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J. Math. Comput. Sci. 11 (2021), No. 6, 8240-8257

<https://doi.org/10.28919/jmcs/6785>

ISSN: 1927-5307

NONLINEAR THERMAL RADIATION AND TEMPERATURE DEPENDENT VISCOSITY EFFECTS ON MHD HEAT AND MASS TRANSFER IN A THIN LIQUID FILM OVER A STRETCHING SURFACE

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Abstract. This paper describes nonlinear thermal radiation effects on MHD heat and mass transfer in a thin liquid film over a permeable unsteady stretching surface taking temperature-dependent fluid viscosity with convective boundary condition. For the non-linearity of the momentum, energy and mass diffusion equations, the problem is solved numerically. At first, Similarity transformations is used to the governing equations to reduce the equations into a set of ordinary differential equations. Then the resulting nonlinear ordinary differential equations are solved using Runge-Kutta-Felberg method with shooting technique. Different physical parameters effects on heat and mass transfer in a thin liquid film are presented graphically. It is found that increase in the unsteadiness parameter leads to increase in the velocity distribution, temperature and concentration gradient. Further, increase in the value of magnetic parameter results in a decrease in the velocity profile and increase in the temperature and concentration gradient. For enhancement of thermal radiation decreases the temperature gradient of the thin film flow. Also, for increase in viscosity variation parameter is to decrease velocity distribution but reverse effects shown in case of temperature and concentration gradient.

Keywords: thermal radiation; thin liquid film; variable viscosity; magnetohydrodynamic; similarity transformation.

2010 AMS Subject Classification: 76N20, 80A20.

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Received September 14, 2021

1. INTRODUCTION

In recent years magnetohydrodynamic heat and mass transfer on a thin liquid film over stretching surface have become more important in number of engineering application, science and technology such as wire and fiber coating, metal and polymer extrusion, cooling of metallic plates, drawing of polymer sheets and thinning of copper wires, aerodynamic extrusion of plastic sheet, artificial fibers, glass fiber, continuous stretching of plastic films.

First, Crane [1] gives an exact solution for the problem of steady two-dimensional boundary layer flow caused by the stretching of a sheet. Wang [2] studied the flow within a thin liquid film over an unsteady stretching surface. Later, Andersson et al. [3] extended Wang's problem to study heat transfer. The effect of variable thermal properties on flow and heat transfer in a liquid film for viscous Newtonian fluid over a unsteady stretching sheet was studied by Dandapat et al. [4]. Lai and Kulacki [5] analyzed the effects of variable viscosity on mixed convection heat transfer along a vertical surface in a saturated porous medium considering Newtonian fluid. The heat transfer in a liquid film on an unsteady stretching surface with viscous dissipation in the presence of external magnetic field was investigated by Abel et al. [6]. Siti et al. [7] investigated hydromagnetic boundary layer flow over stretching surface with thermal radiation. Hazarika et al. [8] studied the effect of variable viscosity and thermal conductivity on MHD flow past a vertical plate. Mohebujjaman et al. [9] considered MHD heat transfer mixed convection flow along a vertical stretching sheet in the presence of magnetic field with heat generation. Agrawal et al. [10] studied MHD flow past a stretching surface embedded in porous medium using Lie similarity analysis along with variable viscosity. Ali [11] observed the effect of variable viscosity on mixed convection heat transfer along a moving surface. Pantokratoras [12] made a theoretical study to investigate the effect of variable viscosity on flow and heat transfer on a continuous moving plate. Mukhopadhaya et al. [13] studied the effect of variable viscosity on the boundary layer flow through a porous medium towards a stretching sheet in the presence of heat generation or absorption. Heat transfer in a thin liquid film over a unsteady stretching sheet in the presence of thermal radiation subject to variable surface heat flux conditions was studied by Liu and Megahed [14]. Cortell [15] analyzed heat transfer and viscoelastic fluid flow over a stretching sheet under the effect of a non uniform heat source, viscous dissipation and thermal

radiation. Pantokratorus and Fung [16] used the Rosseland diffusion approximation to study radiative non-linear heat transfer in different geometries. Aziz et. al. [17] studied heat transfer in a liquid film over a permeable stretching sheet. The variable viscosity with magnetic field on flow and heat transfer to a continuous moving flat plate were reported in Seddek and Salem [18]. Nadeem and Akbar [19] observed the effects of heat transfer on MHD Newtonian fluid with variable viscosity. Benazir and Sivaraj [20] observed the effect of unsteady MHD casson fluid over a vertical cone and flat plate saturated with porous medium and non-uniform heat source/sink. Kumar and Sivaraj [21] investigated heat and mass transfer in MHD viscoelastic fluid flow over a vertical cone and flat plate with variable viscosity.

The motivation of present study is to investigate the influence of non-linear thermal radiation on MHD heat and mass transfer with temperature dependent variable viscosity in a thin liquid film on a permeable unsteady stretching sheet with convective boundary condition. This problem may have useful applications such as wire coating and food processing.

2. FORMULATION OF THE PROBLEM

Consider two dimensional unsteady fluid flow of a Newtonian fluid in a thin liquid film over a permeable stretching surface with variable viscosity and magnetic parameter. It is assumed that the elastic sheet emerges from a narrow slit at the origin of a Cartesian co-ordinate system. The continuous surface aligned with the x -axis at $y=0$ moves in its own plane with a velocity $U(x,t)$ (see Fig. 1). A thin liquid film of uniform thickness $h(t)$ lies on the horizontal surface. The surface heat flux $q_t(x,t)$ at the stretching sheet varies with the power of distance x from the slit and with the inverse power of time factor t as [22]

$$(1) \quad q_t(x,t) = -k \frac{\partial T}{\partial y} = -T_{ref} \frac{dx^2}{(1-at)^2}$$

The surface mass flux $q_m(x,t)$ at the stretching sheet varies with the power of distance x from the slit and with the inverse power of time factor t as [23]

$$(2) \quad q_m(x,t) = -D \frac{\partial C}{\partial y} = -C_{ref} \frac{dx^2}{(1-at)^2}$$

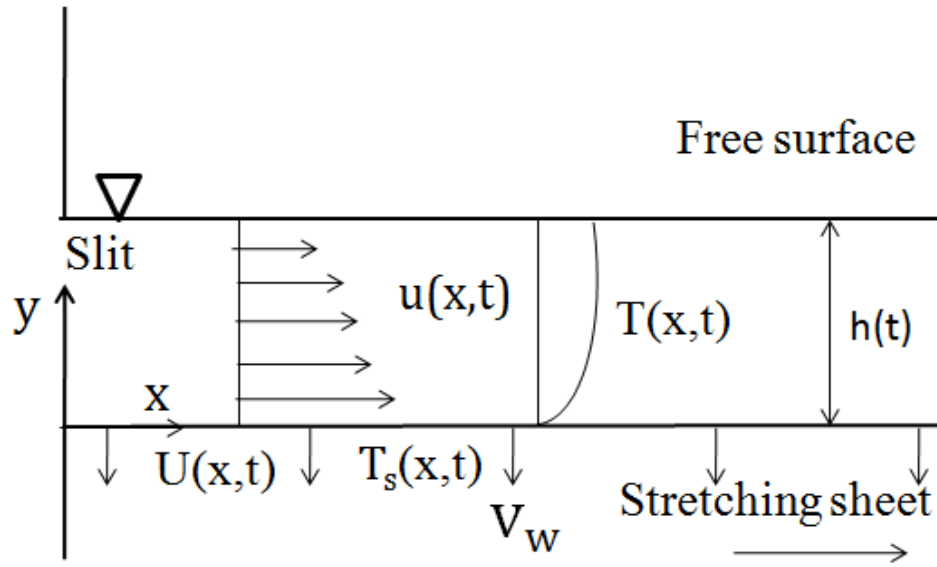


FIGURE 1. Geometrical configuration of the problem.

where k is the thermal conductivity, T_{ref} is reference temperature, C_{ref} is reference concentration, d is a constant. The applied transverse magnetic field $B_1(t)$ is defined by [24]

$$B_1(t) = B_0(1 - at)^{-1/2}.$$

where B_0 is uniform magnetic field. The boundary layer equations mass, momentum and for energy conservation are given by,

$$(3) \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

$$(4) \quad \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) - \frac{\sigma B_1^2}{\rho} u,$$

$$(5) \quad \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{Q}{\rho c_p},$$

$$(6) \quad \frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2},$$

where u and v are components of velocity along the direction of x and y respectively. ρ is the

fluid density, t is time, μ is the variable viscosity of the fluid, q_r called the radiative heat flux and c_p is the specific heat at constant pressure. The term Q is the heat generated (> 0) per unit volume and absorbed (< 0) per unit volume is defined as (Liu and Megahed [22]):

$$(7) \quad Q = \frac{k\rho U}{\mu_h x} B^*(T - T_0),$$

where μ_h is constant viscosity, B^* denotes the temperature dependent heat generation or absorption. That is for the generation of heat B^* is positive and for the absorption of heat B^* is negative within the fluid system. Thus for the present problem the corresponding boundary conditions are:

$$(8) \quad \begin{aligned} u = U(x, t), v = v_w, -k \frac{\partial T}{\partial y} = q_t(x, t), -D \frac{\partial C}{\partial y} = q_m(x, t) \text{ at } y = 0, \\ \frac{\partial u}{\partial y} = \frac{\partial T}{\partial y} = \frac{\partial C}{\partial y} = 0 \text{ at } y = h(t), \\ v = \frac{dh}{dt} \text{ at } y = h(t), \end{aligned}$$

where $U(x, t)$ is the surface velocity of the stretching sheet, h be the thickness of the liquid film. The stretching elastic surface at $y = 0$ moves continuously in x -direction with the velocity:

$$(9) \quad U = \frac{bx}{1 - at},$$

where b and a are both positive constant with dimension per time. The elastic sheet's temperature is assumed to vary both along the sheet and with time accordance with

$$(10) \quad T_s = T_0 - T_{ref} \left(\frac{dx^2}{k \sqrt{\rho b / \mu_h}} \right) (1 - at)^{-\frac{3}{2}},$$

where T_{ref} is the constant reference temperature.

The radiative heat flux q_r is taken according to Rosseland approximation as

$$(11) \quad q_r = -\frac{16\sigma^*}{3k^*} T^3 \frac{\partial T}{\partial y},$$

where σ^* is the Stefan-Boltzman constant, k^* be the mean absorption coefficient.

Now the system of partial differential equations transformed into a system of nonlinear ordinary

differential equation by using the similarity transformations which are given as follows

$$\begin{aligned}
 \eta &= \left(\frac{b}{\mu_h/\rho}\right)^{1/2}(1-at)^{-1/2}y, \\
 u &= bx(1-at)^{-1}f'(\eta), \\
 v &= -\left(\frac{\mu_h b}{\rho}\right)^{1/2}(1-at)^{-1/2}f(\eta), \\
 T &= T_0 - T_{ref}\left(\frac{dx^2}{k\sqrt{\rho b/\mu_h}}\right)(1-at)^{-3/2}\theta(\eta), \\
 C &= C_0 - C_{ref}\left(\frac{dx^2}{D\sqrt{\rho b/\mu_h}}\right)(1-at)^{-3/2}\phi(\eta),
 \end{aligned}
 \tag{12}$$

The dimensionless thin film thickness β is defined by

$$\beta = \left(\frac{b}{\mu_h/\rho}\right)^{1/2}(1-at)^{-1/2}h(t)
 \tag{13}$$

The temperature dependent fluid viscosity is given by (Batchelor [25]),

$$\mu = \mu_h[m + n(T_s - T)]
 \tag{14}$$

where μ_h is the constant value of the coefficient of viscosity far away from sheet and m, n are constants and $n(> 0)$.

This relation can be written in expanded form as, $\mu = \mu_h[m + A(1 - \theta)]$.

where $A = n(T_s - T_0)$, being viscosity variation parameter.

The transformed set of ordinary differential equations are:

$$f''' + (ff'' - f^2 - Sf' - \frac{S}{2}\eta f'' - Mf') - \frac{A}{m + A(1 - \theta)}\theta' f'' = 0,
 \tag{15}$$

$$\begin{aligned}
 \frac{1}{Pr}\theta'' + \frac{1}{Pr}Nr(1 + (\theta_w - 1)\theta)^2[3(\theta_w - 1)\theta'^2 + (1 + (\theta_w - 1)\theta)\theta''] \\
 + [f\theta' - 2f'\theta - \frac{3}{2}S\theta - \frac{S}{2}\eta\theta' + \frac{B^*}{Pr}\theta] = 0,
 \end{aligned}
 \tag{16}$$

$$\phi'' - Sc[\frac{3S}{2}\phi + \frac{S}{2}\eta\phi' + 2f'\phi - f\phi'] = 0,
 \tag{17}$$

subject to the boundary conditions:

$$\begin{aligned}
 f(0) = f_w, f'(0) = 1, \theta'(0) = -1, \phi'(0) = -1, \\
 f''(\beta) = 0, \theta'(\beta) = 0, \phi'(\beta) = 0, \\
 f(\beta) = \frac{S\beta}{2},
 \end{aligned}
 \tag{18}$$

where prime represent differentiation with respect to η , $S = \frac{a}{b}$ be the unsteadiness parameter, $Pr = \frac{\mu_h c_p}{k}$ be the Prandtl number, β be the dimensionless thin film thickness, $Nr = \frac{16\sigma^* T_0^3}{3k^*k}$ be the radiation parameter, $\theta_w = \frac{T_s}{T_0}$ be the temperature ratio parameter, $Sc = \frac{\mu_h/\rho}{D}$ be the Schmidt number, $M = \frac{\sigma B_0^2}{\rho b}$ be the magnetic parameter, f_w being the permeability parameter.

3. NUMERICAL METHOD

The Runge-Kutta-Fehlberg method (RKF45) use to solve initial value problem

$$\frac{dy}{dx} = f(x, y), y(x_i) = y_i
 \tag{19}$$

It has a procedure to determine if the proper step size is being used. At each step, two different approximations are made and compared. If the two results are in close agreement, the approximation is accepted. If the two result do not give the specified accuracy, the step size is reduced. If the result agree to more significant digits than required, the step size is increased. Each step requires the use of following six values since this is fifth order method with six stages that uses all the points of the first one. Then an approximation to the solution of the initial value problem (IVP) is made using a Runge-Kutta method of order 4;

$$y_{k+1} = x_k + \frac{25}{216}K_1 + \frac{1408}{2565}K_3 + \frac{2197}{4104}K_4 - \frac{1}{5}K_5
 \tag{20}$$

where

$$\begin{aligned}
 K_1 &= hf(x_k, y_k) \\
 K_2 &= hf(x_k + \frac{1}{4}h, y_k + \frac{1}{4}K_1) \\
 K_3 &= hf(x_k + \frac{3}{8}h, y_k + \frac{3}{32}K_1 + \frac{9}{32}K_2) \\
 K_4 &= hf(x_k + \frac{12}{13}h, y_k + \frac{1932}{2197}K_1 - \frac{7200}{2197}K_2 + \frac{7296}{2197}K_3)
 \end{aligned}$$

$$K_5 = hf(x_k + h, y_k + \frac{439}{216}K_1 - 8K_2 + \frac{3680}{513}K_3 - \frac{845}{4104}K_4)$$

where the four functional values K_1, K_3, K_4 and K_5 are used. A better value for the solution is determine using Runge-Kutta method of order 5;

$$(21) \quad z_{k+1} = y_k + \frac{16}{135}K_1 + \frac{6656}{12825}K_3 + \frac{28561}{56430}K_4 - \frac{9}{50}K_5 + \frac{2}{55}K_6$$

where,

$$(22) \quad K_6 = hf(x_k + \frac{1}{2}h, y_k - \frac{8}{27}K_1 + 2K_2 - \frac{3544}{2565}K_3 + \frac{1859}{4104}K_4 - \frac{11}{40}K_5)$$

The optimal step size sh is determined by multiplying the scaler s times the current step size h , where the scaler s can be determined from;

$$(23) \quad s = 0.84(\frac{Tolh}{2|z_{k+1} - y_{k+1}|})^{1/4}$$

where Tol is the specified error control tolerance.

The non-linear differential Eqs. (15),(16) and (17) with appropriate boundary conditions (18) are solved numerically by using Runge-Kutta-Fehlberg (RKF) fifth order technique along with shooting method. At a very first step, the higher order non-linear differential equations (15),(16) and (17) are converted into simultaneous differential equation of first order and further they are transformed into initial value problem by applying the shooting technique. Then initial value problem is solved by Runge-Kutta-Fehlberg fifth order method. The ordinary differential equations (15) to (17) which are of third order in f , second order in θ and second order in ϕ are reduced to a system of seven simultaneous equations of first order having seven unknowns. The convergence criterion is employed in the present work based on the difference between the value of the dependent variables of the present and previous iterations. When the absolute values of the difference reaches 10^{-6} which showed that the solution has converged to the desired accuracy then the iteration process is stopped. The governing non-linear ordinary differential equations are reduced to a set of simultaneous first order differential equation as follows,

$$y_1 = f, y_2 = f', y_3 = f'', y_4 = \theta, y_5 = \theta', y_6 = \phi, y_7 = \phi',$$

$$F_1 = y_2, F_2 = y_3, F_3 = -(y_1 y_3 - y_2^2 - S y_2 - \frac{S}{2} \eta y_3 - M y_2) + A(m + A(1 - y_4)) y_5 y_3, F_4 = y_5,$$

$$F_5 = -((3Nr(\theta_w - 1)y_5^2(1 + (\theta_w - 1))y_4)^2 + Pr(y_1 y_5 - 2y_2 y_4 - \frac{3}{2} S y_4 - \frac{3}{2} \eta y_4 + \frac{B^*}{Pr} y_4)) / (1 + Nr(1 + (\theta_w - 1)y_4)^3),$$

$$F_6 = y_7, F_7 = Sc(\frac{3S}{2} y_6 + \frac{S}{2} \eta y_7 + 2y_2 y_6 - y_1 y_7).$$

The boundary condition becomes

$$y_1 = f_w, y_2 = 1, y_5 = -1, y_7 = -1, \text{ at } \eta = 0$$

$$y_3 = 0, y_5 = 0, y_7 = 0 \text{ at } \eta = \beta$$

Since the values of $y_3(0)$, $y_4(0)$, $y_6(0)$ are not prescribed, so we have use the multiple shooting method to find three initial values. Then the resultant system of seven simultaneous equations is solved numerically by fifth-order Runge-Kutta-Fehlberg integration scheme (for detail see Pal and Saha [26])

4. RESULTS AND DISCUSSION

The system of highly non-linear differential equations (15)-(17) subject to the boundary conditions (18) is solved numerically by Runge-Kutta-Fehlberg numerical method with shooting technique. The effects of various important physical parameters such as unsteadiness parameter S , thermal radiation parameter Nr , Schmidt number Sc , temperature ratio parameter θ_w , Prandtl number Pr , magnetic parameter M and viscosity variation parameter A on non dimensional velocity components, temperature gradient, concentration gradient are analyzed and discussed in detail. Fig. 2 highlights the variations of velocity profile for different values of unsteadiness parameter S . From this figure we see that the velocity distribution increases for the increase of the value S . Also, it reduces the thin film thickness η . Fig. 3 represent the effect of unsteadiness parameter S on the temperature gradient of the thin film. For the increase of the value S ,

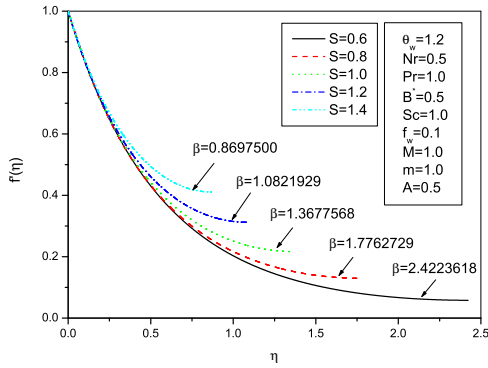


FIGURE 2. The effect of unsteadiness parameter S on velocity profile $f'(\eta)$.

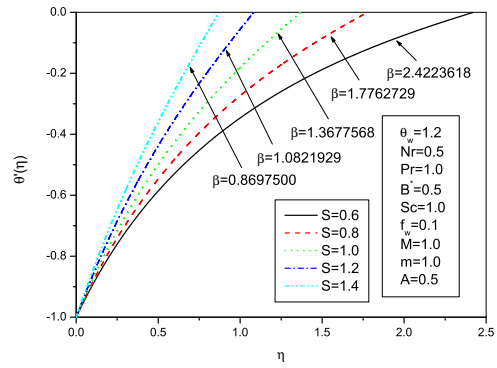


FIGURE 3. The effect of unsteadiness parameter S on temperature gradient $\theta'(\eta)$.

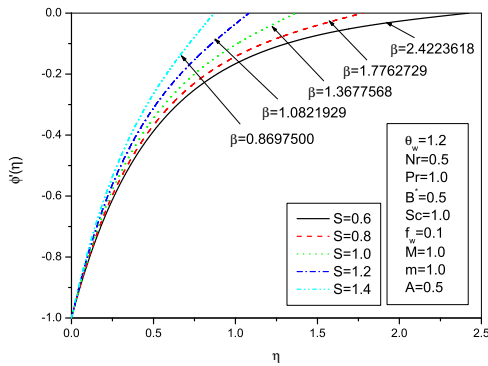


FIGURE 4. The effect of unsteadiness parameter S on concentration gradient $\phi'(\eta)$.

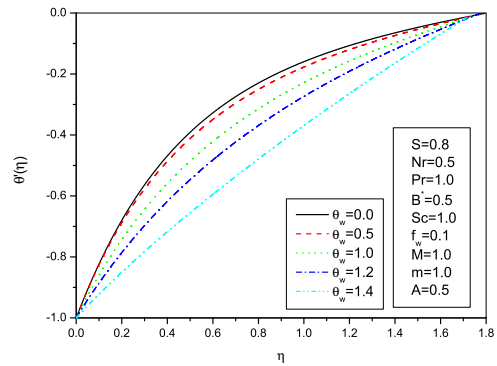


FIGURE 5. The effect of temperature ratio parameter θ_w on temperature gradient $\theta'(\eta)$.

the temperature gradient of the flow also increases, due to reduction in the dimensionless thin film thickness. So, the heat transfer rate increases within the thin liquid film. Fig. 4 shows the effect of unsteadiness parameter S on the concentration gradient of the thin film. It shows that increasing value of S , the concentration gradient of the flow also increase, due to the reduction in dimensionless thin film thickness. So mass transfer rate increases within the thin film liquid. Fig. 5 highlights the effect of temperature ratio parameter θ_w on the temperature gradient of the thin film flow. It is found that increase in the value of θ_w is to decrease in the temperature gradient of the flow. So the heat transfer rate decreases. Since $\theta_w = \frac{T_c}{T_0}$, i.e. θ_w is inversely

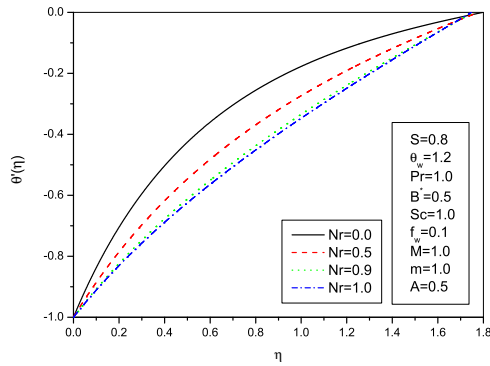


FIGURE 6. The effect parametere Nr on temperature gradient $\theta'(\eta)$.

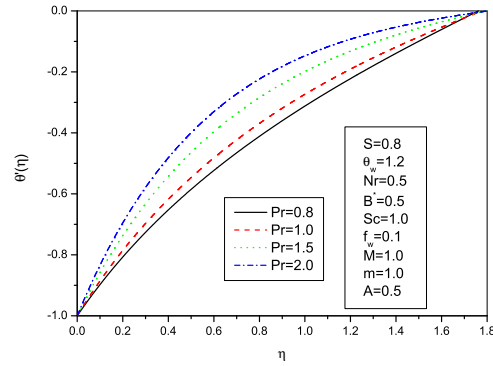


FIGURE 7. The effect of Prandtl number Pr on temperature gradient $\theta'(\eta)$.

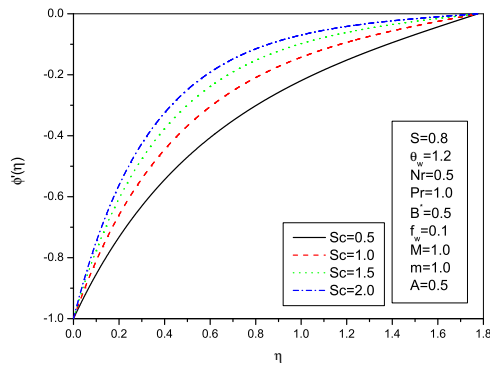


FIGURE 8. The effect of Schmidt number Sc on concentration gradient $\phi'(\eta)$.

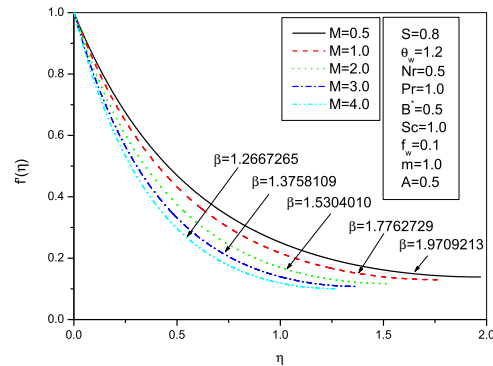


FIGURE 9. The effect of magnetic parametere M on velocity profile $f'(\eta)$.

proportional to T_0 , so when θ_w increase then the value of T_0 is decrease i.e. fluid flow system remain cool, due to the decrease of heat transfer rate. Fig. 6 displays the temperature gradient profile with η for different values of thermal radiation parameter Nr in the presence magnetic field. It is found that the effect of thermal radiation is to decrease the the temperature gradient in the thermal boundary layer. This is due to the fact that when radiation parameter Nr increases then from the expression for $Nr = \frac{16\sigma^* T_0^3}{3k^*k}$, the Rosseland radiative absorption coefficient k^* decreases and therefore the heat flux $q_r (= -\frac{16\sigma^*}{3k^*} T^3 \frac{\partial T}{\partial y})$ decreases.

Fig. 7 represent the variation of temperature gradient for different values of Prandtl number in

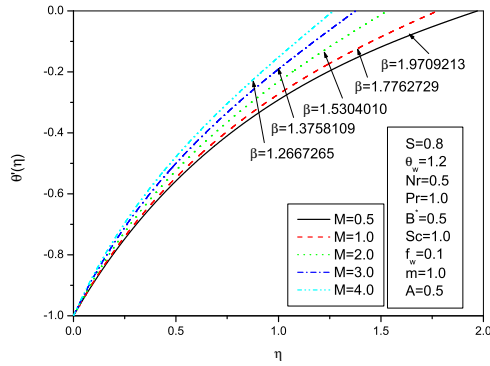


FIGURE 10. The effect of magnetic parameter M on temperature gradient $\theta'(\eta)$.

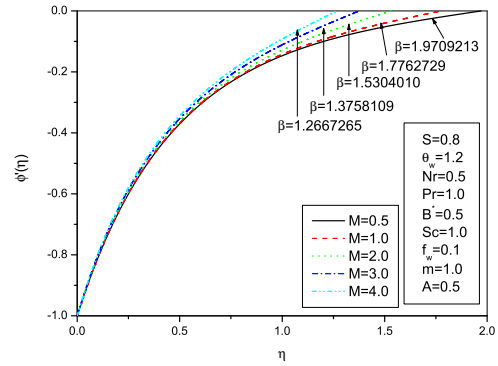


FIGURE 11. The effect of magnetic parameter M on concentration gradient $\phi'(\eta)$.

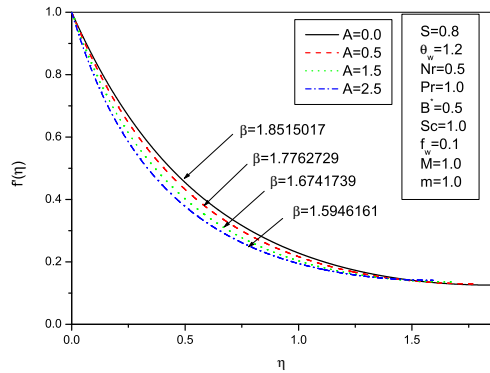


FIGURE 12. The effect of viscosity variation parameter A on velocity profile $f'(\eta)$.

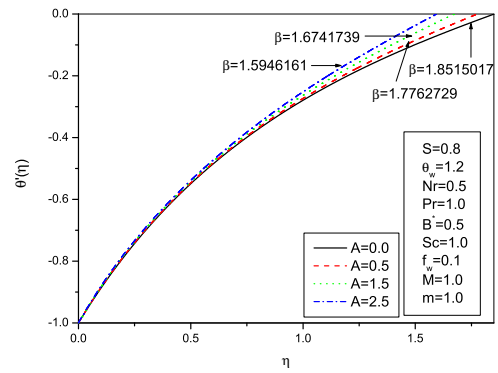


FIGURE 13. The effect of viscosity variation parameter A on temperature gradient $\theta'(\eta)$.

the presence magnetic field. it is found that the effect of increasing values of Prandtl number Pr , results to increase in temperature gradient. Since Pr being the ratio of momentum diffusivity and thermal conductivity for a fluid, so there would be decrease of thermal boundary layer thickness with an increase of Prandtl number. Fig. 8 shows the effect of the Schmidt number Sc on the concentration gradient against η for presence of magnetic field. It is observed that as Schmidt number increase then there is increase in the concentration gradient. This mean there is increase in mass transfer rate. Since the Schmidt number is inversely proportional to the diffusion coefficient D . Hence the concentration decreases with increase of Schmidt number Sc .

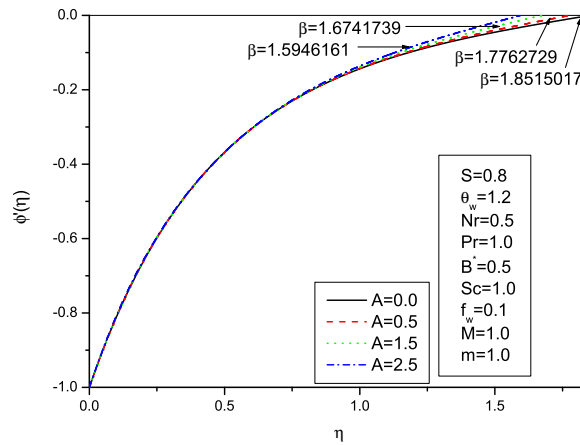


FIGURE 14. The effect of viscosity variation parameter A on concentration gradient $\phi'(\eta)$.

Fig. 9 show the effect of magnetic parameter M on the velocity profile. It is observed that the velocity decreases with η as the values of M increased. Thus the presence and increasing the magnetic field reduces the boundary layer thickness. Physically the presence of the transverse magnetic field gives rise to a drag like force known as Lorentz force, which results in retarding the velocity field. Fig. 10 shows the effect of magnetic parameter M on the temperature gradient. As the value of M increases then the temperature gradient also increase and thin film thickness decreases. So transverse magnetic field contributes to the thickening of the thermal boundary layer. Fig. 11 shows the effect of magnetic parameter M on the concentration gradient. For increasing of the magnetic field a drag force is produce which opposes the flow. Thus as the magnetic parameter M increases there results in increase the concentration gradient.

Fig. 12 shows the effect of the variable viscosity parameter A on the velocity profile. From the expression of A [$A = n(T_s - T_0)$] we see that as variable viscosity parameter A increase the temperature of the thin film which is contact to the sheet increases. i.e. the thin film stay contact to the sheet for much time than previous. So the velocity profile decrease for the increase of A . Fig. 13 shows the effect of the viscosity variation parameter A on the temperature gradient. By increasing A temperature gradient slowly increase. Also, the thin film thickness decreases. Fig. 14 shows the effect of viscosity variation parameter A on the concentration gradient. As

viscosity variation parameter increases, there is no change in the concentration gradient upto some values of η after that concentration gradient slowly increases.

5. CONCLUSION

In this paper the effects of temperature dependent viscosity and external magnetic field on thin liquid film flow of Newtonian fluid over a permeable unsteady stretching sheet in the presence of variable heat flux, non-linear thermal radiation are investigated and following conclusions are drawn.

- (i) The effect of the thermal radiation is to decrease the cooling rate of the thin liquid film, but reverse effect is true with the Prandtl number.
- (ii) Increasing in the magnetic parameter results in decrease in the velocity distribution and increase in the temperature gradients distribution and concentration gradients.
- (iii) The effect of viscosity variation parameter is to decrease velocity distribution in the momentum boundary layer. Also, increase in viscosity variation parameter results in increase in the temperature gradient and concentration gradient.
- (iv) The mass transfer rate increase with increase in the value of the Schmidt number.
- (v) Increase in the unsteadiness parameter results in increase in the velocity distribution due to decrease in the thin film thickness.

6. NOMENCLATURE

a, b, d	positive constant
A	viscosity variation parameter
B^*	temperature dependent heat generation or absorption parameter
B_0	uniform magnetic field
B_1	transverse magnetic field
C	concentration of the fluid
C_p	specific heat at constant pressure
C_{ref}	reference concentration
D	diffusion coefficient
f	dimensionless stream function
f_w	suction parameter
h	thickness of thin liquid film
k	thermal conductivity
k^*	Rosseland mean spectral absorption coefficient
M	magnetic parameter
Pr	Prandtl number
q	heat flux
q_m	surface mass flux at the stretching surface
q_r	radiative heat flux
q_t	surface heat flux at the stretching surface
Q	heat generation or absorption per unit volume
S	unsteadiness parameter
Sc	Schmidt number
t	time
T	temperature of fluid
T_0	temperature at the slit
T_{ref}	reference temperature

T_s elastic sheet temperature

u, v velocity component along x and y direction

U velocity of stretching sheet

v_w permeability parameter

x, y direction along and perpendicular to the plate, respectively

Greek symbols

β dimensionless thin film thickness

η similarity variable

μ variable viscosity of the fluid

μ_h constant viscosity of the fluid

ρ density of the fluid

σ^* Stefan-Boltzman constant

θ dimensionless temperature

θ_w temperature ratio parameter

ϕ dimensionless concentration

Superscripts

' Differentiation with respect to η

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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