Available online at http://scik.org
J. Math. Comput. Sci. 11 (2021), No. 4, 4454-4463
https://doi.org/10.28919/jmcs/5610
ISSN: 1927-5307

ON τ^*-GENERALIZED SEMI CONTINUOUS MULTIFUNCTIONS IN TOPOLOGICAL SPACES

A. PONSURYADEVI, R. SELVI*
Department of Mathematics, Sri Parasakthi College for Women, Courtallam,
Affiliated to Manonmaniam Sundaranar University, Tirunelveli, Tamilnadu, India

Abstract: In this paper, we introduce the concept of τ^*-generalized semi continuous multifunctions in topological spaces and study some of their properties where τ^* is defined by $\tau^* = \{G: cl^*(G^C) = G^C\}$.

Keywords: τ^*-gs open set; τ^*-gs closed set; τ^*-gs continuous.
2010 AMS Subject Classification: 32A12.

1. INTRODUCTION

Continuity is an important concept for the study and investigation in topological spaces. This concept has been extended to the setting of multifunctions and has been generalized by weaker forms of open sets. In 1963, Levine [12] introduced the notions of semi-open sets and semi-continuity in topological spaces.

In 1970, Levine [13] introduced the concepts of generalized closed sets as a generalization of closed sets in topological spaces. Using generalized closed sets, Dunham [8] introduced the concept of the closure operator $C\,l^*$ and a topology τ^*, *where* τ^* is defined by $\tau^* = \{G: cl^*(G^C) = G^C\}$ and studied some of their properties. S.P.Arya [2], P. Bhattacharyya and B.K. Lahiri [6], J. Dontchev[8], A. Pushpalatha et al. [14], S. Eswaran and N. Nagaveni [11] introduced and investigated generalized semi closed sets, semi generalized closed sets and τ^* -

*Corresponding author
E-mail address: suryachairman17@gmail.com
Received February 26, 2021

generalized closed sets, τ^* -generalized semi continuous functions in topological spaces respectively.

By a multifunction $F: X \to Y$, we mean a point to set correspondence from X into Y, also we always assume that $F(x) \neq \emptyset$ for all x∈X. For a multifunction $F: X \to Y$, the upper and lower inverse of any subset $A\ of\ Y$ are denoted by $F^+(A)$ and $F^-(A)$, respectively, Where $F^+(A) = \{x \in X: F(x) \subset A\}$ and $F^-(A) = \{x \in X: F(x) \cap A \neq \emptyset\}$. In particular, $F^-(y) = \{x \in X: y \in F(x)\}$ for each point $Y \in Y$. A multifunction $F: X \to Y$ is said to be a surjection if $F(X) = Y$ [4].

In this paper, we introduce and study the concept of Generalized semi continuous multifunctions in the Topological spaces (X, τ^*), *where* τ^* is defined by $\tau^* = \{G: cl^*(G^C) = G^C\}$

2. PRELIMINARIES

Definition: 2.1

Let A be a subset of a topological space X. Then A is called Semi-open if $A \subseteq cl(int(A))$ and Semi-closed if $int(cl(A)) \subseteq A$.[13]

Definition: 2.2

A subset A of topological space (X, τ) is called Generalized closed (briefly g-closed) if $cl(A) \subseteq G$ whenever $A \subseteq G$ and G is open in X. [14]

Definition:2.3

Semi-generalized closed (briefly sg-closed) if $scl(A) \subseteq G$ whenever $A \subseteq G$ and G is semi-open in X.[8]

Definition: 2.4

For the subset A of a topological $space\ X$, the generalized closure operator Cl^* is defined by the intersection of all g-closed sets containing A.[9]

Definition: 2.5

For the subset A of a topological space X, the topology τ^* is defined by τ^*={G:C$l^*(G^c)$ = G^c}.[9]

Definition: 2.6

A subset A of a topological space X is called τ^* -generalized closed set (briefly τ^*-gclosed) if C$l^* \subseteq G$ whenever $A \subseteq G$ and G is τ^*-open. The complement of τ^*-generalized closed set is called the τ^*-generalized open set (briefly τ^*-gopen).[15]

Definition: 2.7

A subset A of a topological space X is called τ^*-generalized semi-closed (briefly τ^*-gs closed) if

$Cl^*(Scl(A)) \subseteq G$ whenever $A \subseteq G$ and G is τ^*-open. The complement of τ^*-generalized semi-closed set is called the τ^*-generalized semi-open set (briefly τ^*-gs open). [1]

Definition: 2.8

A topological space X is said to be Semi- regular if for each semi-closed set F of X and each $x \notin F$, there exist disjoint semi-open sets U and V such that $F \subset U$ and $x \in V$. [7]

Definition: 2.9

A function $F: X \to Y$ from a topological space X into a topological space Y is called Semi continuous if the inverse image of a closed set in Y is semi closed in X.[13]

Definition: 2.10

A function F:X → Y from a topological space X into a topological space Y is called g -continuous if the inverse image of a closed set in Y is g-closed in X. [5]

Definition: 2.11

A function $F: X \to Y$ from a topological space X into a topological space Y is called gs-continuous if the inverse image of a closed set in Y is gs-closed in X.[8]

Definition: 2.12

A function $F: X \to Y$ from a topological space X into a topological space Y is called τ^*-g continuous if the inverse image of every g-closed set in Y is τ^*-gclosed in X.[10]

Definition: 2.13

A function $F: X \to Y$ from a topological space X into a topological space Y is called τ^*-gs continuous if the inverse image of every gs-open set in Y is τ^*-gopen in X. [11]

3. τ^*-GENERALIZED SEMI-CONTINUOUS MULTIFUNCTION

Definitions: 3.1

A multifunction $F: (X, \tau^*) \to (Y, \sigma)$ is said to be

(i) Upper τ^*-gs continuous at $x \in X$ if for each open set V such that $x \in F^+(V)$, there exists a τ^*-gs open set U containing x such that U$\subset F^+(V)$.

(ii) Lower τ^*-gs continuous at x∈X if for each open set V such that x∈ F^-(V), there exists a τ^*-gs open set U containing x such that $U \subset F^-$(V).

(iii) Upper (lower)τ^*-gs continuous if F has this property at each point of X.

Theorem: 3.2 The following statements are equivalent for a multifunction $F: (X, \tau^*) \to$

(Y,σ)

(i) F *is upper* τ^**-gs continuous;*

(ii) $F^+(V)$ *is a* τ^**-gs open set for any open set* $V \subset Y$ *;*

(iii) $F^-(K)$ *is a* τ^**-gs closed set for any closed set* $K \subset Y$ *;*

(iv) for each $x \in X$ *and for each open set* V *such that* $F(x) \subset V$ *, there exists a* τ^**-gs open set* U *containing x such that if* $y \in U$*, then* $F(y) \subset V$ *;*

(v)for each point x *of* X *and each neighbourhood* V *of* $F(x)$*,* $F^+(V)$ *is a* τ^**-gs neighbourhood of* x*;*

(vi) for each point x *of* X *and each neighbourhood* V *of* $F(x)$*, there exists a* τ^**-gs neighbourhood* U *of x such that* $F(U) \subset V$ *.*

Proof:

$(i) \Leftrightarrow (ii)$ Let $V \subset Y$ be a open set and let $x \in F^+(V)$. From (i), there exists a τ^*-gs open set U containing x such that $U \subset F^+(V)$. It follows that $F^+(V)$ is a τ^*-gs open set.

The converse can be obtained similarly from the definition of τ^*-gs open set.

$(ii) \Leftrightarrow (iii), (i) \Leftrightarrow (iv)$. Obvious.

(ii)$\Rightarrow (v)$. Let $x \in X$ and V be a neighbourhood of $F(x)$. Then there exists an open set G of Y such that $F(x) \subset G \subset V$. Therefore, we obtain $x \in F^+(G) \subset F^+$ (v). Since $F^+(G) \in \tau^*$*-gs O*(X), $F^+(V)$ is a τ^*-gs neighbourhood of x.

$(v) \Rightarrow (vi)$. Let $x \in X$ and V be a neighbourhood of $F(x)$. By (v), $F^+(V)$ is a τ^*-gs neighbourhood of x. Take $U = F^+(V)$. Then $F(U) \subset V$.

$(vi) \Rightarrow (i)$. Let $x \in X$ and V be any open set of Y such that $F(x) \subset V$. Then V is a neighbourhood of $F(x)$. By (vi), there exists a τ^*-gs neighbourhood U of x such that $F(U) \subset V$. Therefore, there exists $G \in \tau^*$*-gs*$O(X)$ such that $x \in G \subset U$ and hence $F(G) \subset F(U) \subset V$. We obtain that F is upper τ^*-gs continuous.

Theorem: 3.3

Let $F: (X,\tau^*) \to. (Y,\sigma)$ *be a multifunction. Then the following statements are equivalent:*

i.F is lower τ^*-gs *continuous multifunction;*

ii.$F^-(V)$ *is a* τ^*-gs *open set for any open set* $V \subset Y$ *;*

iii.$F^+(K)$ *is a* τ^*-gs *closed set for any closed set* $K \subset Y$ *;*

iv.for each $x \in X$ *and for each open set* V *such that* $F(x) \cap V \neq \emptyset$ *, there exists a* τ^*-gs *open* *set U containing x such that if $y \in U$, then $F(y) \cap V \neq \emptyset$.*

Proof.

$(i) \Leftrightarrow (ii)$ Let $V \subset Y$ be a open set and let $x \in F^{-}(V)$. From (i), there exists a τ^*-gs open set U containing x such that $U \subset F^{-}(V)$. It follows that $F^{-}(V)$ is a τ^*-gs open set.

The converse can be obtained similarly from the definition of τ^*-gs open set. $(ii) \Leftrightarrow (iii), (i) \Leftrightarrow (iv)$. Obvious.

Theorem:3.4

Let $F: (X, \tau^*) \to (Y, \sigma)$ be a multifunction from a topological space (X, τ^*) to a topological space (Y, σ) and let $F(X)$ be endowed with subspace topology. If F is upper τ^*-gs continuous multifunction, then $F: (X, \tau^*) \to (F(X), \sigma)$ is upper τ^*-gs continuous multifunction.

Proof:

Since F is upper τ^*-gs continuous, $F^{+}(V) \cap F(X)) = F^{+}(V) \cap F^{+}(F(X)) = F^{+}(V)$ is τ^*-gs open for each open subset V of Y. Hence $F: (X, \tau^*) \to (F(X), \sigma)$ is upper τ^*-gs continuous multifunction.

Definition: 3.5

A subset A of a topological space X is said to be:

(i)α-paracompact if every cover of A by open sets of X is refined by cover of A which consists of open sets of X and locally finite in X.[16]

$(ii)$$\alpha$-regular if for each $a \in A$ and each open set U of X containing a, there exists an open set G of X such that $a \in G \subset cl(G) \subset U$.[12]

Lemma :3.6 [16]

If A is an α-regular α-paracompact set of a topological space X and U is an open neighbourhood of A, then there exists an open set G of X such that $A \subset G \subset cl(G) \subset U$.

Definition :3.7

For a multifunction $F: X \to Y$, a multifunction $cl(F): X \to Y$ is defined as $cl(F)(x) = cl(F(x))$ for each point $x \in X$. Multifunctions α-$cl(F)$, $scl(F)$, δ-$scl(F)$, $pcl(F)$, β-$cl(F)$can be defined similarly.[3]

Lemma :3.8

If $F: X \to Y$ is a multifunction such that $F(x)$ is α-regular of Y, $G^{+}(V) = F^{+}(V)$, where G denotes α-$cl(F)$, $scl(F)$, δ-scl(F), $pcl(F)$,or α-paracompact for each $x \in X$ and Y is semi-

regular, then for each open set V of Y, $G^+(V) = F^+(V)$, *where G denotes* α-$cl(F)$, $scl(F)$, δ-scl(F), $pcl(F)$, *or* β-$cl(F)$.

Theorem:3.9

Let $F: (X, \tau^*) \to (Y, \sigma)$ *be a multifunction such that* $F(x)$ *is* α*-regular* α*-paracompact for each* $x \in X$ *and* Y *is semi-regular. Then the following are equivalent:*

i. F *is upper* τ^*-gs *continuous;*
ii. $scl(F)$ *is upper* τ^*-gs *continuous;*
iii. $pcl(F)$ *is upper* τ^*-gs *continuous;*

Proof:

We take $G = scl(F)$, *or* $pcl(F)$. Suppose that F is upper τ^*-gs continuous. Let $x \in X$ and V be any open set of Y containing $G(x)$. By lemma 3.8, we have $x \in G^+(V) = F^+(V)$ and hence there exists $U \in \tau^*$-gsO (X, x) such that $F(U) \subset V$. Since $F(u)$ is α-regular α-paracompact for each $u \in U$, there exists an open set W such that $F(u) \subset W \subset cl(W) \subset V$, hence $G(u) \subset cl(W) \subset V$ for each $u \in U$. Therefore $G(U) \subset V$. Hence G is upper τ^*-gs continuous. Conversely, suppose that G is upper τ^*-gs continuous. Let $x \in X$ and V be any open set of Y containing $F(x)$. By the previous lemma, we have $x \in G^+(V) = F^+(V)$ and hence $G(x) \subset V$. There exists $U \in \tau^*$-gsO (X, x) such that $G(U) \subset V$. So that $U \subset G^+(V) = F^+(V)$ and hence $F(U) \subset V$. This shows that f is upper τ^*-gs continuous.

Lemma: 3.10

If $F: (X, \tau^*) \to (Y, \sigma)$ *is a multifunction and* Y *is semi-regular, then for each open set* V *of* Y, $G^-(V) = F^-(V)$, *where* G *denotes* α-$cl(F)$, $scl(F)$, δ-$scl(F)$, $pcl(F)$, *or* β-$cl(F)$.

Theorem: 3.11

For a multifunction $F: (X, \tau^*) \to (Y, \sigma)$, *the following are equivalent:*

i. F *is lower* τ^*-gs *continuous;*
ii. $scl(F)$ *is lower* τ^*-gs *continuous;*
iii. $pcl(F)$ *is lower* τ^*-gs *continuous;*

Theorem: 3.12

Let $F: (X, \tau^*) \to (Y, \sigma)$ *be a multifunction from a topological space* (X, τ^*) *to a topological space* (Y, σ). F *is lower* τ^*-gs *continuous multifunction if and only if* $F(\tau^*\text{-gscl}(A)) \subset cl(F(A))$ *for each* $A \subset X$

Proof:

Suppose that F is lower τ^*-gs continuous and $A \subset X$. Since $cl(F(A))$ is a closed set, it follows that $F^{-}(cl\ (F\ (A)))$ is a τ^{*}-gs closed set in X from Theorem 3.3, Since $A\subset F^{-}(cl\ (F\ (A)))$, then τ^{*}-gscl$(A)\subset\tau^{*}$-gscl$(F^{-}(cl\ (F\ (A))))=F^{-}(cl\ (F\ (A)))$. Thus, we obtain that $F\ (\tau^{*}$-gscl $(A))\subset F(F^{-}(cl\ (F(A))))\subset cl\ (F\ (A))$. Conversely, suppose that $F\ (\tau^{*}$ -gscl $(A))\subset cl\ (F(A))\ for\ each\ A\subset X$. Let K be any closed set in Y. Then $F\ (\tau^{*}$-gscl $(F^{-}(K)))\subset cl\ (F(F^{-}(K)))\ and\ cl\ (F\ (F^{-}(K)))\subset cl(K)=K$. Hence, τ^{*}-gscl $(F^{-}(K))\subset F^{-}(K)$ then F is lower τ^{*}-gs continuous multifunction.

Theorem: 3.13

Let $F:(X,\tau^{*})\to(Y,\sigma)$ *be a multifunction from a topological space* (X,τ^{*}) *to a topological space* (Y,σ). F *is lower* τ^{*}-gs *continuous multifunction if and only if* τ^{*}-gscl $(F^{-}(B))\subset F^{-}(cl\ (B))$ *for each* $B\subset Y$.

Proof:

Suppose that F is lower τ^{*}-gs continuous multifunction and $B\subset Y$. Then $F^{-}(cl\ (B))$ is τ^{*}-gs closed in X and $F^{-}(cl\ (B))=\tau^{*}$-gscl $(F^{-}(cl\ (B)))$. Hence, τ^{*}-gscl $(F^{-}(B))\subset F^{-}(cl\ (B))$. Conversely, let K be any closed set in Y. Then τ^{*}-gscl $(F^{-}(K))\subset F^{-}(cl\ (K))=F^{-}(K)\subset\tau^{*}$ -gscl $(F^{-}(K))$. Thus, $F^{-}(K)=\tau^{*}$ -gscl $(F^{-}(K))$. Then F is lower τ^{*}-gs continuous multifunction.

Definition: 3.14

A space X is said to be τ^{*}-gs compact if every τ^{*}-gs open cover of X has a finite sub cover.

Theorem: 3.15

Let $F:(X,\tau^{*})\to(Y,\sigma)$ *be an upper* τ^{*}-gs *continuous surjective multifunction such that* $F\ (x)$ *is compact for each* $x\in X$. *If* X *is a* τ^{*}-gs *compact space, then* Y *is compact.*

Proof:

Let $\{v_{\lambda}:\lambda\in\Lambda\}$ *be a open cover of Y. since F(x) is compact for each* $x\in X$ *,there exists a finite subset* $\Lambda(x)$ *of* Λ *such that* $F(x)\subset\{v_{\lambda}:\lambda\in\Lambda\}$. *Put* $V(x)=\cup\{v_{\lambda}:\lambda\in\Lambda(x)\}$ *. Since F is upper* τ^{*}-gs continuous, there exists a τ^{*}-gs open set U(x) of X containing x such that F(U$(x)\subset$V(x))). Then the family $\{U(x):x\in X\}$ *is a* τ^{*}-gs open cover of X and since X is τ^{*}-gs compact, there exists a finite number of points, say ,x_1 ,x_2,…….x_n in X such that X=$\cup\{\cup(x_i)$: i=1,2,3,…..n}. Hence we have Y=F(x)=F($\bigcup_{i=1}^{n}\cup(x_i)$)=$\bigcup_{i=1}^{n}F(\cup(x_i))\subset\bigcup_{i=1}^{n}V(x_i)=\bigcup_{i=1}^{n}\bigcup_{\lambda\in\Lambda(xi)}V_{\lambda}$. This shows that Y is compact.

Lemma:3 . 16

Let A and X_0 be subset of a space(X, τ^*). If $A \in \tau^*$-gs$O(X)$, and $X_0 \in \tau^*$-gs$O(X)$, then $A \cap X_0 \in \tau^*$-gs$O(X_0)$.

Lemma:3 . 17

Let $A \subset X_0 \subset X. If\ X_0 \in \tau^*$-gs$O(X)$ *and* $A \in \tau^*$-gs$O(X_0)$, *then* $A \in \tau^*$-gs $O(X)$

Theorem:3.18

Let $\{U_\lambda : \lambda \in \Lambda\}$ *be a* τ^*-gs open *cover of a space* X. *Then a multifunction* $F: (X, \tau^*) \to (Y, \sigma)$ *is upper* τ^*-gs *continuous if and only if the restriction* $F\,|U_\lambda : U_\lambda \to Y$ *is upper* τ^*-gs *continuous for each* $\lambda \in \Lambda$.

Proof:

Suppose Let $\lambda \in \Lambda$ *and* V be any open set of Y. Since F is upper τ^*-gs continuous, $F^+(V)$ is τ^*-gs open in X. By Lemma 3.16, $(F\,|U_\lambda)^+(V) = F^+(V) \cap U_\lambda$ *is* τ^*-gs open in U_λ and hence $F\,|U_\lambda$ is upper τ^*-gs continuous. Conversely, Let V be any open set of Y. Since $F\,|_{U\lambda}$ is upper τ^*-gs continuous for each $\lambda \in \Lambda$, $(F\,|U_\lambda)^+(V) = F^+(V) \cap U_\lambda$ is τ^*-gs open in U_λ. By Lemma 3.17, $(F\,|U_\lambda)^+(V)$ is τ^*-gs open in X for each $\lambda \in \Lambda$. So that $F^+(V) = \cup\ (F\,|U_\lambda)^+(V)$ is τ^*-gs open in X. Hence F is upper τ^*-gs continuous $\lambda \in \Lambda$.

Theorem: 3.19

Let $\{U_\lambda : \lambda \in \Lambda\}$ *be a* τ^*-gs open *cover of a space* X. *Then a multifunction* $F: (X, \tau^*) \to (Y, \sigma)$ *is lower* τ^*-gs *continuous if and only if the restriction* $F\,|U_\lambda : U_\lambda \to Y$ *is lower* τ^*-gs *continuous for each* $\lambda \in \Lambda$.

Proof:

Suppose Let $\lambda \in \Lambda$ *and* V be any open set of Y. Since F is lower τ^*-gs continuous, $F^-(V)$ is τ^*-gs open in X. By Lemma 3.16, $(F\,|U_\lambda)^-(V) = F^-(V) \cap U_\lambda$ *is* τ^*-gs open in U_λ and hence $F\,|U_\lambda$ is lower τ^*-gs continuous. Conversely, Let V be any open set of Y. Since $F\,|U_\lambda$ is upper τ^*-gs continuous for each $\lambda \in \Lambda$, $(F\,|U_\lambda)^-(V) = F^-(V) \cap U_\lambda$ is τ^*-gs open in U_λ. By Lemma 3.17, $(F\,|U_\lambda)^-(V)$ is τ^*-gs open in X for each $\lambda \in \Lambda$. So that $F^-(V) = \cup\ (F\,|U_\lambda)^-(V)$ is τ^*-gs open in X. Hence F is lower τ^*-gs continuous $\lambda \in \Lambda$.

Theorem: 3.20

If Y *is normal space and* $F_i : X_i \to Y$ *is upper* τ^*-gs *continuous multifunction such that* F_i *is point closed for* $i = 1, 2$, *then is* τ^*-gs *closed set in* $X_1 \times X_2$

Proof:

L$et\ (x_1, x_2) \in (X_i \times X_i)\backslash K. Then\ F_1(x_1) \cap F_2(x_2) = \emptyset$ Since Y_i is normal and F_i is point closed for i = 1, 2, there exist disjoint open sets $V_{1`}, V_{2`}$ such that $F_i(x_i) \subset V_i\ for\ i = 1, 2$. Since F_i is upper τ^*-gs continuous, $F^+(x_i)$ is τ^*-gs open for i = 1, 2. Put $U = F^+(V_{1`}) \times F^+(V_{2`})$, then U is τ^*-gs open and $(x_1, x_2) \in U \subset (X_{1`} \times X_{2`})\backslash K$. This shows that $(x_1 \times x_2)\backslash K$ is τ^*-gs open and hence K is τ^*-gs closed in $X_1 \times X_2$.

Theorem:3.21

Let $(X, \tau^*), (Y, \sigma)(Z, \omega)$ be topological spaces and let $F: X \to Y$ and $G: Y \to Z$ multifunctions. If $F: X \to Y$ is upper τ^*-gs continuous multifunctions and $G: Y \to Z$ is upper semi-continuous multifunctions, thenG_oF:$X \to Z$ is a upper τ^*-gs continuous multifunctions.

Proof:

let $V \subset Z$ be any open set. From the definition of G o F, we have $(G\circ F)^+$ (V)=$F^+(G^+(V)$. Since G is upper semi-continuous multifunctions it follows that $(G^+(V)$ is a open set. Since F is upper τ^*-gs continuous multifunctions, then $F^+(G^+(V))$ is a τ^*-gs open set. Hence G_oF is a upper τ^*-gs continuous multifunctions.

Theorem:3.22

Let $(X, \tau^*), (Y, \sigma)(Z, \omega)$ be topological spaces and let $F: X \to Y$ and $G: Y \to Z$ be multifunctions. If $F: X \to Y$ is lower τ^*-gs continuous multifunctions and $G: Y \to Z$ is lower semi-continuous multifunctions, then GoF:$X \to Z$ is a lower τ^*-gs continuous multifunctions.

Proof:

Let $V \subset Z$ be any open set. From the definition of $G\circ F$, we have $(G_oF)^-$ (V)=$F^-(G^-(V)$. Since G is lower semi-continuous multifunctions it follows that $(G^-(V)$ is a open set. Since F is lower τ^*-gs continuous multifunctions, it follows that $F^-(G^-(V))$ is a τ^*-gs open set and hence G_oF is a lower τ^*-gs continuous multifunctions.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

[1] R. Selvi, C. Aruna, τ^*- generalized semiclosed sets in topological spaces, Int. J. Eng. Sci. Res. Technol. 5(8) (2016), 621-625.

[2] S.P. Arya, T.M. Nour, Characterizations of s-normal spaces, Indian J. Pure Appl. Math. 21 (1990), 717–719.

[3] T. Banzaru, Multifunctions and M-product space (Romanian), Bul. St. Tehn. Inst. Politeh. “T. Vuia” Timisoara Mat. Fiz. Mec. Teor. Apl. 17(13) (1972), 17–23.

[4] C. Berge, Espaces Topologiques, Fonctions Multivoques, Dunod, Paris, 1959.

[5] H. Maki, P. Sundaram, K. Balachandran, On generalized continuous maps in topological spaces. Mem. Fac. Sci. Kochi Univ. A. 12 (1991), 5–13

[6] P. Bhattacharya, B.K. Lahiri, Semi-generalized closed sets in topology, Indian J. Math. 29 (1987), 375–382.

[7] C. Dorsett, Semi-regular spaces, Soochow J. Math. 8 (1982), 45–53.

[8] J. Dontchev, On generalizing semi-preopen sets, Mem. Fac. Sci. Kochi Univ. Ser. A. Math. 16 (1995), 35–48.

[9] W. Dunham, A new closure operator for non-T_1 topologies, Kyungpook Math. J. 22(1982), 55-60

[10] S. Eswaran, A. Pushpalatha, τ^*-generalized continuous functions in topological spaces. Int. J. Math. Sci. Eng. Appl. 3 (2009), 67-76.

[11] S. Eswaran, N. Nagaveni. τ^* - generalized semi continuous functions in topological spaces, Int. J. Math. Anal. 7 (2013), 2091-2100.

[12] I. Kovac, Subsets and paracompactness, Univ. u Novom Sadu, Zb. Rad. Prirod.-Mat. Fak. Ser. Mat. 14 (1984), 79–87.

[13] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Mon. 70 (1963), 36-41.

[14] N. Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo. 19 (1970) 89–96.

[15] A. Pushpalatha, S. Eswaran, P. Rajarubi, τ^* -generalized closed sets in topological spaces, in Proceedings of the World Congress on Engineering 2009, Vol II WCE 2009, London, 2009.

[16] D. Wine, Locally paracompact spaces, Glasnik Mat. 10(30) (1975), 351–357.