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NEW FORMULA OF DEGREE DISTANCE INDEX FOR SOME COMPLEX NETWORKS

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Abstract. Mathematics plays an important role in various fields, one of them is graph theory. Graphs can be used to model many types of relations and processes in many domains such as solving problems related to mathematical chemistry by using topological indices. A topological index of a graph is a number that quantifies the structure of the graph. It is used for modeling the biological and chemical properties of molecules in QSPR (Qualitative Structure-Property Relationships) and QSAR (Qualitative Structure-Activity Relationships) studies. The Degree Distance index $DD(G)$ is one of the important topological indices. In this paper, we are going to determine $DD(G)$ for some complex graphs like: Star vertex's graph (SV), Star edge's graph (SE), and Path's graph (P).

Keywords: graph theory; topological index; QSPR/QSAR; degree distance index; Wiener index; first Zagreb index; complex graph.

2010 AMS Subject Classification: 05C90.

1. Introduction

In mathematics and computer science, graph theory is a branch of discrete combinatorial mathematics that studies graphs. They are mathematical structures used to model the relationships between objects. They can be adopted to model several types of processes in transport

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networks, biological, mathematical, and physical systems. For example in computer science, graph theory is used to model and study communication networks, data organization, etc. Indeed, the structure of the links of a Web site can be represented by a graph in which the pages are represented by vertices and links between these pages are represented by edges, Essalih [1]. The same approach can be used for social networks, Essalih [1]. In chemistry, molecules are represented by a molecular graph, whose vertices represent atoms and their edges correspond to chemical bonds , Estrada and Bonchev [2]. Some studies have used qualitative structure-activity relationships (QSAR) and qualitative structure-property relationships (QSPR) to define a mathematical relationship between the structure and activity (property) of a molecule, Essalih [1], Zeryouh, El Marraki and Essalih[3], Roy, Kar and Das [4]. Indeed, by using the notion of topological indices, they were able to predict and design chemicals that are more beneficial to health, Roy, Kar and Das [4], Essalih [1]. The concept of topological indices is based on numerical values encoding certain information relating to molecular structure, Essalih [1], Laghridat, Mounir and Essalih [5], Ediz [6], Balaban and Devillers [7]. These values make it possible to establish correlations between the structure of a molecule and its physicochemical properties or its biological activity Essalih [1], Balaban and Devillers [7].

Many types of topological indices have been introduced in theoretical chemistry to measure the topological properties of molecules, such as distance-based topological indices (e.g. Wiener index, Wiener Terminal index, and degree distance index) or those based on the degree (for example Zagreb index and Randic index), Jamil [8], Wiener [9], Das, Xu, and Nam [10], Zeryouh, EL Marraki and Essalih[11].

2. Preliminary Notes

2.1. Graph theory. Let G be a connected graph $G=(V,E)$ where E and V denote respectively the edge set and the vertex set of the graph G. The order of a graph G is its number of vertices $|V|$ denoted by n and the size of graph, denoted by m, is its number of edges $|E|$.

For vertices u, $v \in V(G)$ the distance between two vertices u and v, denoted by $d(u,v)$ is the number of edges of the shortest path connecting these vertices in G.

We denote by $deg(u)$ the degree of the vertex u which is the number of edges incident to u.

The diameter of a graph $G=(V,E)$, denoted by $D(G)$ is the longest distance between two vertices of this graph, Roy, Kar and Das [4], Essalih, EL Marraki and EL Hagri[12], Schmuck [13]. The Wiener index is one of the oldest topological indices, defined as the summation of distances between all vertices of a graph G, Wiener [9], Schmuck [13]: $W(G)=\frac{1}{2}\sum_{u\in V(G)} w(u,G)$ with $w(v,G)=\sum_{u\in V(G)} d(u,v)$. The first Zagreb index $M_1(G)$ is equal to the sum of squares of the degrees of the vertices , defined by $M_1(G)=\sum_{v\in V(G)}(deg(v))^2$ Das, Xu, and Nam [10].

2.2. Complex networks. The study of complex networks is an active and novel area of scientific research, Benuwa, Ghansah and Wornyo [14], Kim and Wilhelm [15] inspired largely by the study of real-world networks such as computer networks, chemical networks, technological networks, and social networks. The adjective "complex" refers to the fact that the properties of the network emerge from unplanned evolutions and interactions of its elements (e.g. sites and links), Barrat [16]. This general definition could be linked to different topological properties. The simplest would be related to the number of nodes, Kim and Wilhelm [15]. Indeed, if we consider for instance social networks, they could be represented as complex graphs by considering the accounts (a person, an organization, an institution, etc.) as vertices and the relationship between them as edges, Steen [17]. The complexity of such networks comes from the fact that they are generally so huge that it is impossible to understand or predict their overall behavior by looking into the behavior of individual nodes or links, Steen [17], Kolaczek [18]. The best known of these systems are Twitter and Facebook. For Twitter, the persons are considered as vertices, while the interactions such as retweets, replies, or favs are considered as the edges. For Facebook, every person or page is a node or vertex and every relationship or interaction (Tag, comment, like, or share) that connects the persons is an edge.

3. Formula of the Degree Distance Index

Prior 1989, the degree distance index of a graph G ($DD(G)$) has been subject to many changes in its expression and name. Since then, it has become a standard index with more or less unified name. Indeed, the $DD(G)$ index formula was introduced in MTI (Molecular topological index), Schultz [19], modified later by Gutman [20], who will name it the invariant of Schultz.

The degree distance index of a graph G was considered first in connection with certain chemical applications by Dobrynin and Kochetova [21] and Gutman [20], as a weighted version of the Wiener index, who named it the Schultz index. This name was adopted by most other authors, Dobrynin [22], Schultz and Schultz [23], Zhou [24].
Recently, the $DD(G)$ index is used in studies of QSAR/QSPR methods as a structure descriptor based on molecular topology, Estrada and Bonchev [2], Essalih, EL Marraki and EL Hagri [12], Essalih, El Marraki and Zeryouh [25].

Our objective is to exploit this tool to characterize some specific complex networks such as Star vertex's graph (SV) , Star edge's graph (SE) and Path's graph (P). Our method relays on the parameter $d_G^u(k)$ which represents the number of pairs of vertices of G that are at distance k from u $(k \leq D(G))$. More precisely, by considering members of a complex network as a set of vertices V(G) and its relations as a set of edges E(G), $d_G^u(1)$ will represent the degree of u, $d_G^u(2)$ will represent the number of reachable vertices from the neighbors of u, etc. Laghridat, Mounir and Essalih [5], Essalih, El Marraki and Zeryouh [25]. This parameter has been used in Essalih, El Marraki and Zeryouh [25] to establish a new formula of the degree distance of a graph G.

Theorem 1. *Let G be a simple connected undirected graph, with n vertices, m edges and* $D(G) \geq 2$. *Then :*

$$DD(G) = 4m(n-1) + \sum_{u \in V(G)} \sum_{k=1}^{D(G)} deg(u)(k-2)d_G^u(k) \tag{1}$$

4. Main Results

In Laghridat, Mounir and Essalih [5], we give a characterisation of some social networks by applying the formula (1). In this work, we improve our results by considering more complex social networks such as : Star vertex's graph (SV), Star edge's graph (SE) and Path's graph (P).

4.1. The star vertex's graphs.

The Star vertex's graph SV is a graph composed of N graphs G_i of order n_i, connected to each other by a vertex s (see Figure 1 (a)).

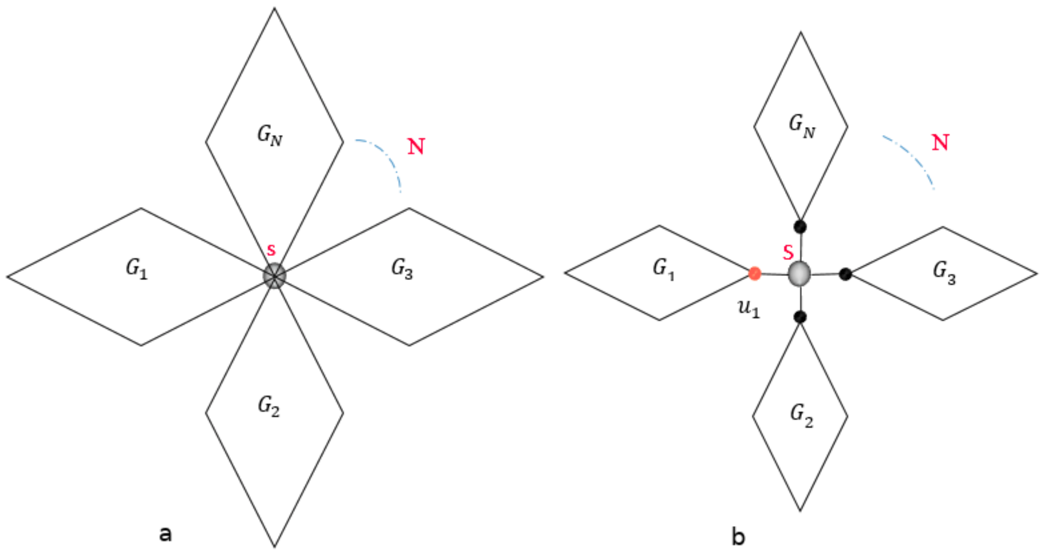

FIGURE 1. (a) Star vertex's graph SV composed of N graphs G_i and (b) Star edge's graph SE composed of N graphs G_i

Lemma 1. *Let SV be a connected simple graph with n vertices and m edges, composed of N graphs G_i, each of them is of order n_i, and has m_i edges, $i \in \{1,2,3,\dots,N\}$. The graphs G_i are connected by a common vertex s. Then :*

- $n = \sum_{i=1}^{N} n_i - (N-1)$,
- $m = \sum_{i=1}^{N} m_i$,
- $D(SV) = \max_{1\leq i,j\leq N}(D(G_i)+D(G_j))$, *with* $i \neq j$

Theorem 2. *let SV be a star vertex's graph with n vertices and m edges.The Degree Distance index of SV is defined as follows ($i \neq j$):*

$$DD(SV) = \sum_{i=1}^{N}[(DD(G_i)) + \sum_{j=1}^{N} G_{ij}]. \tag{2}$$

With:

$$G_{ij} = 4m_i(n_j-1) + 1/2 \sum_{u\in V(G_i)} \sum_{k=1}^{D(SV)} deg(u)(k-2)d^{v}_{G_j}(k).$$

Proof.

$$DD(SV) = 4m(n-1) + \sum_{u\in V(G_1)} \left[\sum_{k=1}^{D(G_1)} deg(u)(k-2)d^{u}_{SV}(k) + \sum_{k=D(G_1)+1}^{D(SV)} deg(u)(k-2)d^{u}_{SV}(k) \right] +$$

$$\sum_{u\in V(G_2)}\left[\sum_{k=1}^{D(G_2)} deg(u)(k-2)d^u_{SV}(k)+\sum_{k=D(G_2)+1}^{D(SV)} deg(u)(k-2)d^u_{SV}(k)\right]+........+\sum_{u\in V(G_n)}$$
$$\left[\sum_{k=1}^{D(G_n)} deg(u)(k-2)d^u_{SV}(k)+\sum_{k=D(S_n)+1}^{D(SV)} deg(u)(k-2)d^u_{SV}(k)\right]$$
$$=4m(n-1)+\sum_{u\in V(G_1)}\left[\sum_{k=1}^{D(G_1)} deg(u)(k-2)d^u_{G_1}(k)+......+\sum_{k=1}^{D(G_1)} deg(u)(k-2)d^u_{G_n}(k)\right.$$
$$\left.+\sum_{k=D(G_1)+1}^{D(SV)} deg(u)(k-2)d^u_{G_2}(k)+......+\sum_{k=D(G_1)+1}^{D(SV)} deg(u)(k-2)d^u_{G_n}(k)\right]+.......+$$
$$\sum_{u\in V(G_n)}\left[\sum_{k=1}^{D(G_n)} deg(u)(k-2)d^u_{G_1}(k)+.......+\sum_{k=1}^{D(G_n)} deg(u)(k-2)d^u_{G_n}(k)\right.$$
$$\left.+\sum_{k=D(G_n)+1}^{D(SV)} deg(u)(k-2)d^u_{G_1}(k)+.......+\sum_{k=D(G_n)+1}^{D(SV)} deg(u)(k-2)d^u_{G_{n-1}}(k)\right]$$
$$=4m_1(n_1-1)+\sum_{u\in V(G_1)}\sum_{k=1}^{D(G_1)} deg(u)(k-2)d^u_{G_1}(k)+......+4m_n(n_n-1)+\sum_{u\in V(G_n)}$$
$$\sum_{k=1}^{D(G_n)} deg(u)(k-2)d^u_{G_n}(k)+4m_1(n_2-1)+\sum_{u\in V(G_1)}\sum_{k=1}^{D(SV)} deg(u)(k-2)d^u_{G_2}(k)+.....+$$
$$4m_1(n_n-1)+\sum_{u\in V(G_1)}\sum_{k=1}^{D(SV)} deg(u)(k-2)d^u_{G_n}(k)+.....+4m_n(n_1-1)+\sum_{u\in V(G_n)}$$
$$\sum_{k=1}^{D(SV)} deg(u)(k-2)d^u_{G_1}(k)+.....+4m_n(n_{n-1}-1)+\sum_{u\in V(G_n)}\sum_{k=1}^{D(SV)}(k-2)d^u_{G_{n-1}}(k)$$

□

4.2. The star edge's graphs SE.

A Star edge's graph SE is a graph composed of N graphs G_i , each of them is of order n_i, and have m_i edges, $i\in\{1,2,3,\dots,N\}$. The graphs G_i are connected by a common vertex s and a set of edges $\{s,u_i\}$, $i\in\{1,2,3,\dots,N\}$ (see Figure 1 (b)).

Lemma 2. *Let SE be a Star edge's graph composed of N graphs G_i, each of them is of order n_i, and has m_i edges, $i\in\{1,2,3,\dots,N\}$.*

Then:

- $n=\sum_{i=1}^{N}(n_i)+1$.
- $m=\sum_{i=1}^{N} m_i+N$.

- *D(SE)* = $\max_{1\leq i,j\leq N}(D(G_i)+D(G_j))+2$, *with* $i\neq j$.

Theorem 3. *Let SE be a star edge's graph with n vertices and m edges. The Degree Distance index of SE denoted by* $DD(SE)$ *is defined as follows* $(i\neq j)$*:*

$$DD(SE)=\sum_{i=1}^{N}[(DD(G_i))+dd_{G_i}(s)+\sum_{j=1}^{N}G_{ij}]+S. \tag{3}$$

With:

$$G_{ij}=4n_i(m_j+1)+1/2\sum_{u\in V(G_i)}\sum_{k=1}^{D(SE)}deg(u)(k-2)d^{v}_{G_j}(k).$$

And:

$$dd_{G_i}(s)=\sum_{k=1}^{D(Gi+1)}deg(S)(k-2)d^{S}_{G_i}(k).$$

$$S=\sum_{i=1}^{N}n_i+\sum_{i=1}^{N}m_i,$$

Proof.

$$DD(SE)=4m(n-1)+\sum_{u\in V(G_1)}\left[\sum_{k=1}^{D(G_1)}deg(u)(k-2)d^{u}_{SE}(k)+\sum_{k=D(G_1)+1}^{D(SE)}deg(u)(k-2)d^{u}_{SE}(k)\right]+$$

$$\sum_{u\in V(G_2)}\left[\sum_{k=1}^{D(G_2)}deg(u)(k-2)d^{u}_{SE}(k)+\sum_{k=D(G_2)+1}^{D(SE)}deg(u)(k-2)d^{u}_{SE}(k)\right]+...+\sum_{u\in V(G_n)}\left[\sum_{k=1}^{D(G_n)}\right.$$

$$\left.deg(u)(k-2)d^{u}_{SE}(k)+\sum_{k=D(G_n)+1}^{D(SE)}deg(u)(k-2)d^{u}_{SE}(k)\right]+\sum_{i=1}^{N}\sum_{k=1}^{D(Gi)+1}deg(s)(k-2)d^{s}_{G_i}(k)$$

$$=4m(n-1)+\sum_{u\in V(G_1)}deg(u)\left[\sum_{k=1}^{D(G_1)}(k-2)d^{u}_{G_1}(k)+...+\sum_{k=1}^{D(G_1)}(k-2)d^{u}_{G_n}(k)+\sum_{k=D(G_1)+1}^{D(SE)}(k-2)\right.$$

$$\left.d^{u}_{G_2}(k)+...+\sum_{k=D(G_1)+1}^{D(SE)}(k-2)d^{u}_{G_n}(k)\right]+...+\sum_{u\in V(G_n)}deg(u)\left[\sum_{k=1}^{D(G_n)}(k-2)d^{u}_{G_1}(k)+....+\right.$$

$$\left.\sum_{k=1}^{D(G_n)}(k-2)d^{u}_{G_n}(k)+\sum_{k=D(G_n)+1}^{D(SE)}(k-2)d^{u}_{G_1}(k)+...+\sum_{k=D(G_n)+1}^{D(SE)}(k-2)d^{u}_{G_{n-1}}(k)\right]+\sum_{i=1}^{N}dd_{G_i}(s)$$

$$=DD(G_1)+......+DD(G_n)+4n_1(m_2+1)+\sum_{u\in V(G_1)}\sum_{k=1}^{D(SE)}deg(u)(k-2)d^{u}_{G_2}(k)+......+$$

$$4n_1(m_n+1)+\sum_{u\in V(G_1)}\sum_{k=1}^{D(SE)} deg(u)(k-2)d^u_{G_n}(k)+....+4n_n(m_1+1)+\sum_{u\in V(G_n)}\sum_{k=1}^{D(SE)} deg(u)$$

$$(k-2)d^u_{G_1}(k)+....+4n_{n-1}(m_{n-1}+1)+\sum_{u\in V(G_n)}\sum_{k=1}^{D(SE)}(k-2)d^u_{G_{n-1}}(k)+\sum_{i=1}^{N} dd_{G_i}(s)+S$$

□

4.3. The Path's Graphs. A Path's graph PG is a graph composed of N graphs G_i each of them is of order n_i, and has m_i edges, $i\in\{1,2,\dots,N\}$. The graphs G_i are connected by a set of edges $\{s_i,s_{i+1}\}$,$i=\{1,2,\dots,N\}$ (see Figure 2).

Lemma 3. *Let PG be a connected simple graph with n vertices and m edges, composed of N graphs G_i, each of them has n_i vertices and m_i edges, $i\in\{1,2,3,\dots,N\}$. Then :*

- $n=\sum_{i=1}^{N}(n_i)$,
- $m=\sum_{i=1}^{N} m_i+(N-1)$,
- *D(PG)* $\geq \max_{1\leq i,j\leq N}(D(G_i)+D(G_j))$, *with* $i\neq j$.

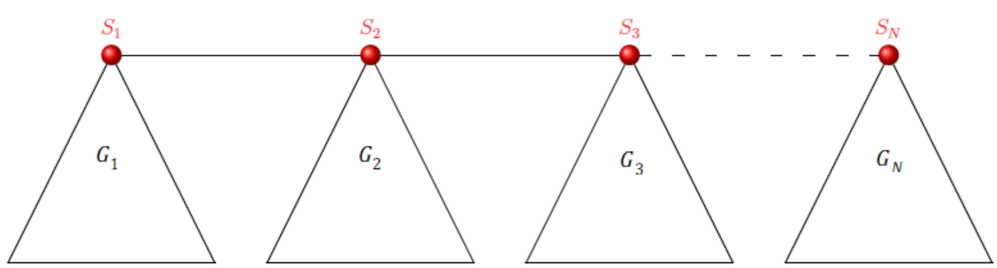

FIGURE 2. The path's graph PG composed by N graphs G_i

Theorem 4. *let PG be a path's graph with n vertices and m edges.The Degree Distance index of the path's graph PG is defined as follows* $(i\neq j)$ *:*

$$DD(PG)=\sum_{i=1}^{N}[(DD(Pi))+\sum_{j=1}^{N}P_{ij}]-P. \tag{4}$$

With:

$$P_{ij}=4n_i(m_j+1)+1/2\sum_{u\in V(P_i)}\sum_{k=1}^{D(PG)} deg(u)(k-2)d^v_{P_j}(k).$$

And:

$$P = 4(N-1).$$

Proof.

$$DD(PG) = 4m(n-1) + \sum_{u\in V(P_1)} \left[\sum_{k=1}^{D(G_1)} deg(u)(k-2)d^u_{PG}(k) + \sum_{k=D(P_1)+1}^{D(PG)} deg(u)(k-2)d^u_{PG}(k) \right]$$

$$+ \sum_{u\in V(G_2)} \left[\sum_{k=1}^{D(P_2)} deg(u)(k-2)d^u_{PG}(k) + \sum_{k=D(G_2)+1}^{D(PG)} deg(u)(k-2)d^u_{PG}(k) \right] + \ldots.. +$$

$$\sum_{u\in V(G_n)} \left[\sum_{k=1}^{D(P_n)} deg(u)(k-2)d^u_{PG}(k) + \sum_{k=D(G_n)+1}^{D(PG)} deg(u)(k-2)d^u_{PG}(k) \right]$$

$$= 4m(n-1) + \sum_{u\in V(G_1)} \left[\sum_{k=1}^{D(G_1)} deg(u)(k-2)d^u_{G_1}(k) + \ldots.. + \sum_{k=1}^{D(G_1)} deg(u)(k-2)d^u_{G_n}(k) \right.$$

$$\left. + \sum_{k=D(G_1)+1}^{D(PG)} deg(u)(k-2)d^u_{G_2}(k) + \ldots.. + \sum_{k=D(G_1)+1}^{D(PG)} deg(u)(k-2)d^u_{G_n}(k) \right] + \ldots.. +$$

$$\sum_{u\in V(G_n)} \left[\sum_{k=1}^{D(G_n)} deg(u)(k-2)d^u_{G_1}(k) + \ldots.. + \sum_{k=1}^{D(G_n)} deg(u)(k-2)d^u_{G_n}(k) + \right.$$

$$\left. \sum_{k=D(G_n)+1}^{D(PG)} deg(u)(k-2)d^u_{G_1}(k) + \ldots.. + \sum_{k=D(G_n)+1}^{D(PG)} deg(u)(k-2)d^u_{G_{n-1}}(k) \right]$$

$$= 4m_1(n_1-1) + \sum_{u\in V(G_1)} \sum_{k=1}^{D(G_1)} deg(u)(k-2)d^u_{G_1}(k) + \ldots.. + 4m_n(n_n-1)$$

$$+ \sum_{u\in V(G_n)} \sum_{k=1}^{D(G_n)} deg(u)(k-2)d^u_{G_n}(k) + 4n_1(m_2+1) + \sum_{u\in V(G_1)} \sum_{k=1}^{D(PG)} deg(u)(k-2)d^u_{G_2}(k)$$

$$+ \ldots. + 4n_1(m_n+1) + \sum_{u\in V(G_1)} \sum_{k=1}^{D(PG)} deg(u)(k-2)d^u_{G_n}(k) + \ldots. + 4n_n(m_1+1) + \sum_{u\in V(G_n)}$$

$$= \sum_{k=1}^{D(PG)} deg(u)(k-2)d^u_{G_1}(k) + \ldots. + 4n_n(m_{n-1}+1) + \sum_{u\in V(G_n)} \sum_{k=1}^{D(PG)} (k-2)d^u_{G_{n-1}}(k)$$

□

5. Practical Cases

As complex networks application, we have calculated the degree distance index for some composed graphs, according to the results presented in Laghridat, Mounir and Essalih [5]. In this paragraph, we will calculate the Degree Distance index $DD(G)$ for some specific complex graphs, that are connected to each others in different ways.

5.1. The Star vertex's graph : Star graphs.

The Star vertex's graph is a graph composed by N users of the same social network connected to each other by a common friend (vertex).

Corollary 1. *Let SS be a Star vertex's graph with n vertices and m edges composed by N same Star's graph S_i such that $|V(S_i)| = n_s$ and $|E(S_i)| = m_s$, $i \in \{1,2,3,\dots,N\}$ (see Figure 3). The graphs S_i are connected by a common vertex s. We have:*

- *For $u \in V(S_i)$ a vertex such that $d(u,s)$ =1 , $i \in \{1,2,3,\dots,N\}$:*

$$d^u_{\mathscr{S}_i}(k) = \begin{cases} N-1 & \text{if } k = 2 \\ (n_s-2)(N-1) & \text{if } k = 3 \end{cases}$$

- *For $v \in V(S_i)$ a vertex such that $d(v,s)$ =2 , $i \in \{1,2,3,\dots,N\}$:*

$$d^v_{\mathscr{S}_i}(k) = \begin{cases} N-1 & \text{if } k = 3 \\ (n_s-2)(N-1) & \text{if } k = 4 \end{cases}$$

Moreover, the Degree Distance index of Star vertex's graph SS is :

$$DD(SS) = N(n_s-1)(3n_s-4) + N(N-1)\left[4m_s(n_s-1) + n_s^2 - n_s + 1\right]. \tag{5}$$

Proof. By applying Lemma 4.1 and the Theorem 4.2 $\square$

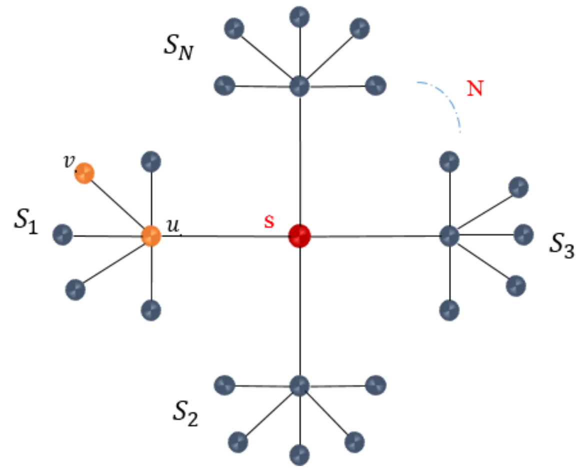

FIGURE 3. Star vertex's graph SS composed of N Star S_i

5.2. The Star edge's graph : Fan graphs. The star edge's graph is a grouping of different social networks (graphs) of the same user (vertex) .

Corollary 2. *Let SF be a Star edge's graph with n vertices and m edges composed by N same Fan graphs $F_i, i \in \{1,2,3,\ldots,N\}$ such that $|V(F_i)| = n_f$ and $|E(F_i)| = m_f$ (see Figure 4). The Fan graphs F_i are connected by a common vertex s and a set of edges $\{s,u_i\}$. We have:*

- $DD(F_i) = 7n_f^2 - 27n_f + 30,$
- *For $u \in V(F_i)$ a vertex such that $d(u,s)$ =1 , $i \in \{1,2,3,\ldots,N\}$:*

$$d^u_{\mathscr{F}_i}(k) = \begin{cases} 1 & \text{if } k = 1 \\ N-1 & \text{if } k = 2 \\ (n_f-1)(N-1) & \text{if } k = 3 \end{cases}$$

- *For $v \in V(F_i)$ a vertex such that $d(v,s)$ =2 , $i \in \{1,2,3,\ldots,N\}$:*

$$d^v_{\mathscr{F}_i}(k) = \begin{cases} 1 & \text{if } k = 2 \\ N-1 & \text{if } k = 3 \\ (n_f-1)(N-1) & \text{if } k = 4 \end{cases}$$

- *For $S \in V(SF)$:*

$$d^S_{\mathscr{S}\mathscr{F}}(k) = \begin{cases} 1 & \text{if } k = 1 \\ N(n_f-1) & \text{if } k = 2 \end{cases}$$

Moreover, the Degree Distance index of Star edge's graph SF is :

$$DD(SF) = N\left[DD(F_i) + m_f + n_f + (N-1)(4n_f(m_f+1) + 3/2n_f^2 - n_f - 5/2)\right]. \tag{6}$$

Proof. By applying Lemma 4.3 and the Theorem 4.4. □

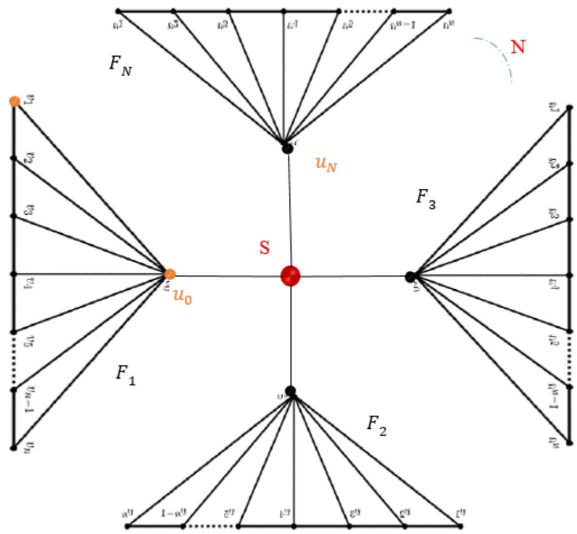

FIGURE 4. Star edge's graph SF composed of N Fan graph F_i

5.3. The Path graph: Star graphs. The path graph is a complex graph that represents the connectivity between each user (vertex) of a social network and his friends. It helps to calculate the social distance between the nodes of the graph (users).

Corollary 3. *Let PS be a Path graph with n vertices and m edges composed by the same N star graphs S_i of order such that $|V(S_i)| = n_s$ and $|E(S_i)| = m_s$, $i \in \{1,2,3,\ldots,N\}$ (see Figure 5). The Star graphs S_i are connected by a common vertex v_i and a set of edges $\{v_i, v_{i+1}\}$, $i \in \{1,2,3,\ldots,N\}$. We have:*

- $DD(S_i) = (n_s-1)(3n_s-4)$,
- *For $v \in V(S_i))$ with i=1,N*

$$d^v_{\mathscr{S}_i}(k) = \begin{cases} 1 & \text{if } k = 1 \text{ and } i=1,N \\ n_s & \text{if } k = 2,..,D(PS)\text{-}2 \text{ and } i=1,N \\ n_s - 1 & \text{if } k = D(PS)\text{-}1 \text{ and } i=1,N \end{cases}$$

- *For* $v \in V(S_i)$ *with i=2,N-1*

$$d^{v}_{\mathscr{S}_i}(k) = \begin{cases} 2 & \textit{if } k = 1 \textit{ and } i{=}2,N{-}1 \\ 2n_s - 1 & \textit{if } k{=}2 \textit{ and } i{=}2,N{-}1 \\ n_s & \textit{if } k = 3,\ldots,D(PS)-3 \textit{ and } i{=}2,N{-}1 \\ n_s - 1 & \textit{if } k{=}\ D(PS){-}2 \textit{ and } i{=}2,N{-}1 \end{cases}$$

- *For* $v \in V(S_i)$ *with* $i \neq 1,2,N-1,N$

$$d^{v}_{\mathscr{S}_i}(k) = \begin{cases} 2 & \textit{if } k = 1 \textit{ and } i \neq 1,2,N-1,N \\ 2n_s & \textit{if } k{=}2 \textit{ and } i \neq 1,2,N-1,N \\ 2n_s - 1 & \textit{if } k = 3 \textit{ and } i \neq 1,2,N-1,N \\ n_s & \textit{if } k = 4,\ldots,D(PS)-4 \textit{ and } i \neq 1,2,N-1,N \\ n_s - 1 & \textit{if } k{=}\ D(PS){-}3 \textit{ and } i \neq 1,2,N-1,N \end{cases}$$

- *For* $u \in V(S_i)$ *with i=1,N*

$$d^{u}_{\mathscr{S}_i}(k) = \begin{cases} 1 & \textit{if } k = 2 \textit{ and } i{=}1,N \\ n_s & \textit{if } k = 3,\ldots,D(PS)-1 \textit{ and } i{=}1,N \\ n_s - 1 & \textit{if } k{=}\ D(PS) \textit{ and } i{=}1,N \end{cases}$$

- *For* $u \in V(S_i)$ *with i=2,N-1*

$$d^{u}_{\mathscr{S}_i}(k) = \begin{cases} 2 & \textit{if } k{=}2 \textit{ and } i{=}2,N{-}1 \\ 2n_s - 1 & \textit{if } k{=}3 \textit{ and } i{=}2,N{-}1 \\ n_s & \textit{if } k{=}\ 4,\ldots,D(PS){-}3 \textit{ and } i{=}2,N{-}1 \\ n_s - 1 & \textit{if } k{=}D(PS){-}2 \textit{ and } i{=}2,N{-}1 \end{cases}$$

- *For* $u \in V(S_i)$ *with* $i \neq 1,2,N-1,N$

$$d^{u}_{\mathscr{S}_i}(k) = \begin{cases} 2 & \textit{if } k = 2 \textit{ and } i \neq 1,2,N-1,N \\ 2n_s & \textit{if } k{=}\ 3 \textit{ and } i \neq 1,2,N-1,N \\ n_s & \textit{if } k = 4,\ldots,D(PS)-3 \textit{ and } i \neq 1,2,N-1,N \\ n_s - 1 & \textit{if } k{=}\ D(PS){-}3 \textit{ and } i \neq 1,2,N-1,N \end{cases}$$

The Degree Distance index of Path graph PS is :

$$DD(PS) = NDD(\mathscr{S}_i) + 4N(N-1)n_s(m_s+1) + 4(N-1)$$

$$+1/2\Big[1/2N^3n_s^3 - 11/2n_s^3N^2 - N^3n_s^2 + 2n_s^3N + 15/2N^2n_s^2 \tag{7}$$

$$-22n_s^2N - 28n_s^3 + n_sN^2 + 22n_s^2 - 17Nn_s + 31n_s + 6N - 9\Big]$$

Proof. By applying Lemma 4.5 and the Theorem 4.6 □

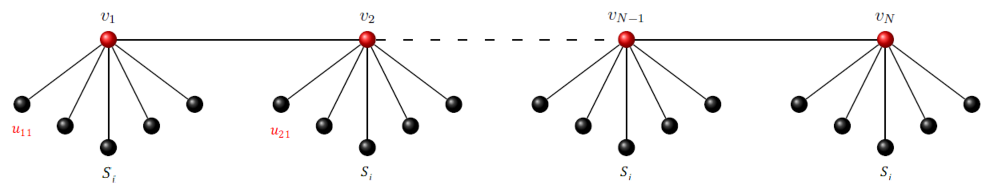

FIGURE 5. The path's graph PS composed by N Star graph S_i

6. CONCLUSION

Nowadays, social networks have a significant impact on society's behavior and human thinking. To accompany the massive use of such communication models many disciplines are built all around. For example, social network analysis has been introduced to describe and characterize these networks. Thanks to many works, such analysis were formalized into graphs where the vertices represent the accounts and the edges represent the relationship between them. Our contribution provides topological properties to characterize some specific social networks. Indeed, by using topological indices we give the formulas to calculate the Degree Distance index of certain complex graphs connected by a vertex (Star vertex's graphs) and others connected by an edge (Star edge's graphs) and those connected by a set of edges (Path's graphs).

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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