Available online at http://scik.org
J. Math. Comput. Sci. 10 (2020), No. 6, 3074-3089
https://doi.org/10.28919/jmcs/5019
ISSN: 1927-5307

CONVERGENCE RATE OF VARIOUS ITERATIONS WITH SM-ITERATION FOR CONTINUOUS FUNCTION

SAVITA RATHEE[1], MONIKA SWAMI[2,*]

[1,2]Department of Mathematics, Maharshi Dayanand University, Rohtak, 124001, India

Abstract. In the present paper, we propose a new iteration process named as SM-iteration and compare its rate of convergence with the already existing iterations such as CR, S, Abbas, Thakur for different type of functions with the support of an example.

Keywords: Convergence theorem; iteration process

2010 AMS Subject Classification: 47H09, 47H10.

1. Introduction

For a nonempty $\mathbb{K}$ closed and convex subset of normed space $\mathbb{N}$ and $T : \mathbb{K} \to \mathbb{K}$.

In 2007, Agarwal et al. [4] introduced the following iterative process:

$$\begin{cases} a_{n+1} = (1-\alpha_n')Ta_n + \alpha_n' T b_n \\ \quad b_n = (1-\beta_n')a_n + \beta_n' T a_n \end{cases} \qquad S_n$$

where $\{\alpha_n'\}$ and $\{\beta_n'\}$ are in (0, 1).

In 2012, Chugh et al. [3] introduced following three step iteration:

*Corresponding author
E-mail address: monikaswami06@gmail.com
Received September 13, 2020

$$\begin{cases} a_{n+1} = (1-\alpha_n')b_n + \alpha_n' T b_n \\ b_n = (1-\beta_n')Ta_n + \beta_n' T c_n \\ c_n = (1-\gamma_n')a_n + \gamma_n' T a_n \end{cases} \qquad CR_n$$

where $\{\alpha_n'\}, \{\beta_n'\}$ and $\{\gamma_n'\}$ are in [0,1] with $\sum_{n=0}^{\infty} \alpha_n' = \infty$.

In 2014, Abbas and Nazir [2] gave the following three-step iterative process:

$$\begin{cases} a_{n+1} = (1-\alpha_n')Tb_n + \alpha_n' T c_n \\ b_n = (1-\beta_n')Ta_n + \beta_n' T c_n \\ c_n = (1-\gamma_n')a_n + \gamma_n' T a_n \end{cases} \qquad A_n$$

where $\{\alpha_n'\}, \{\beta_n'\}$ and $\{\gamma_n'\}$ are in (0, 1).

In 2016, Thakur et al. [1] introduced three step iterative process defined as follow:

$$\begin{cases} a_{n+1} = Tb_n \\ b_n = T((1-\alpha_n')a_n + \alpha_n' z_n) \\ c_n = (1-\beta_n')a_n + \beta_n' T a_n \end{cases} \qquad TH_n$$

where $\{\alpha_n'\}$ and $\{\beta_n'\}$ are in (0, 1).

We introduce the following three step iteration process:

$$\begin{cases} a_{n+1} = T((1-\alpha_n')Tc_n + \alpha_n' T b_n) \\ b_n = T((1-\beta_n')a_n + \beta_n' c_n) \\ c_n = Ta_n \end{cases} \qquad SM_n$$

where $\{\alpha_n'\}$ and $\{\beta_n'\}$ are in [0, 1].

Definition 1.1. [5] *Suppose $\{a_n\}$ and $\{b_n\}$ be two real sequences with limit a and b, respectively. Then $\{a_n\}$ is said to be converges faster than $\{b_n\}$ if*

$$\lim_{n\to\infty} \left| \frac{a_n - a}{b_n - b} \right| = 0.$$

A new class of operators was introduced by Berinde, in [6] on a normed space $\mathbb{N}$ satisfying

$$||Ta - Tb|| \leq \delta ||a-b|| + L||Ta - a|| \qquad \text{(B)}$$

for $a, b \in \mathbb{N}$ for some $\delta \in [0,1), L \geq 0$.

In this article, motivated from Kumar et al. [7], first we establish a general proof to approximate the fixed point of quasi contractive operator in a Banach space with SM iteration. After that we compare the convergence rate of SM iteration with S, CR, Abbas and Thakur iterations.

2. MAIN RESULTS

Theorem 2.1. *Let $\mathbb{K}$ be a non empty closed convex subset of normed space $\mathbb{N}$. Let $T : \mathbb{K} \to \mathbb{K}$ be a operator satisfying $||Ta - Tb|| \leq \delta||a-b|| + L||Ta-a||$. Let $\{a_n\}$ be defined through iterative process $\{SM_n\}$ and $a_0 \in \mathbb{K}$, where $\{\alpha'_n\}$and$\{\beta'_n\}$ are sequences in [0,1] satisfying $\sum_{n=0}^{\infty}\alpha'_n = \infty$. Then $\{a_n\}$ strongly converges to a fixed point of T.*

Proof As $F(T) \neq \phi$ and $w \in F(T)$ then

$$\begin{aligned}
||a_{n+1} - w||| &= ||T((1-\alpha'_n)Tc_n + \alpha'_n Tb_n) - w|| \\
&\leq \delta||(1-\alpha'_n)Tc_n + \alpha'_n Tb_n - w|| \\
&\leq \delta||(1-\alpha'_n)(Tc_n - w) + \alpha'_n(Tb_n - w)|| \\
&\leq \delta(1-\alpha'_n)||c_n - w|| + \alpha'_n||b_n - w|| \\
&\leq \delta\alpha'_n||b_n - w|| + \delta(1-\alpha'_n)||c_n - w|| \qquad (2.1)
\end{aligned}$$

where

$$\begin{aligned}
||b_n - w|| &= ||T((1-\beta'_n)a_n + \beta'_n c_n) - w|| \\
&\leq \delta||(1-\beta'_n)a_n + \beta'_n c_n - w|| \\
&\leq \delta(1-\beta'_n)||a_n - w|| + \delta\beta'_n||c_n - w|| \qquad (2.2)
\end{aligned}$$

$$||c_n - w|| = ||Ta_n - w|| \leq \delta||a_n - w|| \qquad (2.3)$$

use (2.3) in (2.2), we get

$$||b_n - w|| \leq \delta(1-\beta'_n)||a_n - w|| + \delta^2\beta'_n||a_n - w|| \qquad (2.4)$$

use (2.3), (2.4) in (2.1), we have

$$\begin{aligned}
||a_{n+1} - w|| &\leq \delta^2(1-\alpha'_n)||a_n - w|| + \delta^2\alpha_n(1-\beta'_n)||a_n - w|| + \delta^3\alpha'_n\beta'_n||a_n - w|| \\
&= (\delta^2(1-\alpha'_n) + \delta^2\alpha'_n(1-\beta'_n) + \delta^3\alpha_n\beta'_n)||a_n - w|| \\
&= \delta^2(1-(1-\delta)\alpha'_n\beta'_n)||a_n - w|| \qquad (2.5)
\end{aligned}$$

By (2.5) , through induction , we obtain

$$||a_{n+1} - w|| \leq \prod_{k=0}^{n}\delta^{2k}(1-(1-\delta)\alpha'_k\beta'_k)||a_0 - w||$$

from the fact, $0 \leq \delta < 1, 0 \leq \alpha_n', \beta_n' \leq 1$ and $\sum \alpha_n' \beta_n' = \infty$, we get

$$\lim_{n\to\infty} \prod_{k=0}^{n} \delta^{2k}(1-(1-\delta)\alpha_k'\beta_k') = 0,$$

by which (2.5) implies that $\lim_{n\to\infty} ||a_{n+1} - w|| = 0$.

Therefore, $a_n \to w \in F(T)$ and thus the proof.

Now with an example we compare the convergence of SM iteration process with S, CR, Abbas, Thakur iterations.

Example Let $T : [0,1] \to [0,1] := \frac{x}{2}$. Let $\alpha_n' = \frac{4}{\sqrt{n}} = \beta_n' = \gamma_n'$

As, T is a quasi-contractive operator with 0 its unique fixed point. Also it satisfies all the conditions of Theorem 2.1

Now, for

$$\begin{aligned} CR_n &= (1-\alpha_n')b_n + \alpha_n' T b_n \\ &= (1-\alpha_n')((1-\beta_n')Ta_n + \beta_n' T c_n) + \alpha_n' T((1-\beta_n')Ta_n + \beta_n' T c_n) \end{aligned}$$

where

$$c_n = \left(1 - \frac{4}{\sqrt{n}}\right) a_n + \frac{4}{\sqrt{n}} \frac{a_n}{2} = \left(1 - \frac{2}{\sqrt{n}}\right) a_n$$

and

$$b_n = \left(1 - \frac{4}{\sqrt{n}}\right) \frac{a_n}{2} + \frac{4}{2\sqrt{n}} \left(1 - \frac{2}{\sqrt{n}}\right) a_n = \left(\frac{1}{2} - \frac{4}{n}\right) a_n$$

implies

$$\begin{aligned} a_{n+1} &= \left(1 - \frac{4}{\sqrt{n}}\right)\left(\frac{1}{2} - \frac{4}{n}\right) a_n + \frac{4}{2\sqrt{n}} \left(\frac{1}{2} - \frac{4}{n}\right) a_n = \left(1 - \frac{2}{\sqrt{n}}\right)\left(\frac{1}{2} - \frac{4}{n}\right) a_n \\ &= \dots \\ a_{n+1} &= \prod_{i=16}^{n} \left(\frac{1}{2} - \frac{2}{\sqrt{i}} - \frac{4}{i} + \frac{8}{i\sqrt{i}}\right) \end{aligned}$$

$$\begin{aligned}
S_n &= (1-\alpha_n')\frac{a_n}{2} + \alpha_n'\frac{(1-\beta_n')a_n + \beta_n' T a_n}{2} \\
&= \left(1 - \frac{4}{\sqrt{n}}\right)\frac{a_n}{2} + \frac{4}{2\sqrt{n}}\left(1 - \frac{4}{2\sqrt{n}}\right)a_n \\
&= \left(\frac{1}{2} - \frac{4}{\sqrt{n}}\right)a_n \\
&= \ldots \\
&= \prod_{i=16}^{n}\left(\frac{1}{2} - \frac{4}{\sqrt{i}}\right)a_0 \\
A_n &= (1-\alpha_n')\left(1 - \frac{2}{\sqrt{n}}\right)\frac{a_n}{2} + \alpha_n'\left(\frac{1}{2} - \frac{4}{n}\right)\frac{a_n}{2} \\
&= \left(\frac{1}{2} - \frac{2}{\sqrt{n}} + \frac{4}{n} - \frac{8}{n\sqrt{n}}\right)a_n \\
&= \ldots \\
&= \prod_{i=16}^{n}\left(\frac{1}{2} - \frac{2}{\sqrt{i}} + \frac{4}{n} - \frac{8}{i\sqrt{i}}\right)a_0 \\
TH_n &= \frac{1}{2}\left(\frac{1}{2} - \frac{4}{n}\right)a_n \\
&= \left(\frac{1}{4} - \frac{2}{n}\right)a_n \\
&= \ldots \\
&= \prod_{i=16}^{n}\left(\frac{1}{4} - \frac{2}{i}\right)a_0
\end{aligned}$$

and

$$\begin{aligned}
SM_n &= \frac{(1-\alpha_n')Tc_n + \alpha_n' T b_n}{2} \\
&= \left(\frac{1}{8} - \frac{1}{n}\right)a_n \\
&= \ldots \\
&= \prod_{i=16}^{n}\left(\frac{1}{8} - \frac{1}{i}\right)a_0
\end{aligned}$$

Now, for $n \geq 16$, consider

$$\begin{aligned}\left|\frac{SM_n - 0}{S_n - 0}\right| &= \left|\frac{\prod_{i=16}^{n}(\frac{1}{8} - \frac{1}{i})a_0}{\prod_{i=16}^{n}(\frac{1}{2} - \frac{4}{\sqrt{i}})a_0}\right| \\ &= \prod_{i=16}^{n}\left|\frac{i-8}{4(i-8\sqrt{n})}\right| \\ &= \prod_{i=16}^{n}\left|\left(1-\frac{1}{i}\right)\right|\end{aligned}$$

implies

$$\lim_{n\to\infty}\left|\frac{SM_n - 0}{S_n - 0}\right| = \lim_{n\to\infty}\frac{15}{n} \to 0$$

Thus

$$\lim_{n\to\infty}\left|\frac{SM_n - 0}{S_n - 0}\right| = 0.$$

Similarly,

$$\begin{aligned}\left|\frac{SM_n - 0}{CR_n - 0}\right| &= \left|\frac{\prod_{i=16}^{n}(\frac{1}{8} - \frac{1}{i})a_0}{\prod_{i=16}^{n}(\frac{1}{2} - \frac{2}{\sqrt{i}} - \frac{4}{i} + \frac{8}{i\sqrt{i}})a_0}\right| \\ &= \prod_{i=16}^{n}\left|1 - \frac{\frac{7}{8} + \frac{1}{i} - \frac{4}{\sqrt{i}} - \frac{8}{i\sqrt{i}}}{1 - \frac{4}{\sqrt{i}} - \frac{8}{i\sqrt{i}}}\right| \\ &= \prod_{i=16}^{n}\left|1 - \frac{7i\sqrt{i} + 32\sqrt{i} - 2i - 4}{i\sqrt{i} - 4i - 8}\right| \\ &\leq \prod_{i=16}^{n}\left(i - \frac{1}{i}\right)\end{aligned}$$

implies

$$\begin{aligned}&\lim_{n\to\infty}\left|\frac{SM_n - 0}{CR_n - 0}\right| \\ &= \lim_{n\to\infty}\frac{15}{n} \\ &= 0\end{aligned}$$

Hence,

$$\lim_{n\to\infty}\left|\frac{SM_n-0}{CR_n-0}\right|=0$$

Again, Let $n\geq 16$. Then

$$\begin{aligned}\left|\frac{SM_n-0}{A_n-0}\right| &= \left|\frac{\prod_{i=16}^{n}(\frac{1}{8}-\frac{1}{i})a_0}{\prod_{i=16}^{n}(\frac{1}{2}-\frac{2}{\sqrt{i}}+\frac{4}{n}-\frac{8}{i\sqrt{i}})a_0}\right| \\ &= \prod_{i=16}^{n}\left|\frac{(i-8)\sqrt{i}}{4(i\sqrt{i}-4i+8\sqrt{i}-16)}\right| \\ &\leq \prod_{i=16}^{n}\left(i-\frac{1}{i}\right)\end{aligned}$$

implies

$$\begin{aligned}&\lim_{n\to\infty}\left|\frac{SM_n-0}{A_n-0}\right| \\ &= \lim_{n\to\infty}\frac{15}{n} \\ &= 0.\end{aligned}$$

Thus

$$\lim_{n\to\infty}\left|\frac{SM_n-0}{A_n-0}\right|=0$$

Similarly,

$$\begin{aligned}\left|\frac{SM_n-0}{TH_n-0}\right| &= \left|\frac{\prod_{i=16}^{n}(\frac{1}{8}-\frac{1}{i})a_0}{\prod_{i=16}^{n}(\frac{1}{4}-\frac{2}{i})a_0}\right| \\ &= \prod_{i=16}^{n}\left|\frac{\frac{1}{8}-\frac{1}{i}}{\frac{1}{4}-\frac{2}{i}}\right| \\ &= \prod_{i=16}^{n}\left|\frac{(i-8)}{2(i-8)}\right| \\ &= \prod_{i=16}^{n}\frac{1}{2}.\end{aligned}$$

implies

$$\lim_{n\to\infty}\left|\frac{SM_n-0}{TH_n-0}\right|=0$$

3. Application

In this part we compare the convergence rate of S, CR, Abbas, Thakur and new iteration SM - iterative process through example. The result is recorded as Table 1- 4 by taking initial aproximation $a_0 = 0.8$ for all iterative processes.

Decreasing cum sublinear functions

Let $T:[0,1]\to[0,1]$ be described by "$T(a)=(1-a)^m$", m = 7, 8.... Then T is a decreasing function. By taking $m=8$ and "$\alpha_n'=\beta_n'=\gamma_n'=\frac{1}{(1+n)^{\frac{1}{4}}}$", the correlation of convergence of the previously mentioned iterative procedures to the exact fixed point $p=0.18834768$ is listed in Table 1.

Increasing Functions

Let $T:[0,8]\to[0,8]$ be defined as $T(a)=\frac{a^2+9}{10}$. Then T is an increasing function. By taking "$\alpha_n'=\beta_n'=\gamma_n'=\frac{1}{(1+n)^{\frac{1}{2}}}$", the correlation of convergence of the previously mentioned iterative procedures to the exact fixed point $p=1$ is listed in Table 2.

Superlinear functions having different roots

The functions defined by $T(a)=2a^3-7a^2+8a-2$ is a superlinear function with different real roots. By taking "$\alpha_n'=\beta_n'=\gamma_n'=\frac{1}{(1+n)^{\frac{1}{2}}}$", the correlation of convergence of the previously mentioned iterative procedures to the exact point $p=1$ is listed in Table 3.

Oscillatory functions

The function defined by $T(a)=\frac{1}{a}$ is an oscillatory function. By taking "$\alpha_n'=\beta_n'=\gamma_n'=\frac{1}{(1+n)^{\frac{1}{4}}}$", the correlation of convergence of the previously mentioned iterative procedures to the exact fixed point $p=1$ is listed in Table 4.

For detailed study, these functions are executed after changing the values of parameters, values are investigating and recorded in next section.

4. INVESTIGATION

Decreasing cum sublinear functions

(1) For $m = 8$ and $a_0 = 0.8$, the S, CR and Abbas - iterations process converges in 17, 13 and 77 iterations where Thakur and SM iteration process never converges.

(2) For $m = 20$ and $a_0 = 0.8$ the S, CR and Abbas - iterations in 20, 16 and 439 iterations respectively where Thakur and SM iteration process again never converges.

(3) Taking $a_0 = 0.2$ (closer to fixed point), the S- iteration converges in 18 iteration, the CR iteration converges in 13 iteration, the Abbas iteration converges in 73 iteration where Thakur and SM iteration process never converges.

(4) While considering "$\alpha_n' = \beta_n' = \gamma_n' = \frac{1}{(1+n)^{\frac{1}{6}}}$" and $a_0 = 0.8$, we see that the the S, CR, Abbas, Thakur and SM- iterations converges in 22, 21, 16, 23 and 26 iterations accordingly.

Increasing Function

(1) For $a_0 = 0.8$, the S, CR, Abbas, Thakur and SM iterations, converges in 9, 13, 7, 6 and 4 iterations respectively.

(2) Speculating $a_0 = 0.4$ (away from fixed point), the S, CR and Abbas converges in 10, 13 and 8 iterations correspondingly. The Thakur iteration and and the SM iteration process converges in 6 iterations and 4 iterations respectively.

(3) While considering "$\alpha_n' = \beta_n' = \gamma_n' = \frac{1}{(1+n)^{\frac{1}{4}}}$" and $a_0 = 0.8$, we see that the the S, CR, Abbas, Thakur and SM iterations converges in 8, 6, 6, 5 and 4 iterations respectively.

TABLE 1. Decreasing cum sublinear functions

No of Iterations	CR iteration		S iteration		Abbas iteration		Thakur iteration		SM iteration	
n	$T(a_n)$	a_{n+1}	$T(a_n)$	a_{n+1}	$T(a_n)$	a_{n+1}	$T(a_n)$	a_{n+1}	$T(a_n)$	a_{n+1}
0	3.09463E-38	2.56E-06	0.99997952	2.56E-06	3.09463E-38	2.56E-06	2.56E-06	2.56E-06	1	2.56E-06
1	0.235537419	1	0.210230621	3.09463E-38	0.210223479	1	1	0.707106781	0.107508836	0
2	0.139300461	0.116640631	0.215059294	0.151356937	0.165991408	0.151367888	1.2401E-27	0.422466642	0.001594006	0.402560724
3	0.227452758	0.301170591	0.209484063	0.14411023	0.199696573	0.234079942	6.54064E-09	0.452584264	5.47317E-05	0.987318866
4	0.172799754	0.126882104	0.199146793	0.152505335	0.187451608	0.168281902	2.39424E-10	0.518116438	0.000147749	0.99956223
5	0.192260627	0.219222556	0.191679817	0.169208958	0.188178834	0.190017623	7.90103E-14	0.578126287	0.00029853	0.998818618
6	0.187909843	0.181204936	0.188842469	0.182249935	0.188284816	0.188661359	1.67571E-17	0.617059988	0.000510044	0.997614252
11	0.188347681	0.18834767	0.188347692	0.18834775	0.188340113	0.188365595	8.95132E-27	0.710930359	0.002501469	0.984298603
12	0.18834768	0.188347678	0.188347675	0.188347657	0.188341473	0.188361728	5.34916E-28	0.722368087	0.003093654	0.980162582
13	0.18834768	0.18834768	0.188347682	0.188347689	0.18834241	0.188359204	4.03953E-29	0.732531868	0.003756216	0.975517095
14	0.18834768	0.18834768	0.188347679	0.188347676	0.188343085	0.188357464	3.71375E-30	0.741645688	0.004493709	0.970342377
15	0.18834768	0.18834768	0.188347681	0.188347682	0.18834359	0.18835621	4.0391E-31	0.749880918	0.005312118	0.964610692
16	0.18834768	0.18834768	0.18834768	0.188347679	0.18834398	0.188355273	5.08078E-32	0.757371563	0.006219196	0.958284837
17	0.18834768	0.18834768	0.18834768	0.188347681	0.18834429	0.188354549	7.25843E-33	0.764224311	0.007224938	0.951316064
18	0.18834768	0.18834768	0.18834768	0.18834768	0.188344544	0.188353973	1.16022E-33	0.770525397	0.008342278	0.943641158
19	0.18834768	0.18834768	0.18834768	0.18834768	0.188344757	0.188353501	2.04947E-34	0.776345445	0.009588099	0.935178222
74	0.18834768	0.18834768	0.188347679	0.188347679	0.188347679	0.188347682	2.56663E-53	0.88452971	1.0448E-05	0.01536701
75	0.18834768	0.18834768	0.188347679	0.188347679	0.188347679	0.188347681	1.67038E-53	0.885291909	0.098678787	0.999916419
76	0.18834768	0.18834768	0.188347679	0.188347679	0.18834768	0.188347681	1.0933E-53	0.886039211	0.428545666	0.435548719
79	0.18834768	0.18834768	0.188347679	0.188347679	0.18834768	0.18834768	3.16738E-54	0.888196419	0.445392208	0.413369444
80	0.18834768	0.18834768	0.188347679	0.188347679	0.18834768	0.18834768	2.1178E-54	0.888888715	7.80874E-07	0.008951333

TABLE 2. Increasing Functions

No of Iterations	CR iteration		S iteration		Abbas iteration		Thakur iteration		SM iteration	
n	$T(a_n)$	a_{n+1}	$T(a_n)$	a_{n+1}	$T(a_n)$	a_{n+1}	$T(a_n)$	a_{n+1}	$T(a_n)$	a_{n+1}
0	0.9929296	0.964	0.964	0.964	0.998590919	0.964	0.998590919	0.964	0.999718382	0.964
1	0.999388156	0.998590919	0.9929296	0.9929296	0.999940274	0.999718382	0.999966203	0.999718382	0.999998648	0.999943684
2	0.999934172	0.999877669	0.998590919	0.998590919	0.999996272	0.999988055	0.999999009	0.999993241	0.999999992	0.99999973
3	0.999992101	0.999986835	0.999718382	0.999718382	0.999999717	0.999999254	0.999999968	0.999999802	1	0.999999998
4	0.999998985	0.99999842	0.999943684	0.999943684	0.999999976	0.999999943	0.999999999	0.999999994	1	1
5	0.999999863	0.999999797	0.999988737	0.999988737	0.999999998	0.999999995	1	1	1	1
6	0.999999981	0.999999973	0.999997747	0.999997747	1	1	1	1	1	1
7	0.999999997	0.999999996	0.999999549	0.999999549	1	1	1	1	1	1
8	1	0.999999999	0.99999991	0.99999991	1	1	1	1	1	1
9	1	1	0.999999982	0.999999982	1	1	1	1	1	1
10	1	1	0.999999996	0.999999996	1	1	1	1	1	1
11	1	1	0.999999999	0.999999999	1	1	1	1	1	1
12	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1
14	1	1	1	1	1	1	1	1	1	1
15	1	1	1	1	1	1	1	1	1	1

Superlinear functions having different roots

(1) For $a_0 = 0.8$, the S, CR, Abbas, Thakur and SM iterations, converges in 3, 2, 2, 2, 1 iterations commonly.

(2) Speculating $a_0 = 0.6$, the S, CR, Abbas, Thakur and SM iterations, converges in 5, 4, 3, 3, 2 iterations respectively.

(3) While considering "$\alpha'_n = \beta'_n = \gamma'_n = \frac{1}{(1+n)^{\frac{1}{4}}}$" and $a_0 = 0.8$, we see that the the S, CR, Abbas, Thakur and SM iterations converges in 3, 2, 2, 2 and 1 iterations respectively.

Oscillatory functions

(1) For $a_0 = 0.8$, the S, CR, Abbas, Thakur and SM iterations, converges in 12, 7, 12, 15 and 14 iterations respectively.

(2) Speculating $a_0 = 0.6$, the S, CR, Abbas, Thakur and SM iterations, converges in 14,7, 13, 15 and 14 iterations jointly.

(3) Taking "$\alpha'_n = \beta'_n = \gamma'_n = \frac{1}{(1+n)^{\frac{1}{5}}}$" and $a_0 = 0.8$, we see that the the S, CR, Abbas, Thakur and SM iterations converges in 12, 8, 11, 11 and 17 iterations respectively.

TABLE 3. Superlinear Functions having different roots

No of Iterations	CR iteration		S iteration		Abbas iteration		Thakur iteration		SM iteration	
n	$T(a_n)$	a_{n+1}	$T(a_n)$	a_{n+1}	$T(a_n)$	a_{n+1}	$T(a_n)$	a_{n+1}	$T(a_n)$	a_{n+1}
0	0.999987754	0.944	0.996512768	0.944	0.999987754	0.944	0.999987754	0.944	1	0.944
1	1	1	0.999995662	0.999987754	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	1	1

TABLE 4. Oscillatory Function

No of Iterations	CR iteration		S iteration		Abbas iteration		Thakur iteration		SM iteration	
n	a_{n+1}	$T(a_n)$	a_{n+1}	$T(a_n)$	a_{n+1}	$T(a_n)$	a_{n+1}	$T(a_n)$	a_{n+1}	$T(a_n)$
0	1.25	1.25	0.8	1.25	1.25	1.25	1.25	1.25	1.25	1.25
1	0.943760056	0.8	0.912469093	1.25	0.952552091	0.8	0.931801948	0.8	0.863603403	0.8
2	1.004785549	1.059591359	0.98557103	1.095927531	0.999644361	1.049811354	1.013432048	1.073189428	1.012884772	1.15793893
3	1.000000821	0.995237243	0.999982625	1.014640213	0.999956842	1.000355765	1.000089014	0.986745981	0.999911083	0.987279133
4	1.000000029	0.999999179	1.000001834	1.000017375	0.999992128	1.00004316	1.000009401	0.999910994	1.000009477	1.000088925
5	1.000000001	0.999999971	0.999999663	0.999998166	0.999998287	1.000007872	1.000001725	0.999990599	0.999998243	0.999990524
6	1	0.999999999	1.000000082	1.000000337	0.999999592	1.000001713	1.000000421	0.999998275	1.000000434	1.000001757
7	1	1	0.999999976	0.999999918	0.999999898	1.000000408	1.000000123	0.999999579	0.999999871	0.999999566
8	1	1	1.000000008	1.000000024	0.999999974	1.000000102	1.000000041	0.999999877	1.000000044	1.000000129
9	1	1	0.999999997	0.999999992	0.999999993	1.000000026	1.000000015	0.999999959	0.999999984	0.999999956
10	1	1	1.000000001	1.000000003	0.999999998	1.000000007	1.000000006	0.999999985	1.000000007	1.000000016
11	1	1	1	0.999999999	1	1.000000002	1.000000003	0.999999994	0.999999997	0.999999993
12	1	1	1	1	1	1	1.000000001	0.999999997	1.000000001	1.000000003
13	1	1	1	1	1	1	1.000000001	0.999999999	0.999999999	0.999999999
14	1	1	1	1	1	1	1	0.999999999	1	1.000000001
15	1	1	1	1	1	1	1	1	1	1
16	1	1	1	1	1	1	1	1	1	1
17	1	1	1	1	1	1	1	1	1	1
18	1	1	1	1	1	1	1	1	1	1

5. Conclusion

Decreasing cum sublinear function

(1) The Thakur and SM iteration does not converges while the order of the convergence of other iterations are S, CR and Abbas iteration.

(2) On increasing the value of m, convergence rate of all above iterations increases except SM iteration.

(3) If initial guess of fixed point is closer, then iterations show small changes in there convergence rate.

(4) Convergence rate of iterations also depends on α_n' and β_n'. If we increases the value of these sequences, the Thakur and SM iteration starts converging.

Increasing Function

(1) Order of convergence of iterations on decreasing order is CR, S, Abbas, Thakur, SM iterations.

(2) For starting estimation of point near to fixed point we observe that iterations CR, Thakur, SM shows no changes but the S and Abbas iterations rate increases.

(3) On increasing the value of α_n' and β_n', fixed point obtained in least number of iterations.

Superlinear function having different roots

(1) Order of convergence of iterations on decreasing order is S, CR, Abbas, Thakur, SM iterations.

(2) For starting estimation of point, away to fixed point we observe that number of iterations increases for convergence.

(3) On increasing the value of α_n' and β_n' we observe that iteration process show no change.

Oscillatory functions

(1) Order of convergence of iterative process is Thakur, SM, S, Abbas, CR iteration.

(2) For starting estimation of point away to fixed point we observe that number of iterations increases for convergence except Thakur and SM iterations.

(3) On increasing the value of α_n' and β_n' we observe that convergence increases for CR and SM but decreases for S, Abbas and Thakur iterations.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

References

[1] B.S. Thakur, D. Thakur, M. Postolache, A new iterative scheme for numerical reckoning fixed points of Suzuki's generalized nonexpansive mappings. App. Math. Comput. 275(2016), 147–155.

[2] M. Abbas, T. Nazir, A new faster iteration process applied to constrained minimization and feasibility problems. Mat. Vesn. 66(2014), 223–234.

[3] R. Chugh, V. Kumar, S. Kumar, Strong convergence of a new three step iterative scheme in Banach spaces. Amer. J. Comput. Math. 2(2012), 345-357.

[4] R.P. Agarwal, D. O'Regan, D.R. Sahu, Iterative construction of fixed points of nearly asymptotically nonexpansive mappings. J. Nonlinear Convex Anal. 8(2007), 61–79.

[5] V. Berinde, Picard iteration converges faster than Mann iteration for a class of quasi contractive operators. Fixed Point Theory Appl. 2 (2004), 97–105.

[6] V. Berinde, A convergence theorem for some mean value fixed point iteration procedures. Demonstr. Math. 38(1)(2005), 177-184.

[7] V. Kumar, A. Latif, A. Rafiq, N. Hussain, S-iteration process for quasi- contractive mappings. J. Inequal. Appl. 2013(2013), 206.