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HIGHER DIMENSIONAL PERFECT FLUID COSMOLOGICAL MODEL IN GENERAL RELATIVITY WITH QUADRATIC EQUATION OF STATE (EoS)

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Abstract: Higher dimensional Bianchi type-V cosmological model in the general theory of relativity with quadratic equation of state interacting with perfect fluid has been studied. For higher dimensional Bianchi type-V space time, the general solutions of the Einstein's field equations have been obtained under the assumption of quadratic equation of state (EoS) $p = \alpha\rho^2 - \rho$, where $\alpha \neq 0$ is arbitrary constant. The physical and geometrical aspects of the model are discussed.

Keywords: higher dimensions; Bianchi type-V; perfect fluid; quadratic EoS.

2010 AMS Subject Classification: 85A40, 58D30, 83F05.

1. INTRODUCTION

It is now almost proved from theoretical and astronomical observations such as type Ia supernovae (Riess et al. [1], Perlmutter et al. [2], Riess et al. [3]) that the universe is expanding

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with acceleration from the big-bang till today. However, no one can guarantee for forever expansion because there is no final conclusion about the expansion or contraction of universe till today and represents an open question for theoretical physicists which inspired numerous cosmologists and physicists for similarly research within the area of studies on this field to resolve this problem including the modified theory of gravity and possible existence of dark energy (DE). From various literatures and opinions it can be belief that the acceleration of present universe may be accompanied by way of deceleration. In the recent past years, several models in cosmology has been proposed by different authors in order to explain the hidden reasons of expansion of the existing universe with the acceleration in the framework of general theory of relativity and modified theories. Spatially homogeneous and anisotropic Bianchi type cosmological model plays a great role to describe the large-scale behavior of the universe. Modern cosmology revolves around the study about the past and present state of the universe and how it will evolve in future.

In the study of universe, among different models of Bianchi types, Bianchi type-V cosmological models is the natural generalization of the open Friedmann-Robertson-Walker(FRW) model which plays an important role in the description of universe and more interesting is that these models contain isotropic special cases and it may permit small anisotropy levels at some of time. There are theoretical arguments from the recent experimental data which support the existence of an anisotropic phase approaching to isotropic phase leading to the models of the universe with anisotropic background. Cosmological models which are spatially homogeneous and anisotropic play significant roles in the description of the universe at its early stages of evolution. Some authors such as Maartens and Nel [4], Meena and Bali [5] have studied Bianchi type-V models in different contexts. Coley [6] have investigated Bianchi type-V spatially homogeneous universe model with perfect fluid cosmological model which contains both viscosity and heat flow. In recent years, the solution of Einstein's field equation for homogeneous and anisotropic Bianchi type models have been studied by several authors such as Hajj-Boutros [7], Shri Ram [8,9], Pradhan and Kumar [10] by using different generating techniques. Ananda and Bruni [11] discussed the cosmological models by considering different form and non-linear quadratic equation of state. A few models that describe an anisotropic space time and generate particular interest are among Lorenz and Pestzold [12], Singh and Agrawal[13], Marsha [14], Socorro and Medina [15]. Furthermore, from several kinds of literature and findings one can actually locate that the anisotropic model had been taken as possible models to initiate the expansion of the universe. Banerjee and Sanyal [16] have considered Bianchi type-V cosmological models with bulk viscosity and heat flow. Conformally flat tilted Bianchi type-V cosmological models in the presence of a bulk viscous fluid are investigated by Pradhan and Raj [17]. Kandalkar et al. [18] discussed the variation law of Hubble's parameter, average scale factor in spatially homogeneous anisotropic Bianchi type-V space time filled with viscous fluid where the universe exhibits power law and exponential expansion.

In relativity and cosmology, the equation of state is nothing but the relationship among combined matter, pressure, temperature, energy and energy density for any region of space, plays an important role in the study about universe. For the study of dark energy and general relativistic dynamics in different cosmological models, the quadratic EoS plays an important role. Many researchers like Ivanov [19], Sharma and Maharaj [20], Thirukkanesh and Maharaj [21], Varela et al. [22] etc. studied cosmological models with linear and non-linear equation of states. Various authors like Nojiri and Odintsov [23,24,25], Nojiri et al. [26], Capozziello et al. [27], Bamba et al. [28] already discussed dark energy universe with different equations of state(EoS).

The general form of quadratic equation of state(EoS)

$$p = \rho_0 + \alpha\rho + \beta\rho^2$$

is nothing but the first term of the Taylor's expansion of an equation of state of the form $p = p(\rho)$ about the $\alpha = 0$, where ρ_0, α and β are the parameters.

Ananda and Bruni[29] investigated the general relativistic dynamics of Robertson-Walker models considering a non-linear equation of state(EoS) in the form of a quadratic equation $p = \rho_0 + \alpha\rho + \beta\rho^2$. They have shown that in general relativistic theory setting, the anisotropic behavior at the singularity obtained in the Brane Scenario can be reproduced. In the general theory of relativity, they have also discussed the anisotropic homogeneous and inhomogeneous cosmological models with the consideration of quadratic equation of state of the form $p = \alpha\rho + \frac{\rho^2}{\rho_c}$ and attempted to isotropize the model universe in the initial stages when the initial singularity is approached. In this paper, we have considered the quadratic equation of state of the form $p = \alpha\rho^2 - \rho$, where $\alpha \neq 0$ is a constant quantity, here we take $\rho_0 = 0$ to make our calculations easier without affecting the quadratic nature of the equation of state.

Chavanis[30] investigated a four-dimensional Friedmann-Lemaitre-Roberston-Walker (FLRW) cosmological model unifying radiation, vacuum energy and dark energy by considering an equation of state in quadratic nature. Again, by using a quadratic form of equation of state, Chavanis[31] formulated a cosmological model that describes the early inflation, the intermediate decelerating expansion, and the late-time accelerating expansion of the universe. Many authors like Maharaj et al.[32], Rahaman et al.[33], Feroze and Siddiqui[34] have investigated cosmological models on the basis of EoS in quadratic form under different circumstances. Recently, V. U. M. Rao et al.[35], Reddy et al.[36], Adhav et al.[37], MR Mollah and KP Singh[38], MR Mollah et al.[39], Beesham et al.[40] studied different space-time cosmological models with a quadratic EoS in general and modified theories of relativity.

A cosmological model in higher-dimensions performs a crucial role in different aspects of the early phases of the cosmological evolution of the universe and are not observable in real universe. It is not possible to unify the gravitational forces in nature in typical four-dimensional space-times. So the theory in higher dimensions may be applicable in the early evolution. The study on higher-dimensional space-time gives us an important idea about the universe that our universe was much smaller at initial epoch than the universe observed in these days. Due to these reasons studies in higher dimensions inspired and motivated many researchers to enter into such a field of study to explore the hidden knowledge of the present universe. Subsequently, many researchers have already investigated various cosmological models in five dimensional space-time with various Bianchi type models in different aspects [35,38,41,42,43,44].

Inspired by the above studies, here in this article we have investigated the higher dimensional cosmological model in Bianchi type-V space–time considering quadratic equation of state(EoS) interacting with perfect fluid . In this paper, Sec. 2 describes the formulation of problem. Sec.3. gives the solutions of the cosmological problems. In sec.4, some of the important physical and geometrical parameters are derived. The results found are discussed in Sec.5. Finally, in Sec.6, concluding points are provided.

2. THE METRIC AND FIELD EQUATION

We consider the higher dimensional Bianchi type-V cosmological model in the form

$$ds^2 = -dt^2 + a^2dx^2 + e^{-2x}(b^2dy^2 + c^2dz^2 + D^2dm^2) \tag{1}$$

Where a, b, c and D are functions of time only and m is the extra dimension (Fifth dimension) which is space-like.

The energy momentum tensor for the perfect fluid is given by

$$T_{ij} = (p+\rho)u_iu_j + pg_{ij} \tag{2}$$

Where ρ is the energy density, p is the pressure and $u^i = (0,0,0,0,1)$ is the five velocity vector of particles.

Also

$$g_{ij}u^iu^j = 1 \tag{3}$$

We have assumed an equation of state in the general form $p = p(\rho)$ for the matter distribution. In this case, we consider the quadratic form as

$$p = \alpha\rho^2 - \rho \tag{4}$$

Where $\alpha \neq 0$ is arbitrary constant.

The Einstein's field equations in general relativity is given by

$$R_{ij} - \frac{1}{2}g_{ij}R = -T_{ij} \tag{5}$$

If R(t) be the average scale factor, then the spatial volume is given by

$$V = abcD = R^4 \tag{6}$$

Where R(t) is given by

(7) $$R(t) = (abcD)^{\frac{1}{4}}$$

The expansion scalar is defined as

(8) $$\theta = \frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c} + \frac{\dot{D}}{D}$$

The Hubble's parameter is defined as

(9) $$H = \frac{\dot{R}}{R} = \frac{1}{4}\theta = \frac{1}{4}\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c} + \frac{\dot{D}}{D}\right)$$

Also we have

(10) $$H = \frac{1}{4}(H_1 + H_2 + H_3 + H_4)$$

Here, $H_1 = \frac{\dot{a}}{a}$, $H_2 = \frac{\dot{b}}{b}$, $H_3 = \frac{\dot{c}}{c}$ and $H_4 = \frac{\dot{D}}{D}$ are directional Hubble's factors in the directions of x, y, z and m respectively.

Deceleration parameter q is defined as

(11) $$q = -\frac{R\ddot{R}}{R^2}$$

The mean anisotropy parameter is defined as

(12) $$\Delta = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2$$

The shear scalar is defined as

(13) $$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left(\frac{\dot{a}^2}{a^2} + \frac{\dot{b}^2}{b^2} + \frac{\dot{c}^2}{c^2} + \frac{\dot{D}^2}{D^2} - \frac{\theta^2}{4}\right)$$

Using the equations (1)-(3) and (5) we obtain

(14) $$\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\ddot{D}}{D} + \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{b}\dot{D}}{bD} + \frac{\dot{c}\dot{D}}{cD} - \frac{3}{a^2} = -p$$

(15) $$\frac{\ddot{a}}{a} + \frac{\ddot{c}}{c} + \frac{\ddot{D}}{D} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{c}\dot{D}}{cD} - \frac{3}{a^2} = -p$$

(16) $$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{D}}{D} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{b}\dot{D}}{bD} - \frac{3}{a^2} = -p$$

(17) $$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{b}\dot{c}}{bc} - \frac{3}{a^2} = -p$$

(18) $$\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{b}\dot{D}}{bD} + \frac{\dot{c}\dot{D}}{cD} - \frac{6}{a^2} = \rho$$

(19) $$3\frac{\dot{a}}{a} = \frac{\dot{b}}{b} + \frac{\dot{c}}{c} + \frac{\dot{D}}{D}$$

The overhead dots here denote the order of differentiation with respect to time `t’.

The conservational law of the energy momentum tensor is

(20) $$\dot{\rho} + (\rho + p)\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c} + \frac{\dot{D}}{D}\right) = 0$$

3. COSMOLOGICAL SOLUTION

From Equation (19) we get

(21) $$a^3 = kbcD$$

Here, k is the constant of integration. Without loss of generality we can choose $k = 1$ and so we get

(22) $$a^3 = bcD$$

Subtracting (14) from (15), we get

(23) $$\frac{\ddot{a}}{a} - \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{c}}{ac} - \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{a}\dot{D}}{aD} - \frac{\dot{b}\dot{D}}{bD} = 0$$

Subtracting (15) from (16), we get

(24) $$\frac{\ddot{b}}{b} - \frac{\ddot{c}}{c} + \frac{\dot{a}\dot{b}}{ab} - \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{b}\dot{D}}{bD} - \frac{\dot{c}\dot{D}}{cD} = 0$$

Subtracting (16) from (17), we get

(25) $$\frac{\ddot{c}}{c} - \frac{\ddot{D}}{D} + \frac{\dot{a}\dot{c}}{ac} - \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{b}\dot{c}}{bc} - \frac{\dot{b}\dot{D}}{bD} = 0$$

Subtracting (17) from (14), we get

(26) $$\frac{\ddot{D}}{D} - \frac{\ddot{a}}{a} + \frac{\dot{b}\dot{D}}{bD} - \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{c}\dot{D}}{cD} - \frac{\dot{a}\dot{c}}{ac} = 0$$

From the equations (23)-(26) the following equations are obtained

(27) $$\frac{\dot{a}}{a} - \frac{\dot{b}}{b} = \frac{k_1}{abcD}$$

(28) $$\frac{\dot{b}}{b} - \frac{\dot{c}}{c} = \frac{k_2}{abcD}$$

(29) $$\frac{\dot{c}}{c} - \frac{\dot{D}}{D} = \frac{k_3}{abcD}$$

(30) $$\frac{\dot{D}}{D} - \frac{\dot{a}}{a} = \frac{k_4}{abcD}$$

Here $k_1, k_2,$ k_3 and k_4 are the constant of integration. Without loss of generality, we can choose $k_1 = k_2 = k_3 = k_4 = k$(say)

The Equations (27)-(30) yields

(31) $$a = b = c = D$$

We consider negative constant deceleration parameter model defined by

(32) $$q = -\frac{R\ddot{R}}{\dot{R}^2} = \text{constant}$$

The solution of above equation is

(33) $$R = (Lt + M)^{\frac{1}{1+q}}$$

Where $L \neq 0, M$ are constants and $q + 1 \neq 0$

Using (7), (22), (31) and (33) we obtain

(34) $$a = b = c = D = (Lt + M)^{\frac{1}{1+q}}$$

The model is described by the following metric

(35) $$ds^2 = -dt^2 + (Lt + M)^{\frac{2}{1+q}} dx^2 + e^{-2x}(Lt + M)^{\frac{2}{1+q}}(dy^2 + dz^2 + dm^2$$

4. COSMOLOGICAL PARAMETERS

Some of the important Physical and Geometrical Parameters are obtained bellow:

Volume element of the model is

(36) $$V = (Lt + M)^{\frac{4}{1+q}}$$

The Hubble's Parameter is given by

(37) $$H = \frac{L}{(q+1)(Lt+M)}$$

The expansion scalar (θ) is given by

(38) $$\theta = \frac{4L}{(q+1)(Lt+M)}$$

Using equation (4), (34) the equation (20) yields

(39) $$\rho = \frac{q+1}{4\alpha \log(Lt+M) + (q+1)k_5}$$

Where k are arbitrary constant.

From equations (4) and (39) becomes

(40) $$p = -\frac{(q+1)[3\alpha\log(Lt+M)-(q+1)(k_5+\alpha)]}{[3\alpha\log(Lt+M)+(q+1)k_5]^2}$$

The shear scalar is

(41) $$\sigma^2 = 0$$

The average anisotropy parameter is

(42) $$\Delta = 0$$

5. DISCUSSION

The variation of some of the important cosmological parameters (Physical and Geometrical) with respect to time for the model are shown below by taking, $q = -0.5,\ L = M = 1,\ k_5 = 2$ and $\alpha = 1$.

From the equation (36) it is seen that the spatial volume V is finite when time, $t = 0$. With the increases of the time the spatial volume V also increases and it becomes infinitely large as $t \to \infty$, so spatial volume expands as shown in fig.1. This shows that the universe is expanding with the increases of time.

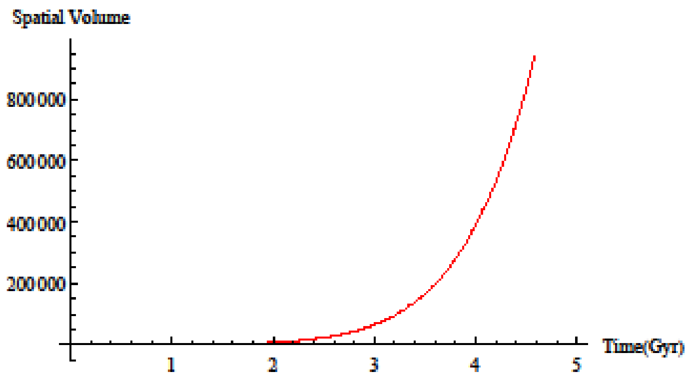

Fig1.Variation of Volume ‘V’ Vs. Time t.

From the equations (37) and (38) we observed that the quantities like Hubble's parameter H and expansion scalar θ are decreasing functions of time and they are finite when, $t = 0$ and are vanishing for infinitely large value of $'t'$ which are shown in the fig2 bellow. Also, since $\frac{dH}{dt} < 0$ which also tells us that our present universe is in the accelerated expanding mode.

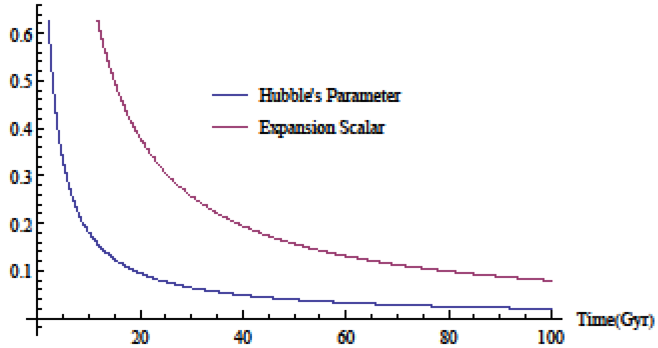

Fig2.Variation of H, θ Vs. Time ‘t’.

We observe from fig.3 or from equation (39), that the evolution of the energy density ρ is large constant at the time $t = 0$ and with increases of time the energy density decreases and finally when time $t \to \infty$ the energy density ρ becomes zero. This shows that the universe is expanding with time.

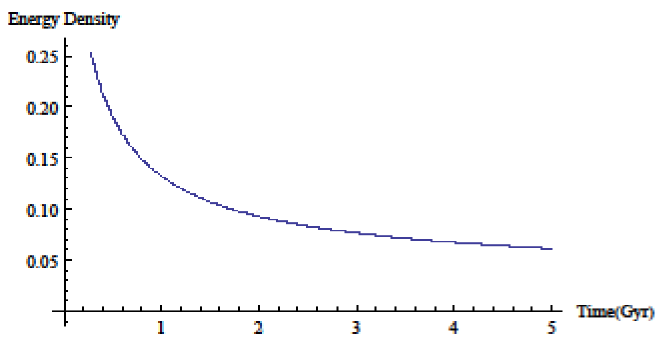

Fig3.Variation of energy density ‘ρ’ Vs. Time ‘t’.

While from the equation (40) it is observed that the isotropic pressure p is a small positive constant at initial epoch, $t = 0$ and it decreases with progresses of time and changes sign from positive to negative as shown in the fig4. Thus the present model universe passes through the transition from matter dominated era to inflationary era.

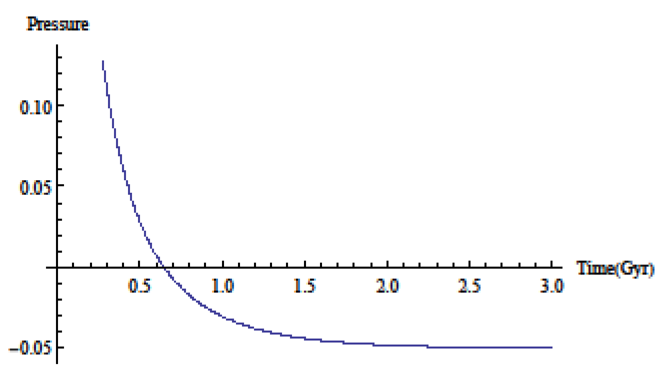

Fig4.Variation of p Vs. Time t.

For this model the value of shear scalar σ is zero throughout the evolution as seen from the equation (41), so the model is shear free. The ratio $\frac{\sigma^2}{\theta^2}$ tends to zero as time tends to infinity, showing that the model approaches isotropy at the late time universe, which agrees with Collins and Hawking [45].

From the equation (42) it is seen that average anisotropy parameter, $\Delta = 0$, so the model is isotropic one and this model also shows the late time acceleration of the universe.

It is observed that the spatial volume, all the three scale factors and all other physical and kinematical parameters are constant at initial epoch, $t = 0$. This shows that the present model is free from the initial singularity.

6. CONCLUSION

Here in this paper we studied a Bianchi type-V cosmological model in the context of general theory of relativity with quadratic EoS interacting with perfect fluid in five-dimensional space-time. The general solutions of the Einstein's field equations have been obtained under the assumption of quadratic EoS, $p = \alpha\rho^2 - \rho$, where $\alpha \neq 0$ is arbitrary constant considering the deceleration parameter as a constant quantity. The model obtained here is expanding with accelerated motion which agrees the recent observational data. The model is isotropic throughout the evolution, non-sharing and free from the initial singularity. We observe from eqn. (39) that the evolution of the energy density ρ is constant at the time $t = 0$ and when time $t \to \infty$ the energy density ρ decreases to a finite constant value. While eqn. (40) indicates that the isotropic pressure (p) is a negative quantity and it decreases when time $t \to \infty$. Such type of solution is consistent with recent observational data like SNe1a. The negative pressure may be a possible cause of the accelerated expansion of the universe. Also, generally the negative pressure can resist the attractive gravity of matter.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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