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SOME FEATURES OF PAIRWISE $\alpha - T_2$ SPACES IN SUPRA FUZZY BITOPOLOGY

MD. HANNAN MIAH*, MD. RUHUL AMIN

Department of Mathematics, Faculty of Science, Begum Rokeya University, Rangpur 5404, Bangladesh

Abstract: Four concepts of supra fuzzy pairwise T_2 bitopological spaces are introduced and studied in this paper. We also establish some relationships among them and study some other properties of these spaces.

Keywords: fuzzy set; supra bitopology; supra fuzzy bitopological space; pairwise continuous; good extension.

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1. INTRODUCTION

American Mathematician Zadeh [16] first time in 1965 introduced the concepts of fuzzy sets. Chang [4] and Lowen [8] developed the theory of fuzzy topological space using fuzzy sets. Next time much research have been done to extend the theory of fuzzy topological spaces in various direction. Lowen [8], Wong [14], Srivastava and Ali [13] have developed the fuzzy topological spaces as well as fuzzy subspace topology. Hossain and Ali [5] worked on T_2 -fuzzy topological spaces.

The research for fuzzy bitopological spaces started in early nineties. The fuzzy bitopological spaces with separation axioms has become attractive as these spaces possesses many desirable

*Corresponding author

E-mail address: hannanmathbrur@gmail.com

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properties and can be found throughout various areas in fuzzy topologies. Recent progress has been made constructing separation axioms on fuzzy bitopological space in [6, 7, 12]. Amin et al [2] have also developed T_2 concepts in fuzzy bitopological spaces in quasi coincidence sense.

In this paper, we study, some features of $\alpha - T_2$-spaces in supra fuzzy bitopological spaces and establish relationship among them.

As usual I=[0, 1] and $I_1 = [0, 1)$.

2. PRELIMINARIES

In this section, we review some concepts, which will be needed in the sequel. Through the present paper X and Y are always presented non -empty sets.

Definition 2.1[16]: For a set X, a function $u: X \to [0, 1]$ is called a fuzzy set in X. For every $x \in X, u(x)$ represents the grade of membership of x in the fuzzy set u. Some authors say u is a fuzzy subset of X. Thus a usual subset of X, is a special type of a fuzzy set in which the ranges of the function is {0, 1}. The class of all fuzzy sets from X into the closed unit interval I will be denoted by I^X .

Definition 2.2[16]: Let X be a nonempty set and A be a subset of X. The function $I_A: X \to [0, 1]\{0, 1\}$ defined by $I_A(x) = \begin{cases} 1 & if\ x \in A \\ 0 & if\ x \notin A \end{cases}$ is called the characterstic function of A. The present authors also write 1_x for the characterstic function of {x}. The characteristic functions of subsets of a set X are referred to as the crisp sets in X.

Definition 2.3[4]: Let X and Y be two sets and $f: X \to Y$ be a function. For a fuzzy subset u in X, we define a fuzzy subset v in Y by

$v(y) = \sup\{u(x)\}\ if\ f^{-1}[\{y\}] \neq \varphi,\ \ x \in X.$

=0; otherwise

Definition 2.4[4]: Let X and Y be two sets and $f: X \to Y$ be a function. For a fuzzy subset v of Y, the inverse image of v under f is the fuzzy subset $f^{-1}(v) = v \circ f$ in X and is defined by $f^{-1}(v)(x) = v(f(x)), for\ x \in X.$

Definition 2.5[4]: Let X be a non empty set and t be the collection of fuzzy sets in I^X. Then t is called a fuzzy topology on X if it satisfies the following conditions:

(i) $1,\ 0 \in t$

(ii) If $u_i \in t$ for each $i \in A$, then $\cup_{i\in\Lambda} u_i \in t$.

(iii) If $u_1,\ u_2 \in t$ then $u_1 \cap u_2 \in t$.

If t is a fuzzy topology on X, then the pair (X, t) is called a fuzzy topological space (fts, in short) and members of t are called t-open(or simply open) fuzzy sets. If u is open fuzzy set, then the fuzzy sets of the form 1-u are called t-closed (or simply closed) fuzzy sets.

Definition 2.6[8]: Let X be a nonempty set and t be the collection of fuzzy sets in I^X such that

(i) $1,\ 0 \in t$

(ii) If $u_i \in t$ for each $i \in$, then $\cup_{i\in\Lambda} u_i \in t$.

(iii) If $u_1,\ u_2 \in t$ then $u_1 \cap u_2 \in t$.

(iv) All constants fuzzy sets in X belong to t.

Then t is called a fuzzy topology on X.

Definition 2.7[9]: Let X be a non empty set. A subfamily t^* of I^X is said to be a supra topology on X if and only if

(i) $1,\ 0 \in t^*$

(ii) If $u_i \in t^*$ for each $i \in \Lambda$, then $\cup_{i\in\Lambda} u_i \in t^*$.

Then the pair (X, t^*) is called a supra fuzzy topological spaces. The elements of t^* are called supra fuzzy open sets in (X, t^*) and complement of a supra open fuzzy set is called supra closed fuzzy set.

Definition 2.8[9]: Let (X, t) and (Y, s) be two topological spaces. Let s^* and t^* are associated supra fuzzy topologies with s and t respectively and $f: (X, s^*) \to (Y, t^*)$ be a function. Then the function f is a supra fuzzy continuous if the inverse image of each

i.e. if for any $v \in t^*, f^{-1}(v) \in s^*$. The function f is called supra fuzzy homeomorphic if and only if f is supra bijective and both f and f^{-1} are supra fuzzy continuous.

Definition 2.9[4]: The function $f:(X,s^*) \to (Y,t^*)$ is called supra fuzzy open if and only if for each supra open fuzzy set u in (X,s^*) f(u) is supra open fuzzy set in (Y,t^*).

Definition 2.10[4]: **T**he function $f:(X,s^*) \to (Y,t^*)$ is called supra fuzzy closed if and only if for each supra fuzzy closed set u in (X,s^*) f(u) is supra fuzzy closed set in (Y,t^*).

Definition 2.11[3]: Let (X,s^*) and (X,t^*) be two supra fuzzy topological spaces. If u_1 and u_2 are supra fuzzy subsets of X and Y respectively, then the Cartesian product $u_1 \times u_2$ is a supra fuzzy subsets of $X \times Y$ defined by $(u_1 \times u_2)(x,y) = \min\,[u_1(x), u_2(y)]$, for each pair $(x,y) \in X \times Y$.

Definition 2.12[15]: Suppose $\{X_i, i \in \Lambda\}$, be any collection of sets and X denoted the Cartesian product of these sets, i.e., $X = \prod_{i\in\Lambda} X_i$. Here X consists of all points $p =< a_i, i \in \Lambda >$, where $a_i \in X_i$. For each $j_0 \in \Lambda$, the authors defined the projection π_{j_0} by $\pi_{j_0}(a_i : i \in \Lambda) = a_{j_0}$. These projections are used to define the product supra fuzzy topology.

Definition 2.13[15]: Let $\{X_\alpha\}_{\alpha\in\Lambda}$ be a family of nonempty sets. Let $X = \prod_{\alpha\in\Lambda} X_\alpha$ be the usual products of $X_\alpha{}'$s and let $\pi_\alpha : X \to X_\alpha$ be the projection. Further, assume that each X_α is a supra fuzzy topological space with supra fuzzy topology t_α^*. Now the supra fuzzy topology generated by $\{\pi_\alpha^{-1}(b) : b_\alpha \in t_\alpha^*, \alpha \in \Lambda\}$ as a sub basis, is called the product supra fuzzy topology on X. Thus if w is a basis element in the product, then there exists $\alpha_1, \alpha_2, \dots \dots . \alpha_n \in \Lambda$ such that $w(x) = \min\{b_\alpha(x_\alpha) : \alpha = 1,2,3, \dots, n\}$, where $x = (x_\alpha)_{\alpha\in\Lambda} \in X$.

Definition 2.14[1]: Let (X, T) be a topological space and T^* be associated supra topology with T. Then a function $f: X \to R$ is lower semi continuous if and only if $\{x \in X : f(x) > \alpha\}$ is open for all $\alpha \in R$.

Definition 2.15[10]: Let (X, T) be a topological space and T^* be associated supra topology with T. Then the lower semi continuous topology on X associated with T^* is $\omega(T^*) = \{\mu : X \to [0,1], \mu$ is supra lsc$\}$. If $\omega(T^*) : (X,T^*) \to [0,1]$ be the set of all lower semi continuous (lsc) functions. We can easily show that $\omega(T^*)$ is a supra fuzzy topology on X.

Definition 2.16[11]: Let (X, s_1^*, t_1^*) and (Y, s_2^*, t_2^*) are two supra fuzzy bitopological spaces and $f: (X, s_1^*, t_1^*) \to (Y, s_2^*, t_2^*)$ be a function. Then the function f is a supra pairwise fuzzy continuous if both the function $f: (X, s_1^*) \to (Y, s_2^*)$ and $f: (X, t_1^*) \to (Y, t_2^*)$ are supra fuzzy continuous.

Definition 2.17[11]: Let (X, s_1^*, t_1^*) and (Y, s_2^*, t_2^*) are two supra fuzzy bitopological spaces and $f: (X, s_1^*, t_1^*) \to (Y, s_2^*, t_2^*)$ be a function. Then the function f is a supra pairwise fuzzy open if both the function $f: (X, s_1^*) \to (Y, s_2^*)$ and $f: (X, t_1^*) \to (Y, t_2^*)$ are supra fuzzy open. i.e. for every open set $u \in s_1^*$, $f(u) \in s_2^*$ and for every $v \in t_1^*, f(v) \in t_2^*$.

Definition 2.18[15]: Let $\{(X_i, s_i, t_i): i \in \Lambda\}$ is a family of fuzzy bitopological spaces. Then the space $(\prod X_i, \prod s_i, \prod t_i)$ is called the product supra fuzzy bitopological space of the family $\{(X_i, s_i, t_i): i \in \Lambda\}$, where $\prod s_i$ *and* $\prod t_i$ denote the usual product fuzzy topologies of the families $\{\prod s_i : i \in \Lambda\}$ and $\{\prod t_i : i \in \Lambda\}$ of the supra fuzzy topologies respectively on X.

Let S^* and T^* be two supra topologies associated with two topologies S and T respectively. Let P be the property of a supra bitopological space (X, S^*, T^*) and FP be its supra fuzzy topological analogue. Then FP is called a 'good extension' of P 'if and only if the statement (X, S^*, T^*) has P if and only if $(X, \omega(S^*), \omega(T^*))$ has FP" holds good for every supra topological space (X, S^*, T^*).

3. $\boldsymbol{\alpha - T_2(I), \alpha - T_2(II), \alpha -}$ $\mathbf{T_2(III)}$ AND $\boldsymbol{T_2(IV)}$ SPACES IN SUPRA FUZZY BITOPOLOGICAL SPACE

In this section, we have given some new notions of $\alpha - T_2$ such as $\alpha - T_2(i), \alpha - T_2(ii), \alpha - \mathrm{T}_2(\mathrm{iii})$ and $T_2(iv)$ spaces in supra fuzzy bitopological spaces. We also discuss some properties of them and establish relationships among them by using these concepts.

Definition 3.1: Let (X, s^*, t^*) be a fuzzy bitopological space and $\alpha \in I_1$, then

(a) (X, s^*, t^*) is a pairwise $\alpha - T_2(i)$ space if and only if for all distinct elements $x, y \in X$, there exists $u \in s^*$ and there exists $v \in t^*$ such that $u(x) = 1 = v(y)$and $u \cap v \leq \alpha$.

(b) (X, s^*, t^*) is a pairwise $\alpha - T_2(ii)$ space if and only if for all distinct elements $x, y \in X$, there exists $u \in s^*$ and there exists $v \in t^*$ such that $u(x) > \alpha, v(y) > \alpha$ and $u \cap v = 0$.

(c) (X, s^*, t^*) is a pairwise $\alpha - T_2(iii)$ space if and only if for all distinct elements $x, y \in X$, there exists $u \in s^*$ and there exists $v \in t^*$ such that u$(x) > \alpha, v(y) > \alpha$and $u \cap v \leq \alpha$.

(d) (X, s^*, t^*) is a pairwise $T_2(iv)$ space if and only if for all distinct elements $x, y \in X$, there exists $u \in s^*$ and there exists $v \in t^*$ such that $u(x) > 0, v(y) > 0$and $u \cap v = 0$.

Lemma 3.1: Suppose (X, s^*, t^*) is a bitopological space and $\alpha \in I_1$. Then the following implications are true:

(a) (X, s^*, t^*) is a pairwise $\alpha - T_2(i)$ implies (X, s^*, t^*) is a pairwise $\alpha - T_2(iii)$ implies (X, s^*, t^*) is a pairwise $T_2(iv)$.

(b) (X, s^*, t^*) is a pairwise $\alpha - T_2(ii)$ implies (X, s^*, t^*) is a pairwise $\alpha - T_2(iii)$ implies (X, s^*, t^*) is a pairwise $\alpha - T_2(iv)$.

Proof: Suppose that (X, s^*, t^*) is a pairwise $\alpha - T_2(i)$. We have to prove that (X, s^*, t^*) is a pairwise $\alpha - T_2(iii)$. Let x and y be two distinct elements in X. Since (X, s^*, t^*) is a pairwise $\alpha - T_2(i)$, for $\alpha \in I_1$,by definition there exists $u \in s^*$ and there exists $v \in t^*$ such that $u(x) = 1 = v(y)$and $u \cap v \leq \alpha$, which shows that there exists $u \in s^*$ and there exists $v \in t^*$ such that $u(x) > \alpha$, $v(y) > \alpha$ and $u \cap v \leq \alpha$ .Hence by definition (c), (X, s^*, t^*) is a pairwise$\alpha - T_2(iii)$. Also we see that $u(x) > 0$, $v(y) > 0$ and $u \cap v \leq 0$. Hence (X, s^*, t^*) is a pairwise$\alpha - T_2(iv)$.

Suppose (X, s^*, t^*) is a pairwise $\alpha - T_2(ii)$. Then for $x, y \in X$, $x \neq y$ there exists $u \in s^*$ and there exists $v \in t^*$ such that $u(x) > \alpha, v(y) > \alpha$and $u \cap v = 0$, for $\alpha \in I_1$. Which shows that that $u(x) > \alpha, v(y) > \alpha$and $u \cap v \leq \alpha$. Hence by definition (c), (X, s^*, t^*) is a pairwise $\alpha - T_2(iii)$ and hence (X, s^*, t^*) is a pairwise $\alpha - T_2(iv)$.

The following examples show the non-implications among pairwise $\alpha - T_2(i), \alpha - T_2(ii), \alpha - T_2(iii)$ and $T_2(iv)$ spaces.

Example 3.1: Let X= $\{x, y\}$ and $u, v \in I^X$ are defined by $u(x) = 0.52, u(y) = 0$ and $v(x) = 0, v(y) = 0.52$. The supra fuzzy topologies s^* and t^* on X are generated by {0, u, 1, constants} and {0, v, 1, constants} respectively. For $\alpha = 0.42$, we have $u(x) = 0.52 > 0.42, v(y) > 0.42$ and $u \cap v = 0$. This according to the definition (X, s^*, t^*) is a pair wise $T_2(ii)$ but (X, s^*, t^*) is not a pairwise $\alpha - T_2(i)$.

Example 3.2: Let X= $\{x, y\}$ and $u, v \in I^X$ are defined by $u(x) = 1, u(y) = 0.43$ and $v(x) = 0.43, v(y) = 1$. The supra fuzzy topologies s^* and t^* on X are generated by {0, u, 1, constants} and {0, v, 1, constants} respectively. For $\alpha = 0.78$, we have $u(x) = 1, v(y) = 1$ and and $u \cap v \leq \alpha$. This according to the definition (X, s^*, t^*) is a pairwise $\alpha - T_2(i)$ but (X, s^*, t^*) is not a pairwise $\alpha - T_2(ii)$.

Example 3.3: Let X= $\{x, y\}$ and $u, v \in I^X$ are defined by $u(x) = 0.93, u(y) = 0.45$ and $v(x) = 0.32, v(y) = 0.93$. Consider the supra fuzzy topologies s^* and t^* on X are generated by {0, u, 1, constants} and {0, v, 1, constants} respectively. For $\alpha = 0.60$, it can easily show that (X, s^*, t^*) is a pairwise $\alpha - T_2(iii)$ but (X, s^*, t^*) is not a pairwise $\alpha - T_2(i)$ and (X, s^*, t^*) is not a pair wise $\alpha - T_2(ii)$.

Example 3.4: Let X= $\{x, y\}$ and $u, v \in I^X$ are defined by $u(x) = 0.46, u(y) = 0$ and $v(x) = 0, v(y) = 0.36$. Let the supra fuzzy topologies s^* and t^* on X are generated by $\{0, u, 1, constants\}$ and $\{0, v, 1, constants\}$ respectively. For $\alpha = 0.52$ it can be easily shown that (X, s^*, t^*) is pair wise $\alpha - T_2(iv)$ but (X, s^*, t^*) not pair wise $\alpha - T_2(i)$ and (X, s^*, t^*) not pair wise $\alpha - T_2(ii)$ and (X, s^*, t^*) not pair wise $\alpha - T_2(iv)$. This completes the proof.

Lemma 3.2: Let (X, s^*, t^*) is a supra fuzzy bitopological space and $\alpha, \beta \in I_1$ with $0 \leq \alpha \leq \beta < 1$, then

(a) (X, s^*, t^*) is a pairwise $\alpha - T_2(i)$ implies (X, s^*, t^*) is a pairwise $\beta - T_2(i)$.

(b) (X, s^*, t^*) is a pairwise $\beta - T_2(ii)$ implies (X, s^*, t^*) is a pairwise $\alpha - T_2(ii)$.

(c) (X, s^*, t^*) is apairwise $0 - T_2(ii)$ implies (X, s^*, t^*) is a pairwise $0 - T_2(iii)$.

Proof: (a)Suppose (X, s^*, t^*) is a pairwise $\alpha - T_2(i)$. We have to show that (X, s^*, t^*) is a pairwise $\beta - T_2(i)$. Let any two distinct points $x, y \in X$. Since (X, s^*, t^*) is a pairwise $\alpha - T_2(i)$, for $\alpha \in I_1$, there exists $u \in s^*$ and there exists $v \in t^*$ such that $u(x) = 1 = v(y)$and $u \cap v \leq \alpha$ .This implies that t $u(x) = 1 = v(y)$and $u \cap v \leq \beta$, since $0 \leq \alpha \leq \beta < 1$. Hence by definition (X, s^*, t^*) is$a$ pairwise $\beta - T_2(i)$.

(b)Suppose (X, s^*, t^*) is a pairwise $\beta - T_1(ii)$. We have to show that (X, s^*, t^*) is a pairwise $\alpha - T_1(ii)$. Then for $x, y \in X, x \neq y$, there exists $u \in s^*$ and there exists $v \in t^*$ such that $(x) > \beta, v(y) > \beta$ and $u \cap v = 0$, for $\beta \in I_1$. This implies that $(x) > \alpha, v(y) > \alpha$ and $u \cap v = 0, as \leq \alpha \leq \beta < 1$. Hence by definition (X, s^*, t^*) is a pairwise $\alpha - T_2(ii)$.

(c) The proof is trivial.

Example 3.5: Let X= $\{x, y\}$ and $u, v \in I^X$ are defined by $u(x) = 1, u(y) = 0.61$ and $v(x) = 0.81, v(y) = 1$. . Let the supra fuzzy topologies s^* and t^* on X are generated by {0, u, 1, constants} and {0, v, 1, constants} respectively. Then by definition for $\alpha = 0.34$ and $\beta = 0.85$; (X, s^*, t^*) is a pairwise $\beta - T_2(i)$ but (X, s^*, t^*) is not a pairwise $\alpha - T_2(i)$.

Example 3.6: Let X= $\{x, y\}$ and $u, v \in I^X$ are defined by $u(x) = 0, u(y) = 0.74$ and $v(x) = 0.86, v(y) = 0$. . Let the supra fuzzy topologies s^* and t^* on X are generated by {0, u, 1, constants} and {0, v, 1, constants} respectively. Then by definition for $\alpha = 0.35$ and $\beta = 0.84$; (X, s^*, t^*) is a pairwise $\alpha - T_2(ii)$ but (X, s^*, t^*) is not a pairwise $\beta - T_2(ii)$.

Theorem 3.1: Suppose (X, S^*, T^*) is a supra fuzzy bitopological space and $\alpha \in I_1$. Suppose the following statements:

(1) (X, S^*, T^*) be a pairwise T_2 space.

(2) $(X, \omega(S^*), \omega(T^*))$ be a pairwise $\alpha - T_2(i)$ space.

(3) $(X, \omega(S^*), \omega(T^*))$ be a pairwise $\alpha - T_2(ii)$ space.

(4) $(X, \omega(S^*), \omega(T^*))$ be a pairwise $\alpha - T_2(iii)$ space.

(5) $(X, \omega(S^*), \omega(T^*))$ be a pairwise $T_2(iv)$ space.

The following implications are true:

(a) $(1) \Longrightarrow (2) \Longrightarrow (4) \Longrightarrow (5) \Longrightarrow (1)$.

(b) $(1) \Longrightarrow (3) \Longrightarrow (4) \Longrightarrow (5) \Longrightarrow (1)$.

Proof: Suppose (X, S^*, T^*) be a T_2 bitopological space. We have to prove that $(X, \omega(S^*), \omega(T^*))$ be a pairwise $\alpha - T_2(i)$ space. Suppose x and y are two distinct elements in X. Since (X, S^*, T^*) be a pairwise T_2 space, there exists $U \in S^*$ and there exists $V \in T^*$such that $x \in U, y \in V$ and $U \cap V = \varphi$. By the definition of lsc, we have $I_U \in \omega(S^*)$ and $I_V \in \omega(T^*)$and $I_U(x) = 1, I_V(y) = 1$ and $I_U(x) \cap I_V(y) = 0$. If $I_U \cap I_V \neq 0$, then there exists $z \in X$ such that $I_U(x) \cap I_V(y) \neq 0$ implies $z \in U, z \in V$ implies $z \in U \cap V \Longrightarrow U \cap V \neq \varphi$, a contradiction. So that $I_U \cap I_V = 0$, and consequently $(X, \omega(S^*), \omega(T^*))$ be a pairwise $\alpha - T_2(i)$. Also we see that $(X, \omega(S^*), \omega(T^*))$ be a pairwise $\alpha - T_2(ii)$.

Further it is easy to show that $(2) \Longrightarrow (3)$, $(3) \Longrightarrow (4)$ and $(4) \Longrightarrow (5)$.

We therefore prove that $(5) \Longrightarrow (1)$. Suppose $(X, \omega(S^*), \omega(T^*))$ be a pairwise $T_1(iv)$ space. We have to prove that (X, S^*, T^*) be a pairwise T_2 space. Let $x, y \in X$, and $x \neq y$. Since $(X, \omega(S^*), \omega(T^*))$ be a pairwise $T_1(iv)$, there exists $u \in \omega(S^*)$ and there exists $v \in \omega(T^*)$ such that $u(x) > \alpha, v(x) > \alpha$ and $u \cap v \leq \alpha$. We have $u^{-1}(\alpha, 1] \in S^*$ and $v^{-1}(\alpha, 1] \in T^*$, $\alpha \in I_1$ and $x \in u^{-1}(\alpha, 1], y \in v^{-1}(\alpha, 1]$. Moreover $u^{-1}(\alpha, 1] \cap v^{-1}(\alpha, 1] = \varphi$. For if $z \in u^{-1}(\alpha, 1] \cap v^{-1}(\alpha, 1]$, then $z \in u^{-1}(\alpha, 1]$ and $z \in v^{-1}(\alpha, 1]$ implies that $u(z) > \alpha$ and $v(z) > \alpha$ implies $(u \cap v)(z) > \alpha$, a contradiction as $(u \cap v)(z) \leq \alpha$. Hence (X, S^*, T^*) be a pairwise T_2 space. Thus it seen that pair wise $\alpha - T_2(p)$ is a good extension of its bitopological counterpart (p=I, ii, iii, iv).

Theorem 3.2: Let (X, s^*, t^*) be a supra fuzzy bitopological space, $\alpha \in I_1$ and let $I_\alpha(s^*) = \{u^{-1}(\alpha, 1): u \in s^*\}$ and $I_\alpha(t^*) = \{v^{-1}(\alpha, 1): v \in t^*\}$, then

(a) (X, s^*, t^*) is a pairwise $\alpha - T_2(i)$ implies $(X, I_\alpha(s^*), I_\alpha(t^*))$ is a pairwise T_2.

(b) (X, s^*, t^*) is a pairwise $\alpha - T_2(ii)$ implies $(X, I_\alpha(s^*), I_\alpha(t^*))$ is pairwise a T_2.

(c) (X, s^*, t^*) is pairwise $\alpha - T_2(iii)$ if and only if $\left(X, I_\alpha(s^*), I_\alpha(t^*)\right)$ is a pairwise T_2.

Proof: (a) Let (X, s^*, t^*) be a supra fuzzy bitopological space and (X, s^*, t^*) is a pairwise $\alpha - T_2(i)$. Suppose x and y be two distinct elements in X. Then for $\alpha \in I_1$, there exists $u \in s^*$ and there exists $v \in t^*$ such that $u(x) = 1 = v(y)$ and $u \cap v \leq \alpha$. But for every $\alpha \in I_1$, $u^{-1}(\alpha, 1) \in I_\alpha(s^*)$, $v^{-1}(\alpha, 1) \in I_\alpha(t^*)$ $and\ y \in v^{-1}(\alpha, 1)$, $x \in u^{-1}(\alpha, 1)$ and $u^{-1}(\alpha, 1) \cap v^{-1}(\alpha, 1) = \varphi$ as $u \cap v \leq \alpha$. We have $(X, I_\alpha(s^*), I_\alpha(t^*))$ is a pairwise T_2 space.

(b) (X, s^*, t^*) is a pairwise $\alpha - T_2(ii)$. Then for any two distinct elements in X, there exists $u \in s^*$ and there exists $v \in t^*$ such that $u(x) > \alpha, v(y) > \alpha$and $u \cap v = 0$ for $\alpha \in I_1$. But for every $\alpha \in I_1$, $u^{-1}(\alpha, 1) \in I_\alpha(s^*)$, $v^{-1}(\alpha, 1) \in I_\alpha(t^*)$ and $y \in v^{-1}(\alpha, 1)$, $x \in u^{-1}(\alpha, 1)$ and $u^{-1}(\alpha, 1) \cap v^{-1}(\alpha, 1) = \varphi$ as $u \cap v = 0$. We have that $(X, I_\alpha(s^*), I_\alpha(t^*))$ is a pairwise T_2 space.

(c)Suppose (X, s^*, t^*) is a pairwise $\alpha - T_2(iii)$. We have to prove that (X, s^*, t^*) is a pairwise T_2 space. Let $x, y \in X, x \neq y$, then for $\in I_1$, there exists $u \in s^*$ and there exists $v \in t^*$ such that $u(x) > \alpha, v(y) > \alpha$ and $u \cap v \leq \alpha$. But for every $\alpha \in I_1$, $u^{-1}(\alpha, 1) \in I_\alpha(s^*)$, $v^{-1}(\alpha, 1) \in I_\alpha(t^*)$ and $y \in v^{-1}(\alpha, 1)$, $x \in u^{-1}(\alpha, 1)$ and $u^{-1}(\alpha, 1) \cap v^{-1}(\alpha, 1) = \varphi$ as $u \cap v \leq \alpha$. Hence it is clear that (X, s^*, t^*) is a pairwise T_2 space.

Conversely suppose that $(X, I_\alpha(s^*), I_\alpha(t^*))$ is a pairwise T_2 space. Let $x, y \in X, x \neq y$. Since $(X, I_\alpha(s^*), I_\alpha(t^*))$ is a pairwise T_2 space, there exist $U \in I_\alpha(s^*)$ and there exists $V \in I_\alpha(t^*)$ such that $x \in U, y \in V$ and $U \cap V = \varphi$. Again since $U \in I_\alpha(s^*)$ and $V \in I_\alpha(t^*)$, so we get $u \in s^*$ and $v \in t^*$ such that $U = u^{-1}(\alpha, 1)$ and $V = v^{-1}(\alpha, 1)$. This implies that $u(x) > \alpha, v(y) > \alpha$ and $u^{-1}(\alpha, 1) \cap v^{-1}(\alpha, 1) = \varphi \implies (u \cap v)^{-1}(\alpha, 1] = \varphi$ i.e., $u \cap v \leq \alpha$. So we see that (X, s^*, t^*) is a pairwise $\alpha - T_2(iii)$.

This completes the proof.

Example 3.7: Let X= $\{x, y\}$ and $u, v \in I^X$ are defined by $u(x) = 0.72, u(y) = 0.24$ and $v(x) = 0.34, v(y) = 0.68$. . Let the supra fuzzy topologies s^* and t^* on X are generated by

$\{0,\ u,\ 1,\ \text{constants}\}$ and $\{0,\ v,\ 1,\ \text{constants}\}$ respectively. Then by definition for $\alpha = 0.53$ (X, s^*, t^*) is not a pairwise $\alpha - T_2(i)$. Now let $I_\alpha(s^*) = \{X, \varphi, \{x\}\}$ and let $I_\alpha(t^*) = \{X, \varphi, \{y\}\}$. Then we see that $I_\alpha(s^*)$ and $I_\alpha(t^*)$ are supra topology on X and $(X, I_\alpha(s^*), I_\alpha(t^*))$ is a pairwise T_2 space. This completes the proof.

Similarly it can easily prove that (X, s^*, t^*) is a pairwise $\alpha - T_2(ii)$ implies $(X, I_\alpha(s^*), I_\alpha(t^*))$ is pairwise a T_2.

Theorem 3.3: Let (X, s^*, t^*) be a supra fuzzy bitopological space. $A \subseteq X$ and $s_A^* = \{u/A : u \in s^*\}$ and $t_A^* = \{v/A : v \in t^*\}$ .Then

(a) (X, s^*, t^*) is a pairwise $\alpha - T_2(i)$ implies (A, s_A^*, t_A^*) is a pairwise $\alpha - T_2(i)$.

(b) (X, s^*, t^*) is a pairwise $\alpha - T_2(ii)$ implies (A, s_A^*, t_A^*) is a pairwise $\alpha - T_2(ii)$

(c) (X, s^*, t^*) is a pairwise $\alpha - T_2(iii)$ implies (A, s_A^*, t_A^*) is a pairwise $\alpha -\ T_2(iii)$.

Proof: (a) Suppose that (X, s^*, t^*) is a supra fuzzy bitopological space and (X, s^*, t^*) is a pairwise $\alpha - T_2(i)$ space. Let $x, y \in A$ with $x \neq y$. So that $x, y \in X$ as $A \subseteq X$. Since (X, s^*, t^*) is a pairwise $\alpha - T_2(i)$, for $\alpha \in I_1$. Then for $\alpha \in I_1$, there exists $u \in s^*$ and there exists $v \in t^*$ such that $u(x) = 1 = v(y)$and $u \cap v \leq \alpha$ .For $A \subseteq X$, we have $u/A \in s_A^*, v/A \in t_A^*$ and $(u/A)(x) = 1, (v/A)(y) = 1$, and $u/A \cap v/A \leq \alpha$. Hence by definition (A, s_A^*, t_A^*) is a pairwise $\alpha - T_2(i)$.

(b)Suppose (X, s^*, t^*) is a pairwise $\alpha - T_2(ii)$ space. Let $x, y \in A$ with $x \neq y$. So that $x, y \in X$ as $A \subseteq X$. Since (X, s^*, t^*) is a pairwise $\alpha - T_2(ii)$, for $\alpha \in I_1$, Then for $\alpha \in I_1$, there exists $u \in s^*$ and there exists $v \in t^*$ such that $u(x) > \alpha, v(y) > \alpha$ and $u \cap v = 0$. .For $A \subseteq X$, we have $u/A \in s_A^*, v/A \in t_A^*$ and $(u/A)(x) > \alpha, (v/A)(y) > \alpha$ and $u/A \cap v/A = 0$. Hence by definition(A, s_A^*, t_A^*) is a pairwise $\alpha - T_2(ii)$.

(c)Suppose (X, s^*, t^*) is a pairwise $\alpha - T_2(iii)$ space. Let $x, y \in A$ with $x \neq y$. So that $x, y \in X$ as $A \subseteq X$. Since (X, s^*, t^*) is a pairwise $\alpha - T_2(iii)$, for $\alpha \in I_1$, Then for $\alpha \in I_1$, there exists $u \in s^*$ and there exists $v \in t^*$ such that $u(x) > \alpha, v(y) > \alpha$ and $u \cap v \leq \alpha$. .For $A \subseteq X$, we have $u/A \in s_A^*, v/A \in t_A^*$ and $(u/A)(x) > \alpha, (v/A)(y) > \alpha$ and $u/A \cap v/A \leq \alpha$. Hence by definition(A, s_A^*, t_A^*) is a pairwise $\alpha - T_2(iii)$

Theorem 3.4: Suppose $\{ (X_i, s_i^*, t_i^*), i \in \Lambda\}$ is a family of supra fuzzy bitopological spaces and $(\prod X_i, \prod s_i^*, \prod t_i^*) = (X, s^*, t^*)$ be the product topological space on X, then

(a) $\forall\, i \in \Lambda$, (X_i, s_i^*, t_i^*) is a pairwise $\alpha - T_2(i)$ if and only if (X, s^*, t^*) is a pairwise $\alpha - T_2(i)$.

(b) $\forall\, i \in \Lambda$, (X_i, s_i^*, t_i^*) is a pairwise $\alpha - T_2(ii)$ if and only if (X, s^*, t^*) is a pairwise $\alpha - T_2(ii)$.

(c) $\forall\, i \in \Lambda$, (X_i, s_i^*, t_i^*) is a pairwise $\alpha - T_2(iii)$ if and only if (X, s^*, t^*) is a pairwise $\alpha - T_2(iii)$.

Proof: (a) Suppose $\forall\, i \in \Lambda$, (X_i, s_i^*, t_i^*) is a pairwise $\alpha - T_2(i)$. Let $x, y \in X$ with $x \neq y$, then $x_i \neq y_i$, for some $i \in \Lambda$. Since (X_i, s_i^*, t_i^*) is α pair wise $- T_1(i)$, for $\alpha \in I_1$,there exists $u_i \in s_i^*$, $v_i \in t_i^*$, $i \in \Lambda$ such that $u_i(x_i) = 1 = v_i(y_i)$ and $u_i \cap v_i \leq \alpha$. But we have $\pi_i(x) = x_i$ and $\pi_i(y) = y_i$. Thus $u_i(\pi_i(x)) = 1 = v_i(\pi_i(y))$ and $(u_i \cap v_i) \circ \pi_i \leq \alpha$. Hence $(u_i \circ \pi_i)(x) = 1 = (v_i \circ \pi_i)(y)$ and $(u_i \circ \pi_i) \cap (v_i \circ \pi_i) \leq \alpha$. Put $u = u_i \circ \pi_i$, $v = v_i \circ \pi_i$, then $u \in s^*$, $v \in t^*$ with $u(x) = 1 = v(y)$ and $u \cap v \leq \alpha$. Hence by definition (X, s^*, t^*) is a pairwise $\alpha - T_2(i)$.

Conversely, suppose that (X, s^*, t^*) is a pairwise $\alpha - T_2(i)$. For some $\in \Lambda$. Let a_i be a fixed point in X_i and $A_i = \{x \in X = \prod_{i \in \Lambda} X_i : x_j = a_i \text{ , for some } i \neq j\}$. Thus A_i is a subset of X and hence $(A_i, s_{A_i}^*, t_{A_i}^*)$ is also a subspace of (X, s^*, t^*). Since (X, s^*, t^*) is a pairwise $\alpha - T_2(i)$, $(A_i, s_{A_i}^*, t_{A_i}^*)$ is also a pairwise $\alpha - T_2(i)$. Now we have A_i is homeomorphic image of X_i. Thus (X_i, s_i^*, t_i^*) , $i \in \Lambda$ is a pairwise $\alpha - T_2(i)$.

(b) Suppose $\forall\, i \in \Lambda$, (X_i, s_i^*, t_i^*) is a pairwise $\alpha - T_2(ii)$. Let $x, y \in X$ with $x \neq y$, then $x_i \neq y_i$, for some $i \in \Lambda$. Since (X_i, s_i^*, t_i^*) is α pair wise $- T_2(ii)$, for $\alpha \in I_1$,there exists $u_i \in s_i^*$, $v_i \in t_i^*$, $i \in \Lambda$ such that $u_i(x_i) > \alpha, v_i(y_i) > \alpha$ and $u_i \cap v_i = 0$. But we have $\pi_i(x) = x_i$

and $\pi_i(y) = y_i$. Thus $u_i(\pi_i(x)) > \alpha, v_i(\pi_i(y)) > \alpha$ and $(u_i \cap v_i) \circ \pi_i = 0$. Hence $(u_i \circ \pi_i)(x) > \alpha, (v_i \circ \pi_i)(y) > \alpha$ and $(u_i \circ \pi_i) \cap (v_i \circ \pi_i) = 0$. Put $u = u_i \circ \pi_i$, $v = v_i \circ \pi_i$, then $u \in s^*$, $v \in t^*$ with $u(x) > \alpha, v(y) > \alpha$ and $u \cap v = 0$. Hence by definition (X, s^*, t^*) is a pairwise $\alpha - T_2(ii)$.

Conversely, suppose that (X, s^*, t^*) is a pairwise $\alpha - T_2(ii)$. For some $i \in \Lambda$ .let a_i be a fixed point in X_i and $A_i = \{x \in X = \prod_{i \in \Lambda} X_i : x_j = a_i$, for some $i \neq j\}$. Thus A_i is a subset of X and hence $(A_i, s^*_{A_i}, t^*_{A_i})$ is also a subspace of (X, s^*, t^*). Since (X, s^*, t^*) is a pairwise $\alpha - T_2(ii)$, $(A_i, s^*_{A_i}, t^*_{A_i})$ is also a pairwise $\alpha - T_2(ii)$. Furthermore A_i is homeomorphic image of X_i. Thus (X_i, s^*_i, t^*_i), $i \in A$ is a pairwise $\alpha - T_2(ii)$.

(c) Suppose $\forall\, i \in \Lambda$, (X_i, s^*_i, t^*_i) is a pairwise $\alpha - T_2(iii)$. Let $x, y \in X$ with $x \neq y$, then $x_i \neq y_i$, for some $i \in \Lambda$. Since (X_i, s^*_i, t^*_i) is α pair wise $- T_2(iii)$, for $\alpha \in I_1$,there exists $u_i \in s^*_i$, $v_i \in t^*_i$, $i \in \Lambda$ such that $u_i(x_i) > \alpha, v_i(y_i) > \alpha$ and $u_i \cap v_i < \alpha$. But we have $\pi_i(x) = x_i$ and $\pi_i(y) = y_i$. Thus $u_i(\pi_i(x)) > \alpha, v_i(\pi_i(y)) > \alpha$ and $(u_i \cap v_i) \circ \pi_i \leq \alpha$. Hence $(u_i \circ \pi_i)(x) > \alpha, (v_i \circ \pi_i(y)) > \alpha$ and $(u_i \circ \pi_i) \cap (v_i \circ \pi_i) \leq \alpha$. Put $u = u_i \circ \pi_i$, $v = v_i \circ \pi_i$, then $u \in s^*$, $v \in t^*$ with $u(x) > \alpha, v(y) > \alpha$ and $u \cap v \leq \alpha$. Hence by definition (X, s^*, t^*) is a pairwise $\alpha - T_2(iii)$.

Conversely, suppose that (X, s^*, t^*) is a pairwise $\alpha - T_2(iii)$. For some $i \in \Lambda$ let a_i be a fixed point in X_i and $A_i = \{x \in X = \prod_{i \in \Lambda} X_i : x_j = a_i$, for some $i \neq j\}$. Thus A_i is a subset of X and hence $(A_i, s^*_{A_i}, t^*_{A_i})$ is also a subspace of (X, s^*, t^*). Since (X, s^*, t^*) is a pairwise $\alpha - T_2(iii)$, $(A_i, s^*_{A_i}, t^*_{A_i})$ is also a pairwise $\alpha - T_2(iii)$. Furthermore A_i is homeomorphic image of X_i. Thus (X_i, s^*_i, t^*_i), $i \in \Lambda$ is a pairwise $\alpha - T_2(iii)$.

Theorem 3.5: Let (X, s^*_1, t^*_1) and (Y, s^*_2, t^*_2) be two supra fuzzy bitopological spaces. $f: X \to Y$ be one-one, onto and open map, then

(a) (X, s^*_1, t^*_1) is a pairwise $\alpha - T_2(i)$ implies (Y, s^*_2, t^*_2) is a pairwise $\alpha - T_2(i)$.

(b) (X, s^*_1, t^*_1) is a pairwise $\alpha - T_2(ii)$ implies (Y, s^*_2, t^*_2) is a pairwise $\alpha - T_2(ii)$.

(c) (X, s_1^*, t_1^*) is a pairwise $\alpha - T_2(iii)$ implies(Y, s_2^*, t_2^*) is a pairwise $\alpha - T_2(iii)$.

Proof: (a) Suppose (X, s_1^*, t_1^*) is a pairwise $\alpha - T_2(i)$. We have to prove that (Y, s_2^*, t_2^*) is a pair wise $\alpha - T_2(i)$. Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$, there exist $x_1, x_2 \in X$ with $f(x_1) = y_1, f(x_2) = y_2, s$ ince f is onto and $x_1 \neq x_2$ as f is one-one. Again since (X, s_1^*, t_1^*) is a pair wise $\alpha - T_2(i)$, $\alpha \in I_1$, there exists $u \in s_1^*$ and there exists $v \in t_1^*$ such that $u(x_1) = 1 = v(x_1)$ and $u \cap v \leq \alpha$.

Now $f(u)(y_1) = \{\sup u(x_{1)}: f(x_1) = y_1\}$

$=1.$

$f(v)(y_2) = \{\sup v(x_{2)} : f(x_2) = y_2\}$

$= 1,$

and $f(u \cap v)(y_1) = \{\sup(u \cap v)(x_{1)}: f(x_1) = y_1\}$

$f(u \cap v)(y_2) = \{\sup(u \cap v)(x_{2)} : f(x_2) = y_2\}$

Hence $f(u \cap v) \leq \alpha \Longrightarrow f(u) \cap f(v) \leq \alpha$.

Since f is open , $f(u) \in s_2^*$ as $u \in s_1^*$, $f(v) \in t_2^*$,as $v \in t_1^*$. We observe that $f(u) \in s_2^*$, $f(v) \in t_2^*$ such that $f(u)(y_1) = 1$, $f(v)(y_2) = 1$ and $f(u) \cap f(v) \leq \alpha$ .Hence by definition (Y, s_2^*, t_2^*) is a pair wise $\alpha - T_2(i)$.

(b) Suppose (X, s_1^*, t_1^*) is a pairwise $\alpha - T_2(ii)$. We have to prove that (Y, s_2^*, t_2^*) is a pair wise $\alpha - T_2(ii)$. Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$, there exist $x_1, x_2 \in X$ with $f(x_1) = y_1, f(x_2) = y_2, s$ ince f is onto and $x_1 \neq x_2$ as f is one-one. Again since (X, s_1^*, t_1^*) is a pair wise $\alpha - T_2(ii)$, $\alpha \in I_1$, there exists $u \in s_1^*$ and there exists $v \in t_1^*$ such that $u(x_1) > \alpha, v(x_1) > \alpha$ and $u \cap v = 0$.

Now $f(u)(y_1) = \{\sup u(x_{1)}: f(x_1) = y_1\}$

$> \alpha.$

$f(v)(y_2) = \{\sup v(x_{2)} : f(x_2) = y_2\}$

$> \alpha,$

and $f(u \cap v)(y_1) = \{\sup(u \cap v)(x_{1)}: f(x_1) = y_1\}$

$f(u \cap v)(y_2) = \{\sup(u \cap v)(x_{2)}: f(x_2) = y_2\}$

Hence $f(u \cap v) = 0 \Longrightarrow f(u) \cap f(v) = 0$.

Since f is open , $f(u) \in s_2^*$ as $u \in s_1^*$, $f(v) \in t_2^*$,as $v \in t_1^*$ We observe that $f(u) \in s_2^*$, $f(v) \in t_2^*$ such that $f(u)(y_1) > \alpha$, $f(v)(y_2) > \alpha$ and $f(u) \cap f(v) = 0$ .Hence by definition (Y, s_2^*, t_2^*) is a pair wise $\alpha - T_2(ii)$.

Similarly (c) can be proved.

Theorem 3.6: Let (X, s_1^*, t_1^*) and (Y, s_2^*, t_2^*) be two supra fuzzy bitopological spaces. $f: X \to Y$ be continuous and one-one map, then

(a) (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_2(i)$ implies (X, s_1^*, t_1^*) is a pairwise $\alpha - T_2(i)$.

(b) (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_2(ii)$ implies (X, s_1^*, t_1^*) is a pairwise $\alpha - T_2(ii)$.

(c) (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_2(iii)$ implies (X, s_1^*, t_1^*) is a pairwise $\alpha - T_2(iii)$.

Proof: (a) Let (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_2(i)$. We have to prove that (X, s_1^*, t_1^*) is a pairwise $\alpha - T_2(i)$. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$ in Y, since f is one-one. Also since (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_2(i)$, $\alpha \in I_1$, there exists $u \in s_2^*$ and there exists $v \in t_2^*$ such that $u(f(x_1)) = 1 = v(f(x_2))$ and $u \cap v \leq \alpha$. This implies that $f^{-1}(u)(x_1) = 1, f^{-1}(v)(x_2) = 1$ and $f^{-1}(u \cap v) \leq \alpha$ implies $f^{-1}(u) \cap f^{-1}(v) \leq \alpha$. since $u \in s_2^*$,$v \in t_2^*$ and f is continuous, then $f^{-1}(u) \in s_1^*$, $f^{-1}(v) \in t_1^*$.

Now it is clear that there exists $f^{-1}(u) \in s_1^*$ and there exists $f^{-1}(v) \in t_1^*$ such that $f^{-1}(u)(x_1) = 1, f^{-1}(v)(x_2) = 1$ and $f^{-1}(u) \cap f^{-1}(v) \leq \alpha$. Hence (X, s_1^*, t_1^*) is a pairwise $\alpha - T_2(i)$.

(b) Let (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_2(ii)$. We have to prove that (X, s_1^*, t_1^*) is a pairwise $\alpha - T_2(ii)$. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$ in Y, since f is one-one. Also since (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_2(ii)$,$\alpha \in I_1$, there exists $u \in s_2^*$ and there exists $v \in t_2^*$ such that $u(f(x_1)) > \alpha, v(f(x_2)) > \alpha$ and $u \cap v = 0$. This implies that $f^{-1}(u)(x_1) > \alpha, f^{-1}(v)(x_2) >$

α and $f^{-1}(u \cap v) = 0$ implies $f^{-1}(u) \cap f^{-1}(v) = 0$. since $u \in s_2^*$, $v \in t_2^*$ and f is continuous, then $f^{-1}(u) \in s_1^*$, $f^{-1}(v) \in t_1^*$.

Now it is clear that there exists $f^{-1}(u) \in s_1^*$ and there exists $f^{-1}(v) \in t_1^*$ such that $f^{-1}(u)(x_1) > \alpha, f^{-1}(v)(x_2) > \alpha$ and $f^{-1}(u) \cap f^{-1}(v) = 0$. Hence (X, s_1^*, t_1^*) is pairwise $\alpha - T_2(ii)$.

Similarly (c) can be proved.

CONCLUSION

One of the important results of this paper is introducing some new notions of supra fuzzy pairwise $\alpha - T_2$ bitopological spaces. We represent their good extension, hereditary, productive and projective properties. These concepts would be very helpful for future research work in supra fuzzy bitopological spaces.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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