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CONTROLLING HYPERCHAOS IN NON-IDENTICAL SYSTEMS USING ACTIVE CONTROLLED HYBRID PROJECTIVE COMBINATION-COMBINATION SYNCHRONIZATION TECHNIQUE

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Abstract. The current manuscript investigates a hybrid projective combination-combination synchronization scheme (HPCCSS) in four different hyperchaotic (HC) systems via active control technique (ACT). In view of Lyapunov stability theory (LST), the discussed strategy determines asymptotic stability globally. Also, numerous synchronization schemes, namely, projective synchronization, chaos control issue, combined synchronization, and many more turn into specific kinds for combination-combination (C-C) synchronization. Also, a comparison analysis within the given HPCCSS and prior published literature has been done. The proposed scheme has applications in secure communication as well as encryption. Additionally, numerical simulation is executed to check the efficiency of considered scheme using MATLAB software.

Keywords: active control; combination-combination synchronization; hybrid projective synchronization; hyperchaotic system; Lyapunov stability theory.

2020 AMS Subject Classification: 34K23, 34K35, 37N35.

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1. INTRODUCTION

Interestingly, chaotic dynamics deals with phenomena that varies with evolution of time. Nonlinear dynamical systems are found in many areas of science, ranging from engineering to life sciences. These systems may, in a wide range of cases, be described by ordinary or partial differential equations or difference equations and mostly present in nature featuring the high sensitiveness to the initial conditions. This feature has been famously reported as ”butterfly effect” in chaos literature, the term defined in 1963 by Lorenz [25]. Analytical and numerical methods are extremely essential tools in the analysis of chaotic systems. In recent years, chaotic dynamics is an intriguing topic for researchers and scientists from different fields. They include the fields of mathematics [53], physics [7], chemistry [8], biology [32, 36, 37], mechanical engineering [28], electrical engineering [39], secure communication [22], neural networks [2], civil engineering, aeronautic and aerospace [51] and economics [47], among others.

Historically, chaos phenomenon dating back to the seminal research carried out by H. Poincare [31] in 1890s when dealing with celestial 3-body problem consisting of moon, sun and earth for stabilizing the solar system. Despite the significant chaos phenomenon was first observed by Poincare [31], the first formal description for chaos in deterministic nonlinear dynamical system/ model was introduced in 1963 by Edward N. Lorenz [25]. Lorenz reported that simplified meteorological models exhibited sensitive dependency on initial conditions which showed that chaos synchronization is hugely applicable among interdisciplinary areas. Nevertheless, the most significant work in synchronization for chaotic systems was established by Fujisaka and Yamada [6], nevertheless the rapid rise in chaos theory was observed after the classical research done in 1990 by Ott et al. [40]. After all, the key idea for synchronization theory was firstly proposed in 1990 by Pecora and Carroll [29], where both of them attained synchronization method in same chaotic systems using master-slave framework. Afterwards, numerous researchers put forward the pioneering work established by Pecora and Carroll. Effectively, it is proved that synchronization process is further achievable in different chaotic systems having entirely distinct initial conditions.

The existing literature on chaos theory exhibits a huge variety in synchronization techniques proposed and applied on several chaotic models to achieve stability. They are listed as lag [19], complete [41], hybrid [17, 42], anti [14, 16], projective [5], modified projective [21], hybrid projective [9, 27], combination [13, 49], combination difference [10, 15], C-C synchronization [9, 45], compound [44] etc. By now, various control approaches have been reported in chaos control theory, viz., time-delay feedback control [23], active control [4,12,15,17], back stepping design [33], feedback control [18], sliding mode control (SMC) [48], adaptive control [9,12,14], impulsive control [20], optimal control [11] and so on. HC system is traditionally defining as chaotic system that contains two or more +ve Lyapunov exponents. More importantly, Rossler [34] reported in 1979 the first classic HC system. Over these past decades, a wide spectrum of classic HC systems has been emerged, namely, Lorenz system, Sundarapandian system, Liu system, Pehlivan system, Lu system, Tigan system, Cai system, Nikolov system, Liu system, Vaidyanathan system, Chen system, and many more.

Synchronization procedure in chaotic Lorenz systems via ACT was reported firstly by Longren and Bai [1] in 1997. Mainieri and Rehacek [26] first described projective synchronization scheme for synchronizing the chaotic systems in 1999. Li and Xu [22] studied secure communication via projective chaos synchronization. Further, Runzi et al. [35] introduced combination synchronization scheme between three chaotic systems in 2012. Also, complex projective synchronization was discussed by Wu et al. [54] in complex chaotic systems. Wu and Fu [55] described combination synchronization among chaotic systems having different order via active backstepping scheme. Interestingly, Sun et al. [45] for the first time initiated C-C synchronization methodology among 4 different or identical systems by applying ACT in 2012. Later on, many detailed studies [9, 43, 45, 46, 56, 58] are carried out by using different control techniques and C-C synchronization scheme.

Considering these aforementioned discussions with regard to C-C synchronization scheme, we propose for investigating the hybrid projective C-C synchronization scheme (HPCCSS) in four distinct HC systems via active control technique (ACT). In addition, using Lyapunov stability Theory (LST), we formulate the active control law to achieve stability. If chaos synchronization in the considered systems could be controlled effectively, it would be simply possible to attain greater quality achievements significantly in image encryption and secure communication. The HPCCSS between 4 distinct HC systems via ACT has not been yet explored till date. It establishes the novelty in the presented work.

The main findings in this work are summarized as: Sect. 2 consists of few necessary notations as well as basic definitions to be utilized in the coming up sections. In Sect. 3, the basic features of discussed HC systems are outlined. Sect. 4 investigates in detail the HPCCSS technique via ACT methodology. Further, numerical simulation results utilizing MATLAB environment are executed to ensure the efficiency in the obtained computational results of suggested approach in Sect. 5. In addition, a detailed comparative analysis of the synchronization error stability time for different synchronization techniques and for different integer orders systems has been mentioned. Lastly, conclusion is presented in Sect. 6.

2. Mathematical Preliminaries

Let the first master system be

$$\dot{y}_{m1} = H_1(y_{m1}), \tag{1}$$

and the second master system be

$$\dot{y}_{m2} = H_2(y_{m2}). \tag{2}$$

Let the first slave system be

$$\dot{y}_{s1} = H_3(y_{s1}) + W_1(y_{m1}, y_{m2}, y_{s1}, y_{s2}), \tag{3}$$

and the second slave system be

$$\dot{y}_{s2} = H_4(y_{s2}) + W_2(y_{m1}, y_{m2}, y_{s1}, y_{s2}), \tag{4}$$

where $y_{m1} = (y_{m11}, y_{m12}, \ldots, y_{m1n})^T \in R^n$, $y_{m2} = (y_{m21}, y_{m22}, \ldots, y_{m2n})^T \in R^n$, $y_{s1} = (y_{s11}, y_{s12}, \ldots, y_{s1n})^T \in R^n$ and $y_{s2} = (y_{s21}, y_{s22}, \ldots, y_{s2n})^T \in R^n$ are state vectors for master and slave systems (1), (2), (3) and (4) respectively, $H_1, H_2, H_3, H_4 : R^n \to R^n$ are continuous vector functions possessing non linearity, $W_1 = (W_{11}, W_{12}, \ldots, W_{1n})^T$: $R^n \times R^n \times R^n \times R^n \to R^n$, $W_2 = (W_{21}, W_{22}, \ldots, W_{2n})^T$: $R^n \times R^n \times R^n \times R^n \to R^n$ are controllers.

Definition 1. *Suppose there exist constant matrices $P, Q, R, S \in R^n \times R^n$ and $R \neq 0$ or $S \neq 0$ showing:*

$$\lim_{t\to\infty} \|e(t)\| = \lim_{t\to\infty} \|(Py_{s2}(t) + Qy_{s1}(t) - Ry_{m2}(t) - Sy_{m1}(t))\| = 0,$$

the combination of 2 master systems (1)-(2) is said to attain C-C synchronization with combination of 2 slave systems (3)-(4) and $\|\cdot\|$ indicates vector norm.

Remark 1. *If $P = Q = -\zeta I$ and $R = S = -I$, for $\zeta = 1$ it turns into C-C complete synchronized technique and for $\zeta = -1$ it will reduces to C-C anti-synchronized technique. Specifically, combination of anti-synchronization and complete makes hybrid projective C-C synchronization scheme (HPCCSS). Hence, the HPCCSS error is formulated as:*

$$e(t) = (y_{s2}(t) + y_{s1}(t)) - \zeta(y_{m2}(t) + y_{m1}(t)), \tag{5}$$

where $\zeta = diag(\zeta_1, \zeta_2, \zeta_3, \ldots, \zeta_n)$.

3. Model Analysis

Wang et al. [50] announced the given HC system

$$\begin{cases} \dot{y}_{m11} = p_1(y_{m12} - y_{m11}) + y_{m14} \\ \dot{y}_{m12} = r_1 y_{m11} - y_{m12} - y_{m11}y_{m13} \\ \dot{y}_{m13} = y_{m11}y_{m12} - q_1 y_{m13} \\ \dot{y}_{m14} = -y_{m12}y_{m13} + s_1 y_{m14}, \end{cases} \tag{6}$$

where $y_{m11}, y_{m12}, y_{m13}, y_{m14}$ are states variables for eq. (6) and for $p_1 = 10$, $q_1 = 8/3$, $r_1 = 28$, $s_1 = -1$, the system eq. (6) exhibits HC behaviour as shown in Figures 1(a) and 2(a).

Lu et al. [3] pronounced the given HC system

$$\begin{cases} \dot{y}_{m21} = p_2(y_{m22} - ym21) + y_{m24} \\ \dot{y}_{m22} = -y_{m21}y_{m23} + r_2 y_{m22} \\ \dot{y}_{m23} = y_{m21}y_{m22} - q_2 y_{m23} \\ \dot{y}_{m24} = y_{m21}y_{m23} + s_2 y_{m24}, \end{cases} \tag{7}$$

where $y_{m21}, y_{m22}, y_{m23}, y_{m24}$ are states variables for eq. (7) and for $p_2 = 36$, $q_2 = 3$, $r_2 = 20$, $s_2 = 1.3$, the system eq. (7) exhibits HC behaviour as depicted in Figures 1(b) and 2(b).

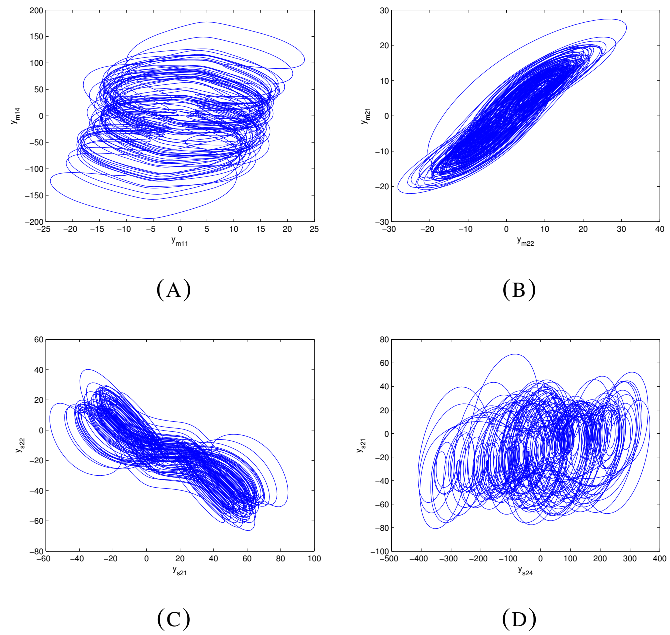

FIGURE 1. Phase graphs for HC systems (6) to (9) respectively in (a) $y_{m11} - y_{m13}$ plane, (b) $y_{m21} - y_{m22}$ plane, (c) $y_{s11} - y_{s13}$ plane, (d) $y_{s23} - y_{s24}$ plane

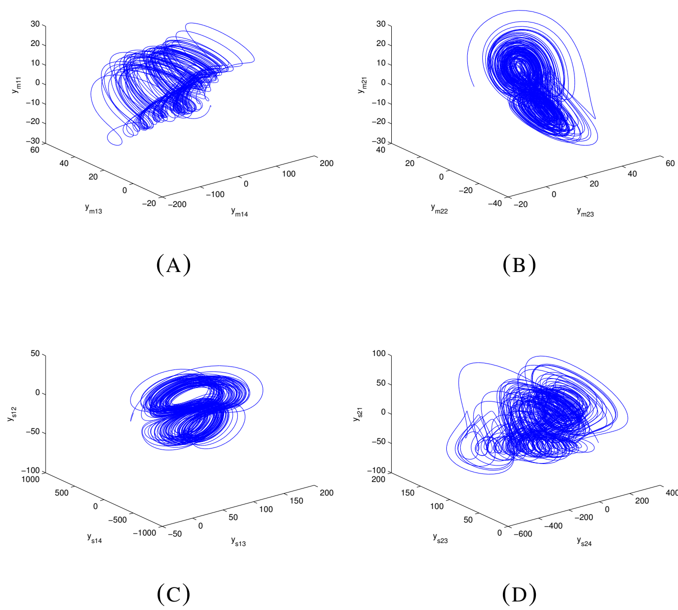

FIGURE 2. Phase graphs for HC systems in (6) to (9) respectively (a) $y_{m11} - y_{m12} - y_{m13}$ space, (b) $y_{m21} - y_{m22} - y_{m23}$ space, (c) $y_{s11} - y_{s12} - y_{s13}$ space, (d) $y_{s21} - y_{s22} - y_{s23}$ space

Zheng et al. [57] reported the HC system

$$
\begin{cases}
\dot{y}_{s11} = -y_{s12} - y_{s13} \\
\dot{y}_{s12} = y_{s11} + p_3 y_{s12} + y_{s14} \\
\dot{y}_{s13} = q_3 + y_{s11} y_{s13} \\
\dot{y}_{s14} = -r_3 y_{s13} + s_3 y_{s14},
\end{cases}
\tag{8}
$$

where $y_{s11}, y_{s12}, y_{s13}, y_{s14}$ are the states variables for eq. (8) and for the parameters $p_3 = 0.25$, $q_3 = 3$, $r_3 = 0.5$, $s_3 = 0.05$, the system eq. (8) showing HC behaviour as shown in Figure 1(c) and Figure 2(c).

Wei et al. [52] proposed HC system

$$
\begin{cases}
\dot{y}_{s21} = p_4(y_{s22} - y_{s21}) \\
\dot{y}_{s22} = -y_{s21} y_{s23} - r_4 y_{s22} + s_4 y_{s24} \\
\dot{y}_{s23} = q_4 + y_{s21} y_{s22} \\
\dot{y}_{s24} = -h_4 y_{s22},
\end{cases}
\tag{9}
$$

where $y_{s21}, y_{s22}, y_{s23}, y_{s24}$ are the states variables for eq. (9) and for the parameters $p_4 = 10$, $q_4 = 25$, $r_4 = -2.5$, $s_4 = 1$, $h_4 = 1$ the system eq. (9) depicts HC behaviour as depicted in Figure 1(d) and Figure 2(d).

4. SYNCHRONIZATION PHENOMENA

We discuss, in this section, the ACT to stabilize these given four HC systems. Conveniently, the systems (6)-(7) are selected as master systems. Further, systems (8)-(9) are selected as slave systems having controllers would be described as:

$$
\begin{cases}
\dot{y}_{s11} = -y_{s12} - y_{s13} + W_{11} \\
\dot{y}_{s12} = y_{s11} + p_3 y_{s12} + y_{s14} + W_{12} \\
\dot{y}_{s13} = q_3 + y_{s11} y_{s13} + W_{13} \\
\dot{y}_{s14} = -r_3 y_{s13} + s_3 y_{s14} + W_{14},
\end{cases}
\tag{10}
$$

$$
\begin{cases}
\dot{y}_{s21} = p_4(y_{s22} - y_{s21}) + W_{21} \\
\dot{y}_{s22} = -y_{s21}y_{s23} - r_4 y_{s22} + s_4 y_{s24} + W_{22} \\
\dot{y}_{s23} = q_4 + y_{s21}y_{s22} + W_{23} \\
\dot{y}_{s24} = -h_4 y_{s22} + W_{24},
\end{cases} \tag{11}
$$

where W_{11}, W_{12}, W_{13}, W_{14}, W_{21}, W_{22}, W_{23}, W_{24} are active control inputs.

Defining the HPCCSS errors by

$$
\begin{cases}
e_1 = y_{s21} + y_{s11} - \zeta_1(y_{m21} + y_{m11}) \\
e_2 = y_{s22} + y_{s12} - \zeta_2(y_{m22} + y_{m12}) \\
e_3 = y_{s23} + y_{s13} - \zeta_3(y_{m23} + y_{m13}) \\
e_4 = y_{s24} + y_{s14} - \zeta_4(y_{m24} + y_{m14}).
\end{cases} \tag{12}
$$

Now, HPCCSS error dynamics acquires the form

$$
\begin{cases}
\dot{e}_1 = \dot{y}_{s21} + \dot{y}_{s11} - \zeta_1(\dot{y}_{m21} + \dot{y}_{m11}) \\
\dot{e}_2 = \dot{y}_{s22} + \dot{y}_{s12} - \zeta_2(\dot{y}_{m22} + \dot{y}_{m12}) \\
\dot{e}_3 = \dot{y}_{s23} + \dot{y}_{s13} - \zeta_3(\dot{y}_{m23} + \dot{y}_{m13}) \\
\dot{e}_4 = \dot{y}_{s24} + \dot{y}_{s14} - \zeta_4(\dot{y}_{m24} + \dot{y}_{m14}).
\end{cases} \tag{13}
$$

Substituting the Master systems eqs. (6)-(7) and slave systems eqs. (10)-(11) in eq. (13), given error dynamics turns into

$$
\begin{cases}
\dot{e}_1 = -y_{s12} - y_{s13} - \zeta_1 p_1(y_{m12} - y_{m11}) - \zeta_1 y_{m14} + p_4(y_{s22} - y_{s21}) \\
\qquad\quad -\zeta_1 p_2(y_{m22} - y_{m21}) - \zeta_1 y_{m24} + W_{11} + W_{21} \\
\dot{e}_2 = y_{s11} + p_3 y_{s12} + y_{s14} - \zeta_2(r_1 y_{m11} - y_{m12} - y_{m11}y_{m13}) - y_{s21}y_{s23} - r_4 y_{s22} \\
\qquad\quad +s_4 y_{s24} - \zeta_2(-y_{m21}y_{m23} + r_2 y_{m22}) + W_{12} + W_{22} \\
\dot{e}_3 = q_3 + y_{s11}y_{s13} - \zeta_3(y_{m11}y_{m12} - q_1 y_{m13}) - q_4 + y_{s21}y_{s22} \\
\qquad\quad -\zeta_3(y_{m21}y_{m22} - q_2 y_{m23}) + W_{13} + W_{23} \\
\dot{e}_4 = -r_3 y_{s13} + s_3 y_{s14} - \zeta_4(-y_{m12}y_{m13} + s_1 y_{m14}) - h_4 y_{s22} \\
\qquad\quad -\zeta_4(y_{m21}y_{m23} + s_2 y_{m24}) + W_{14} + W_{24}.
\end{cases} \tag{14}
$$

The active controllers are described as:

$$
\begin{cases}
W_{11}+W_{21}=y_{s12}+y_{s13}+\zeta_1 p_1(y_{m12}-y_{m11})+\zeta_1 y_{m14}-k_1e_1-p_4(y_{s22}-y_{s21}) \\
\qquad\qquad +\zeta_1 p_2(y_{m22}-y_{m21})+\zeta_1 w_{m24} \\
W_{12}+W_{22}=-y_{s11}-p_3y_{s12}-y_{s14}+\zeta_2(r_1y_{m11}-y_{m12}-y_{m11}y_{m13}) \\
\qquad\qquad -k_2e_2+y_{s21}y_{s23}+r_4y_{s22}-s_4y_{s24}+\zeta_2(-y_{m21}y_{m23}+r_2y_{21}) \\
W_{13}+W_{23}=-q_3-y_{s11}y_{s13}+\zeta_3(y_{m11}y_{m12}-q_1y_{m13})-k_3e_3+q_4-y_{s21}y_{s22} \\
\qquad\qquad +\zeta_3(y_{m21}y_{m22}-q_2y_{m23}) \\
W_{14}+W_{24}=r_3y_{s13}-s_3y_{s14}+\zeta_4(-y_{m12}y_{m13}+s_1y_{m14}) \\
\qquad\qquad -k_4e_4+h_4y_{s22}+\zeta_4(y_{m21}y_{m23}+s_2y_{m14}),
\end{cases}
\tag{15}
$$

where $k_i > 0$ $(i = 1,2,3,4)$ are known as gain constants.

Putting controllers eq. (15) into the error dynamics eq. (14), one may get

$$
\begin{cases}
\dot{e_1}=-k_1e_1 \\
\dot{e_2}=-k_2e_2 \\
\dot{e_3}=-k_3e_3 \\
\dot{e_4}=-k_4e_4.
\end{cases}
\tag{16}
$$

Next, we take the preferred choice for Lyapunov function V which is given by

$$
V=\frac{1}{2}[e_1^2+e_2^2+e_3^2+e_4^2],
\tag{17}
$$

which is positive definite function in R^4.

Derivative for V along with trajectories of (17), we deduce

$$
\dot{V}=e_1\dot{e_1}+e_2\dot{e_2}+e_3\dot{e_3}+e_4\dot{e_4}.
\tag{18}
$$

Theorem 1. *The HC non-identical systems eqs.* (6)-(7) *and eqs.* (10)-(11) *are asymptotically hybrid projective combination-combination synchronized globally for each initial states with respect to the defined active control law eq.* (15)*, where* k_i $(i = 1,2,3,4)$ *are positive gaining constants.*

Proof. The functional V as described in eq. (17) is positive definite in R^4. Substituting the eq. (16) in eq. (18), the derivative for V is attained as:

$$\dot{V} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2.$$

This depicts that $\dot{V} < 0$ and hence $\dot{V}$ is -ve definite in R^4.

Therefore, in accordance with LST [30, 38], we deduce that given HPCCSS error $e(t) \to 0$ globally and asymptotically as $t \to \infty$ to each initial values $e(0) \in R^4$. □

5. Numerical Simulations

We demonstrate, in this section, the simulations for checking the efficiency of investigated approach. In order to achieve HPCCSS, 4^{th}-order Runge-Kutta (R-K) methodology is utilized. Initial conditions for master systems (6)-(7) and slave systems (10)-(11) are specifically taken as $(-1,1,-1,1)$, $(5,8,-1,-3)$, $(5,4,7,2)$, $(0.2,0.1,0.75,-2)$. The control gaining are selected as $k_i = 4$ for $i = 1,2,3,4$. Here, we attain HPCCSS of master systems (6)-(7) and slave systems (10)-(11) by choosing the scaling matrix ζ with $\zeta_1 = 2$, $\zeta_2 = -3$, $\zeta_3 = 3$, $\zeta_4 = -2$. Here, Figure 3(a-d) display the path followed by master systems (6)-(7) and slave systems (10)-(11) having control inputs.

In addition, initial states for considered error system are written as $(e_1,e_2,e_3,e_4) = (-2.8,31.1,13.75,-4)$. The synchronized error $e(t)$ is convergent to 0 for t approaching infinity which is presented in Figure 3(e) which ensuring that the HPCCSS in systems (6)-(7) and (10)-(11) is achieved via ACT. Consequently, the proposed HPCCSS approach in master as well as slave systems is assured computationally.

A Comparative Analysis. In [45], the authors used nonlinear ACT to discuss C-C synchronization in 4 similar Lu systems and dissimilar chaotic systems in which the considered scheme is attained at $t = 5$ (approx.) and $t = 4.8$ (approx.) respectively. Further, authors investigated SMC strategy in [43] to obtain C-C synchronization in 4 distinct systems, in which synchronization errors converge to 0 at $t = 2.4$ (approx.). Also, author [58] discussed C-C synchronization of 4 nonlinear chaotic complex models in which synchronized state is confirmed at $t = 5$ (approx.).

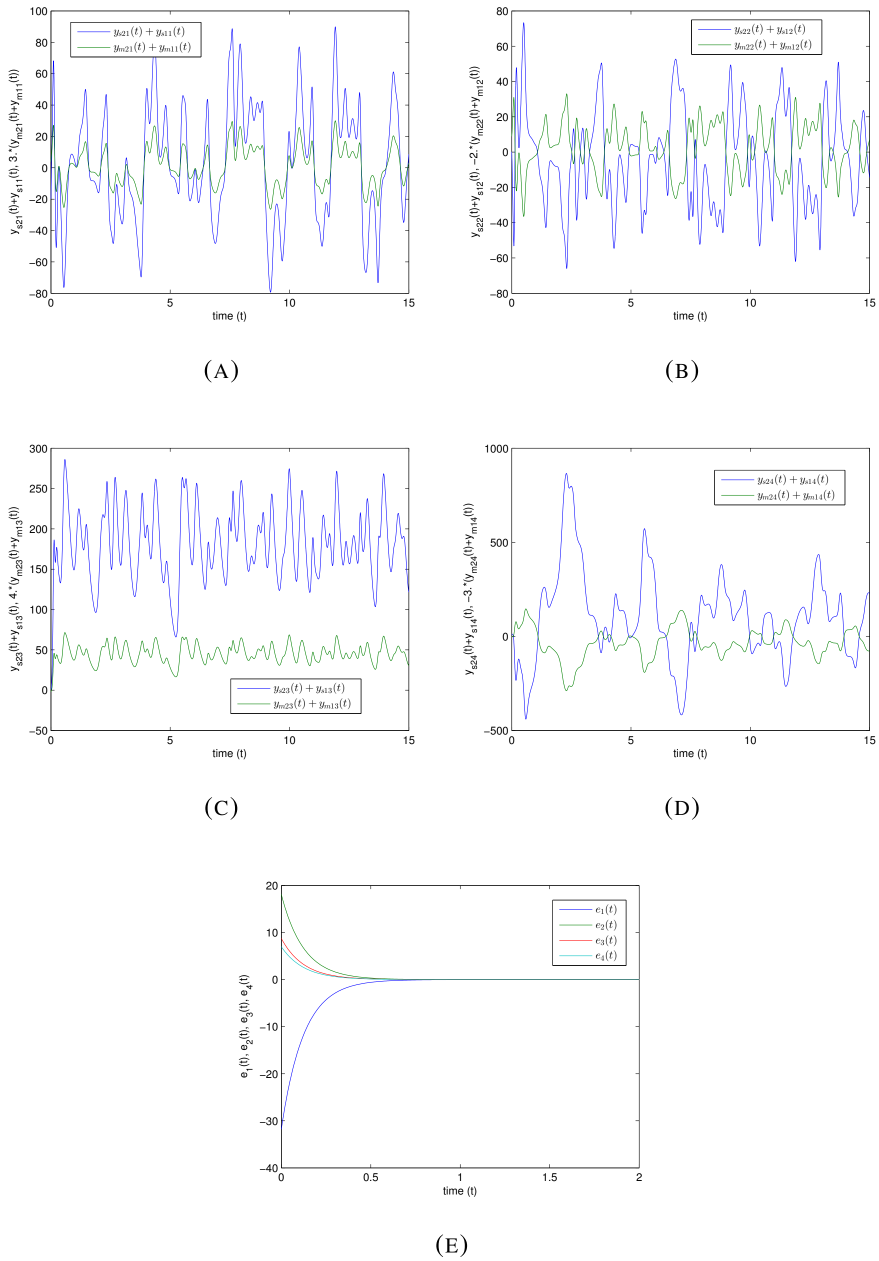

FIGURE 3. HPCCSS of HC systems (a) between $y_{m11}(t)+x_{m21}(t)$ and $y_{s11}(t)+y_{s21}(t)$, (b) between $y_{m12}(t)+y_{m21}(t)$ and $y_{s12}(t)+y_{s22}(t)$ (c) between $y_{m13}(t)+y_{m23}(t)$ and $y_{s13}(t)+y_{s23}(t)$, (d) between $y_{m14}(t)+y_{m24}(t)$ and $y_{s14}(t)+y_{s24}(t)$, (e) HPCCSS errors (e_1,e_2,e_3,e_4)

Further, authors [56] studied phase synchronization in different chaotic fractional complex systems, using this approach the C-C synchronization technique has been conducted at $t =$

4.5(approx.). Moreover in [24], authors initiated adaptive control approach to investigate modified hybrid complex function projective synchronization scheme in chaotic complex models, in which desired synchronized state is achieved at $t = 4.5$ (approx.). Addition, authors [9] proposed the HPCCS technique in 4 different HC systems utilizing adaptive control strategy, where it is seen that synchronized state occurred at $t = 1.2$ (approx.). Apart from these mentioned research works, we have investigated HPCCSS between 4 different HC systems by utilizing ACT, where the synchronized state occurs at $t = 0.6$ (approx.) which is displayed in Figure 4. Therefore, in comparison to aforementioned schemes, the time period for synchronization errors to converge in discussed technique is much lesser that in turn depicts the superiority and effectiveness of our discussed strategy.

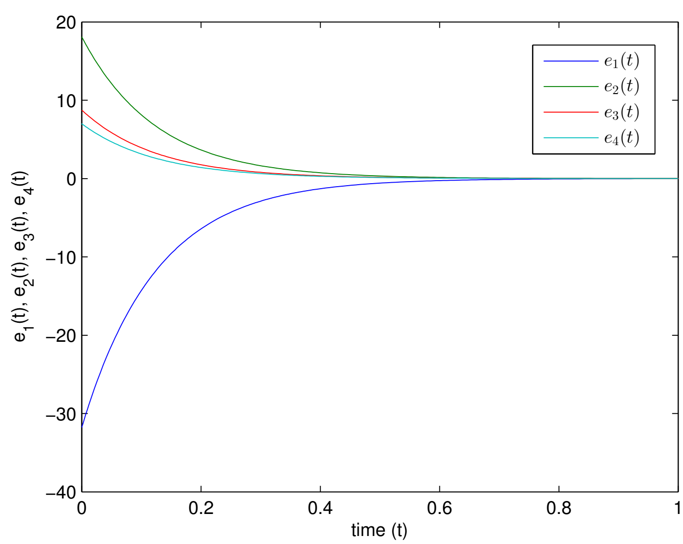

FIGURE 4. HPCCSS error convergence graph

6. DISCUSSION AND CONCLUSION

The presented research study investigated the HPCCSS between four distinct hyperchaotic systems via active control technique. Analytical expressions of active controllers are attained in view of Lyapunov stability theory. Simulation outcomes using MATLAB environment are carried out for checking the efficiency of theoretical results described. Exceptionally, both the numerical and theoretical outcomes are in excellent congeniality. Various synchronization schemes, namely, projective synchronization, chaos control issue, combination synchronization, and many more turn into particular types of C-C synchronization. Also, a comparative study between our considered approach and earlier published work has been done. In this research, the synchronized error taking less time in converging to zero exhibits that our proposed technique is more approved over prior existing literature.

The discussed synchronization strategy would be utilized in secure communication and encryption alongside various applications in the field of social, biological, and physical models. Further, we perceive that the considered HPCCSS can be generalized by applying other controls techniques on the systems which are interfered with model disturbances and uncertainties.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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