hybrid structures like fuzzy soft sets, rough soft sets, intuitionistic fuzzy soft sets, generalised fuzzy soft sets etc. Application of soft sets in many areas like decision making, medical diagnosis, texture classification, data analysis, forecasting etc. has been shown by many researchers [4, 8, 9, 14, 15, 18] in recent years. In this paper a new hybrid set has been introduced by combining soft sets and multisets which are denoted by soft multisets (SMS) and some properties of it under several operations are studied. Also a distance based measure of similarity among two soft multisets has been introduced and its properties studied. An application of soft multisets in student evaluation has been given.

2. Preliminaries

In this section some definitions and results regarding soft sets and multisets are stated.

Definition 2.1 [6] Let U be an initial universal set and E be a set of parameters. Let $P(U)$ denote the power set of U. A pair (F, A) is called a soft set over U iff F is a mapping given by $F: A \rightarrow P(U)$, *where* $A \subset E$.

An example of a soft set is given below:

Example 2.2 As an illustration, consider the following example. Suppose a soft set (F, A) describes attractiveness of the shirts which the authors are going to wear. Here U = the set of all shirts under consideration = $\{x_1, x_2, x_3, x_4, x_5\}$ & A = {colorful, bright, cheap, warm} $= \{e_1, e_2, e_3, e_4\}$.

Let $F(e_1) = \{x_1, x_2\}$, $F(e_2) = \{x_1, x_2, x_3\}$, $F(e_3) = \{x_4\}$, $F(e_4) = \{x_2, x_5\}$.

So, the soft set (F, A) is a subfamily $\{F(e_i), i = 1,2,3,4\}$ of $P(U)$, which represents the attractiveness of shirts w. r. t the parameters given.

The notion of subset, equality, union and intersection and complement defined by maji et. al. were as follows:

Definition 2.3 [6] For two soft sets (F,A) and (G,B) over a common universe U, we say that (F,A) is a soft subset of (G,B) if $(i) A\subset B,\ (ii)\forall\varepsilon\in A, F(\varepsilon)$ is a subset of $G(\varepsilon)$.

Definition 2.4 [6] (Equality of two soft sets) Two soft sets (F,A) and (G,B) over a common universe U are said to be soft equal if (F,A) is a soft subset of (G,B) and (G,B) is a soft subset of (F,A).

Definition 2.5 [6] Union of two soft sets (F,A) and (G,B) over a common universe U is the soft set (H,C), where $C = A\cup B,$ and $\forall e\in C,$

$$H(e)=F(e),\ e\in A-B,\ \ =G(e), e\in B-A,\ =F(e)\cup G(e), e\in A\cap B.$$

This is denoted by $(F,A)\ \tilde{\bigcup}\ (G,B)$.

Definition 2.6 [6] Intersection of two soft sets (F,A) and (G,B) over a common universe U is the soft set (H,C), where $C = A\cap B$ and $\forall e\in C,\ H(e)=F(e)\ \cap\ G(e).$ This is denoted by $(F,A)\ \tilde{\bigcap}\ (G,B)$.

Definition 2.7 [9] The complement of a soft set (F,A) is denoted by $(F,A)^c$ and is defined by $(F,A)^c=(F^c,A),$ where $F^c: A\to P(U)$ is a mapping given by $F^c(\alpha)=U-F(\alpha), \forall\alpha\in A.$

Definition 2.8 [12] Let $U=\{x_1,x_2,.....,x_n\}$ is the universal set. A crisp bag or multiset $M\ of\ U$ is characterized by a function $C_M(.)$ (called count of M) whereby a natural number including zero $N=\{0,1,2,...\}$ corresponds to each $x\in U(C_M: X\to N).$ It is expressed as

$$M=\{\frac{k_1}{x_1},\frac{k_2}{x_2},.....,\frac{k_n}{x_n}\}\ s.t.\ x_i\ appears\ k_i\ times$$
$$in\ M\ (i=1,2,...,n).$$

3. Soft Multisets

The idea of soft multisets was first introduced in [2]. Here a new definition of soft multisets is given and some operations on them are defined.

Let U be the universal set and E be the set of parameters and J be the set of all non-negative integers. Let $P^*(U)$ be the collection of all crisp multisets defined in U. Also let $A \subset E$.

Definition 3.1 A soft multiset is a triple $< F, A, C_F >$, characterized by its soft count function $C_F : A \to J^U$ and which is defined as follows: $C_F(e) = c_F^e \in J^U$, where $c_F^e : U \to J$ is the parametrized count function. And $F : A \to P^*(U)$ is defined such that corresponding to each $e \in E$, every element x occurs exactly $c_F^e(x)$ times in $F(e)$.

For sake of simplicity, we assume the universal set and the parameter set to be finite.

Example 3.2 Let the universal set and the parameter set be as follows: Let $U = \{x, y, z, w\}$ & $E = \{p, q, r\}$. Define a mapping $F : E \to P^*(U)$ as follows:

$$F(p) = \{\frac{2}{x}, \frac{3}{y}, \frac{1}{z}, \frac{4}{w}\}, F(q) = \{\frac{4}{x}, \frac{4}{y}, \frac{5}{w}\}$$
$$\& \ F(r) = \{\frac{2}{x}, \frac{1}{y}, \frac{1}{z}\}.$$

Then $< F, E, C_F >$ is a soft multiset where the soft count function C_F is given by the parametrized count functions $c_F^p, c_F^q, c_F^r : U \to J$, which are defined as follows:

$$c_F^p(x) = 2, c_F^p(y) = 3, c_F^p(z) = 1, c_F^p(w) = 4;$$
$$c_F^q(x) = 4, c_F^q(y) = 4, c_F^q(z) = 0, c_F^q(w) = 5$$
$$\& c_F^r(x) = 2, c_F^r(y) = 1, c_F^r(z) = 1, c_F^r(w) = 0.$$

The following are some basic relations and operations for soft multisets and they are given in terms of the parametrized count function.

Definition 3.3 For any two soft multisets $< F, A, C_F >$ and $< G, B, C_G >,$ the following operations are defined as follows:

(A) (Inclusion):

$$< F, A, C_F > \subseteq < G, B, C_G > \Leftrightarrow (i) A \subseteq B \,\&$$
$$(ii) c_F^e(x) \le c_G^e(x) \forall x \in U, e \in A \cap B.$$

(B) (Equality):

$$< F, A, C_F > = < G, B, C_G > \Leftrightarrow (i) A = B \,\&$$
$$(ii) c_F^e(x) = c_G^e(x) \forall x \in U, e \in A = B.$$

$$i.e. \quad < F, A, C_F > = < G, B, C_G > \Leftrightarrow$$
$$(i) < F, A, C_F > \subseteq < G, B, C_G > \&$$
$$(ii) < F, A, C_F > \supseteq < G, B, C_G >.$$

(C) (Union):

$$< F, A, C_F > \cup < G, B, C_G > = < H, A \cup B, C_H >,$$
$$where\ C_H(e) = c_H^e \ \& \ c_H^e(x) = c_F^e(x) \vee c_G^e(x),$$
$$\forall e \in A \cup B, x \in U.$$

(D) (Intersection):

$$< F, A, C_F > \bigcap < G, B, C_G > = < H, A \cap B, C_H >,$$
$$where\ C_H(e) = c_H^e \ \& \ c_H^e(x) = c_F^e(x) \wedge c_G^e(x),$$
$$\forall e \in A \cap B, x \in U.$$

(E) (Restricted Sum):

$$< F, A, C_F > \oplus < G, B, C_G > = < H, A \cap B, C_H >,$$
$$where\ C_H(e) = c_H^e \ \& \ c_H^e(x) = c_F^e(x) + c_G^e(x),$$
$$\forall e \in A \cap B, x \in U.$$

(F) (Cartesian product):

$$< F, A, C_F > \otimes < G, B, C_G > = < H, A \times B, C_H >,$$
$$where\ H : A \times B \to P(U^2)$$

$$s.t. H(\alpha, \beta) = F(\alpha) \times G(\beta)$$
$$\& C_H : A \times B \to J^{A \times B} \ s.t. C_H(e, f) = c_H^{(e,f)}$$

$$where\ c_H^{(e,f)}(x, y) = c_F^e(x).c_G^f(y),$$
$$\forall (e, f) \in A \times B, (x, y) \in F(e) \times G(f).$$

An example of Cartesian product is given below.

Example 3.4 Let the universal set and the parameter set be as follows:

$U=\{x,y\}\ \&\ E=\{p,q\}$.

Define two soft multisets $<F,E,C_F>$ and $<G,E,C_G>$ as follows:

$$F(p)=\{\frac{1}{x},\frac{2}{y}\},F(q)=\{\frac{4}{y}\}\ \&$$
$$G(p)=\{\frac{4}{x},\frac{3}{y}\},G(q)=\{\frac{2}{x},\frac{1}{y}\}.$$

Then their Cartesian product

$<F,E,C_F>\otimes<G,E,C_G>=<H,E^2,C_H>, where\ H:E^2\to P(U^2)$ is a soft multiset where the soft count function C_H is given by the parametrized count functions $c_H^{p\times p},c_H^{p\times q},c_H^{q\times p},c_H^{q\times q}:U^2\to J$, which are defined as follows:

$$c_H^{p\times p}(x,x)=4,c_H^{p\times p}(x,y)=3,c_H^{p\times p}(y,x)=8,$$
$$c_H^{p\times p}(y,y)=6;c_H^{p\times q}(x,x)=2,c_H^{p\times q}(x,y)=1,$$
$$c_H^{p\times q}(y,x)=4,c_H^{p\times q}(y,y)=2;c_H^{q\times p}(x,x)=0,$$
$$c_H^{q\times p}(x,y)=0,c_H^{q\times p}(y,x)=16,c_H^{q\times p}(y,y)=12;$$
$$c_H^{q\times q}(x,x)=0,c_H^{q\times q}(x,y)=0,c_H^{q\times q}(y,x)=8,$$
$$c_H^{q\times q}(y,y)=4.$$

Theorem 3.5 For any three soft multisets $<F,A,C_F>,<G,B,C_G>\ \&<H,C,C_H>,$ the following holds:

1. $C_{F\cup G}=C_{G\cup F}$
2. $C_{F\cap G}=C_{G\cap F}$
3. $C_{(F\cup G)\cup H}=C_{F\cup(G\cup H)}$

4. $C_{(F\cap G)\cap H} = C_{F\cap(G\cap H)}$
5. $C_{F\cup F} = C_F$
6. $C_{F\cap F} = C_F$
7. $C_{F\cup(G\cap H)} = C_{(F\cup G)\cap(F\cup H)}$
8. $C_{F\cap(G\cup H)} = C_{(F\cap G)\cup(F\cap H)}$,

where C_F *is the soft count function of*
$F =< F, A, C_F >$.

Proof. Follows from definition.

Let (F, A) be a soft set over a finite universe U. Then the cardinality of (F, A) is defined as

$$Card(F) = \sum_{e\in A} |F(e)|.$$

Next we define the cardinality of a soft multiset on a finite universe.

Definition 3.6 Let (F, A) be a soft set over U. Then the cardinality of $< F, A, C_F >$ with soft count function C_F, & $C_F(e) = c_e^F, e \in A$ as parameterized count function, its cardinality is defined as

$$Card(F) = \sum_{e\in A}\sum_{x\in U} c_e^F(x).$$

The cardinality of the soft multiset defined in Example 3.2 is 27.

A soft multiset can be expressed in form of a table or as a matrix as follows:

Tabular representation of the SMS defined in Example 3.2

$< F, A, C_F >$	P	q	r
x	2	4	2
y	3	4	1
z	1	0	1
w	4	5	0

Also the matrix representation of the SMS defined in example 3.2 is as follows:

$$< F, A, C_F > = \begin{pmatrix} 2 & 4 & 2 \\ 3 & 4 & 1 \\ 1 & 0 & 1 \\ 4 & 5 & 0 \end{pmatrix}.$$

4. Distance between two soft multisets

In this section we introduce the notion of distance between two soft multisets.

Let χ be the collection of all soft multisets over the soft universe (U, E).

For the sake of simplicity we will henceforth denote a soft multiset

$< F, A, C_F > by\ F.$

Definition 4.1 Let $< F, A, C_F >$ and $< G, B, C_G >$, be two soft multisets. Then the distance between them, is denoted by $d(F, G)$, and is defined by: $d(F,G) = \sum_{e\in E}\sum_{x\in U}\left|c_e^F(x) - c_e^G(x)\right|$, assuming $\forall x \in U, c_e^F(x) = 0\ if\ e \notin A \,\&\, c_e^G(x) = 0\ if\ e \notin B.$ This distance is the Hamming distance between two soft multisets.

Example 4.2 Consider the two soft multisets $< F, A, C_F >$ and $< G, B, C_G >$ given by the following matrices:

$$\begin{pmatrix} 1 & 1 & 1 \\ 3 & 0 & 1 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{pmatrix} and \begin{pmatrix} 1 & 3 & 1 \\ 4 & 1 & 1 \\ 3 & 0 & 0 \\ 2 & 1 & 3 \end{pmatrix}$$

Then the distance between them is given by:

$d(F, G) = 14.$

Proposition 4.3 Let F and G be two soft multisets with finite cardinalities $m \,\&\, n$ respectively. Then $0 \le d(F, G) \le \max\{m, n\}.$

Theorem 4.4 Here (χ, d) is a metric space.

Proof. Let $F, G, H \in \chi.$

(P1) Obviously $d(F,G) \geq 0.$

(P2) Now $d(F,G) = 0$ iff

$c_e^F = c_e^G \ \forall e \in E, x \in U \Leftrightarrow F = G.$

(P3) From the definition of distance it is clear that $d(F,G) = d(G,F).$

(P4) Triangle inequality will also hold as:

$$\left|c_e^F(x) - c_e^H(x)\right| = \left|(c_e^F(x) - c_e^G(x)) + (c_e^G(x) - c_e^H(x))\right|$$
$$\leq \left|c_e^F(x) - c_e^G(x)\right| + \left|c_e^G(x) - c_e^H(x)\right|, \forall x \in U, e \in E$$
$$\Rightarrow d(F,H) \leq d(F,G) + d(G,H) \ \forall F,G,H \in \chi.$$

5. Similarity between two soft multisets

It is very natural to measure the degree of similarity between two soft multisets. The notion of similarity between two soft sets was introduced in [9]. Here we have introduced a measure of similarity based on Hamming distance defined in the earlier section.

Definition 5.1 Let $F,G \in \chi \,\&\, d(F,G)$ be the Hamming distance between two soft multisets, then the similarity measure between them is defined as: $s(F,G) = \dfrac{1}{1+d(F,G)}.$

Example 5.2 Consider the Example 4.2. The similarity measure of the two soft multisets given there will be:

$$s(F,G) = \frac{1}{1+d(F,G)} = \frac{1}{1+14} = \frac{1}{15} \approx 0.07.$$

Now the following properties of this similarity measure follows:

Proposition 5.3 For any two soft multisets

$< F, A, C_F > \& < G, B, C_G >$ the following holds:

(1) $0 \le s(F,G) \le 1$
(2) $s(F,G) = s(G,F)$
(3) $s(F,G) = 1 \Leftrightarrow d(F,G) = 0.$

Proof. Trivially follows from definition.

6. An Application

Soft multisets can also be used in solving decision making problems. Here we have shown a simple application of SMS in student evaluation. Consider the students of a class who gave tests in Mathematics (M), Physics (P), Chemistry (C), Biology (B) and Language (L). In each subject, they are tested for their "knowledge", "skill", "Reasoning ability" and "confidence". Their performance in each category is either 'good' (g) or 'satisfactory' (s) or 'bad' (b). A typical student X has performed as follows:

Table1 Performance table

Subjects	Performance			
	Know -ledge	Skill	Reaso-n ing	Confi- dence
Mathematics	g	s	b	g
Physics	s	s	s	b
Chemistry	s	b	g	g
Biology	g	s	b	s
Language	s	s	b	g

This information can be stored in form of a SMS, where the set of parameters are the subjects $E = \{M, P, C, B, L\}$ & $U = \{g, s, b\}$ is the universal set. The matrix representation of the associated SMS will be

$$X = \begin{pmatrix} 2 & 0 & 2 & 1 & 1 \\ 1 & 3 & 1 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Now theoretically the best performance of a student M is the one where he performs ‘good’ in all categories of each subject. So the corresponding SMS will be as follows:

$$M = \begin{pmatrix} 4 & 4 & 4 & 4 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Now the score of the student X can be determined by the degree of similarity between X & M. Here

$$d(X, M) = 28,$$
$$So\ Score(X) = S(X, M) = \frac{1}{1+28} = \frac{1}{29}.$$

7. Conclusions

In this paper we have given a new notion of soft multisets and defined several operations on them. Distance and similarity of two soft multisets have also been defined. Soft multisets are more generalised structures than crisp or fuzzy multisets due to the presence of parameters. These seem to have natural applications in many areas of mathematics and computer science, such as information retrieval on the web, pattern recognition and image processing, coding theory etc.

Acknowledgements

The present work is partially supported by Faculty Development Program (FDP) of UGC, New Delhi, India [Grant No. F.TF. WB1-010-01/10-11(ERO)].

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