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NON-NEWTONIAN BLOOD FLOW MODEL WITH THE EFFECT OF DIFFERENT GEOMETRY OF STENOSIS

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Abstract: The objective of this paper is to present a non-Newtonian blood flow model with the effect of different geometry of stenosis on various flow quantities. The Power-law model is considered to explore the non-Newtonian property of blood. Two-point Gauss quadrature formula is applied to obtain the numerical expressions of dimensionless flow resistance, skin-friction and flow rate. The variation of dimensionless flow resistance, skin-friction and flow rate with degree of stenosis, axial distance and power-law index is shown graphically. Moreover, the power-law index is adjusted to explore the non-Newtonian characteristics of blood. The importance of the present work has been carried out by comparing the results with other theories both numerically and graphically. It has been found that resistance to flow becomes maximum with total blockage of artery for different shape of stenosis.

Keywords: non-Newtonian fluid; flow behavior index; Gauss quadrature formula; stenotic geometry.

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1. INTRODUCTION

Diseases in human blood vessels and in heart such as heart attack and brain stroke are the main cause of death worldwide. The major reason for such diseases is the constriction in artery or narrowing of the artery. In medical term, it is called stenosis. Stenosis is a pathological constriction of an artery, generally due to deposition of fat, cholesterol and unusual development of tissues. The fatty substances reduce the cross-sectional area of blood vessels and hence each body part cannot receive an adequate quantity of blood. Therefore, the analytical study of blood flow through a stenosed artery is beneficial for a better understanding and diagnosis of the artery diseases [3,14]. It has been observed that some researchers work on blood by treating it as Newtonian fluid [5, 12]. When blood flows at a high shear rate with a larger diameter arteries, then it is considered as Newtonian blood. While blood flows at a low shear rate with smaller diameter arteries, then it would be treated as non-Newtonian blood [9]. Ishikawa et al. [4] observed that non-Newtonian behavior of flow has high stability than Newtonian flow because non-Newtonian characteristics of blood affect the flow quantities. It reduces wall shear stress, vortex size and wall pressure. The Power-law [8, 9], Herschel-Bulkley [7, 11] and Casson model [10, 16, and 17] of non-Newtonian fluid are frequently used by many researcher. The Herschel-Bulkley model and Power-law model have more benefits than Casson model. The flow-behavior index of Herschel-Bulkley and Power-law model can be adjusted to a preferred value, while the flow-behavior index of Casson model is fixed. Furthermore, Easthope and Brooks [1] observed that power-law model has more significant role in modelling of non-Newtonian flow.

Misra and shit [7] studied the generalized model of blood flow by taking it as a Casson and Herschel-Bulkley fluid. Singh et al. [15] considered a blood flow model to analyze the impact of shape parameter and stenosis length on the wall shear stress and flow resistance with axially symmetric but radially non-symmetric stenosed artery. The impact of different parameters of stenosis shape on various flow quantities with slip conditions studied by Haldar [2], Singh [13] and Srivastava [6]. A non-Newtonian blood flow model with effect of stenosis shape and slip velocity at wall is considered by Bhatnagar et al. [18]. They found that axial velocity and flow rate

is increased with slip but it decreased with yield stress.

Kamanger et al. [19] investigated the blood flow behavior with severity of blockage area with different geometries like trapezium, triangular and elliptical shape. They conducted their study to different size of stenosis in terms of 70%, 80% and 90% area blockage of artery. They observed that wall shear stress is high for trapezoidal shape as compared to triangular and elliptical shape. Sriyab [20] considered a non-Newtonian blood flow model with various geometry of shape of stenosis like bell and cosine shape. They observed that cosine shape displays higher pressure, flow resistance and flow rate than bell shape. Owasiit and Sriyab [21] considered non-Newtonian blood flow model with various geometry of stenosis in stenosed artery. They studied power-law model of two-dimensional blood flow having vertically asymmetric and symmetric stenosis.

It has been observed that geometry of stenosis has a significant role in study of blood flow. In our present work, we presented a graphical and conceptual study of non-Newtonian blood flow model with effect of different geometry of stenosis, like rectangular, cosine and trapezoidal shape. Furthermore, the impact of power-law index, depth of stenosis, and non-Newtonian behavior of blood on the various flow quantities for rectangular, cosine and trapezoidal shape are shown graphically.

2. STENOTIC ARTERY MODEL

We considered a mathematical model for different geometry of arterial stenosis, such as rectangular, cosines and trapezoidal stenosis shape. The geometrical representation of these shapes are shown in figure (1), (2) and (3) respectively.

The mathematical representation of the geometry of the rectangular shape stenosis is given as follows,

$$R(z) = R_0(1 - S) \quad \text{or} \quad \frac{R(z)}{R_0} = 1 - S \quad (1)$$

Where R_0 indicates normal artery radius and $R(z)$ represents stenosed artery radius and S

indicates degree of stenosis, $S = \frac{R_0 - R_{min}}{R_0}$. We considered here that for rectangular shape, R is

constant and equal to R_{min} .

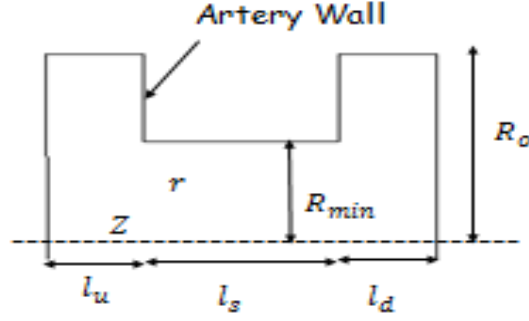


Fig.1. Geometry of axially symmetric rectangular shaped stenosed artery.

The mathematical representation for geometry of cosine shape stenosis is defined as follows,

$$R(z) = R = R_0 \left\{ 1 - \frac{S}{2} \left[1 - \frac{\cos \pi(z - l_u)}{l_s} \right] \right\} \quad \text{for } l_u \leq z \leq l_u + l_s \quad (2)$$

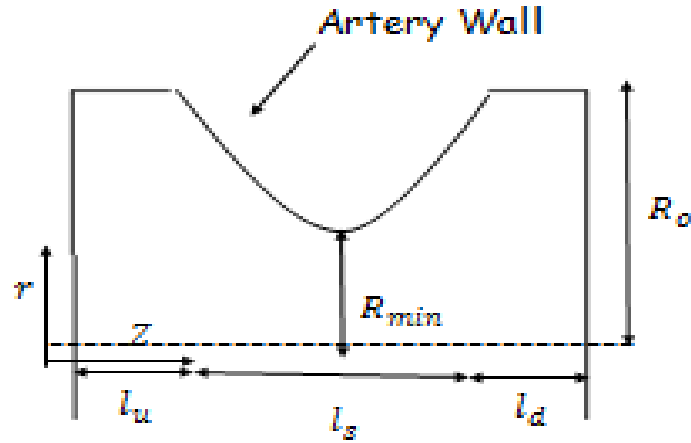


Fig.2. Geometry of axially symmetric cosine shaped stenosed artery.

The mathematical representation for geometry of Trapezoidal shape stenosis is defined as

$$R(z) = R_0 - (z - l_u) \tan \alpha_u \quad \text{for } l_u \leq z \leq (l_u + SR_0 / \tan \alpha_u) \quad 3(a)$$

$$R(z) = R_0 [1 - S] \quad \text{for } (l_u + SR_0 / \tan \alpha_u) \leq z \leq (l_u + l_s - SR_0 / \tan \alpha_d) \quad 3(b)$$

$$\text{And } R(z) = \frac{R_0 - (l_u + l_s - z) \tan \alpha_d}{R_0} \quad \text{for } (l_u + l_s - SR_0 / \tan \alpha_d) \leq z \leq (l_u + l_s) \quad 3(c)$$

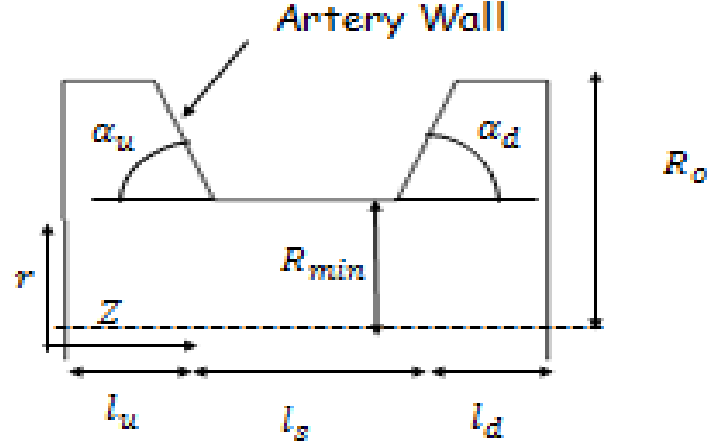


Fig.3. Geometry of axially symmetric trapezoidal shaped stenosed artery.

Where, α_u indicates upstream angle 30° and α_d represents downstream angle 45° .

3. FORMULATION OF THE PROBLEM

In this mathematical model, we considered blood flow in stenosed artery is laminar, steady (independent of time), fully-developed, one-dimensional and blood is taken as non-Newtonian and incompressible fluid. Cylindrical coordinate (r, z, θ) are taken to study this model, where r represents radial and z indicates axial direction. The governing equation of motion is given [19].

$$\frac{-dp}{dz} = \frac{1}{r} \frac{d}{dr}(r\tau) \quad (4)$$

Where, p represents pressure and τ indicates shear stress. The power-law model defined the non-Newtonian characteristics of blood as

$$\frac{-du}{dr} = f(\tau) = \left(\frac{\tau}{\mu} \right)^{\frac{1}{\alpha}} \quad (5)$$

Where, u represents blood velocity, α indicates index of power-law and μ shows viscosity of plasma. The boundary conditions to solve eq. (4) and (5) are considered as

$$u = 0 \quad \text{at } r = R(z) \quad (6)$$

$$\tau \text{ is finite at } r = 0 \quad (7)$$

4. MATHEMATICAL ANALYSIS

To study non-Newtonian blood flow model for various geometry of stenosis, we have to find analytical expressions for various flow quantities like skin-friction, flow resistance, and flow rate. In order to find the skin-friction, we integrate eq. (4) with respect to r and using boundary condition (7), we have,

$$\tau = \frac{-r}{2} \frac{dp}{dz} \quad (8)$$

The skin-friction, τ_R is obtained by
$$\tau_R = \frac{-R}{2} \frac{dp}{dz} \quad (9)$$

The flow rate is defined by
$$Q = \frac{\pi R^3}{\tau_R^3} \int_0^{\tau_R} \tau^2 f(\tau) d\tau \quad (10)$$

Using eq. (5) into (10), we get
$$Q = \frac{\pi R^3}{\tau_R^3} \int_0^{\tau_R} \tau^2 \left(\frac{\tau}{\mu} \right)^{\frac{1}{\alpha}} d\tau \quad (11)$$

Integrating eq. (11) with respect to τ , we get
$$Q = \frac{\pi R^3 \tau_R^{\frac{1}{\alpha}} \alpha}{\mu^{\frac{1}{\alpha}} (3\alpha + 1)} \quad (12)$$

Now, from eq. (12), we get the expression of skin-friction

$$\tau_R = \mu \left[\frac{Q(3\alpha + 1)}{\pi R^3 \alpha} \right]^{\alpha} \quad (13)$$

The resistance of blood flow (λ) is defined as
$$\lambda = \frac{P_2 - P_1}{Q} \quad (14)$$

Where, P_1 represents input pressure and P_2 indicates output pressure of blood in the artery.

Using eq. (13) into eq. (9), we obtain,
$$-\frac{dp}{dz} = \frac{2\mu}{R} \left[\frac{Q(3\alpha + 1)}{\pi R^3 \alpha} \right]^{\alpha} \quad (15)$$

Integrating eq. (15) along artery length and using, $p = P_1$ at $z=0$ and $p = P_2$ at $z = L = (l_u + l_s + l_d)$. We obtain

$$P_1 - P_2 = 4\mu \left[\frac{Q(3\alpha+1)}{\pi\alpha} \right]^\alpha \left(l_u + l_d + \int_{l_u}^{l_u+l_s} \frac{1}{R^{3\alpha+1}} dz \right) \quad (16)$$

Since $R = R(z)$ represents the stenosis area geometry. Hence, flow resistance for rectangular

shape is given by $\lambda_R = \frac{P_1 - P_2}{Q}$ and by applying two point Gauss quadrature formula for solution

of eq. (16) for different shape of stenosis. We have,

$$\lambda_R = 4\mu \left[\frac{(3\alpha+1)}{\pi\alpha} \right]^\alpha \cdot Q^{\alpha-1} \left[l_u + l_d + \frac{1}{[R_0(1-S)]^{3\alpha+1}} l_s \right] \quad (17)$$

Now when the geometry is of cosines shape, flow resistance is given by $\lambda_c = \frac{P_1 - P_2}{Q}$

$$\lambda_c = 4\mu \left[\frac{(3\alpha+1)}{\pi\alpha} \right]^\alpha \cdot Q^{\alpha-1} \cdot \left[l_u + l_d + \frac{l_s}{2R_0^{3\alpha+1}} \left\{ \left[\left(1 - \frac{S}{2} \left(1 - \cos \pi \left(\frac{\sqrt{3}-1}{2\sqrt{3}} \right) \right) \right] \right]^{-(3\alpha+1)} \right. \right. \right. \\ \left. \left. \left. + \left[1 - \frac{S}{2} \left(1 - \cos \pi \left(\frac{\sqrt{3}+1}{2\sqrt{3}} \right) \right) \right] \right]^{-(3\alpha+1)} \right\} \right] \quad (18)$$

Now, flow resistance for trapezoidal shapes given by $\lambda_T = \frac{P_1 - P_2}{Q}$

$$\lambda_T = 4\mu \left[\frac{(3\alpha+1)}{\pi\alpha} \right]^\alpha \cdot Q^{\alpha-1} \\ \left[l_u + l_d + \frac{SR_0 \tan \alpha_u}{2} \left[\left[R_0 - \frac{SR_0(\sqrt{3}-1)}{2\sqrt{3}} \right]^{-(3\alpha+1)} + \left[R_0 - \frac{SR_0(\sqrt{3}+1)}{2\sqrt{3}} \right]^{-(3\alpha+1)} \right] \right. \\ \left. + \frac{l_s}{[R_0(1-S)]^{3\alpha+1}} - \frac{SR_0 \tan \alpha_d}{2} \left[\left[1 - \frac{S(\sqrt{3}-1)}{2\sqrt{3}} \right]^{-(3\alpha+1)} + \left[1 - \frac{S(\sqrt{3}+1)}{2\sqrt{3}} \right]^{-(3\alpha+1)} \right] \right] \quad (19)$$

We have obtained the numerical expressions for shear stress, flow rate and flow resistance. Now, we have to analyse the impact of different geometry of stenosis and non-Newtonian characteristics of blood on these parameters.

When the constriction is not present in artery, the flow resistance defined for normal artery is

$$\lambda_N = 4\mu \left(\frac{(3\alpha+1)}{\pi\alpha} \right)^\alpha Q^{\alpha-1} \left[l_u + l_d + \frac{l_s}{R_0^{3\alpha+1}} \right] \quad (20)$$

Now, the non-dimensional flow resistance ($\Lambda_R, \Lambda_C, \Lambda_T$) due to stenosed artery for rectangular, cosines and trapezoidal shape is given in equations (21), (22) and (23) respectively.

$$\Lambda_R = \frac{\lambda_R}{\lambda_N} = \frac{\left[l_u + l_d + \frac{l_s}{[R_0(1-S)]^{3\alpha+1}} \right]}{\left[l_u + l_d + \frac{l_s}{R_0^{3\alpha+1}} \right]} \quad (21)$$

$$\Lambda_C = \frac{\lambda_C}{\lambda_N} = \frac{\left[l_u + l_d + \frac{l_s}{2R_0^{3\alpha+1}} \left[\left(1 - \frac{S}{2} \left\{ 1 - \cos \left(\frac{\sqrt{3}-1}{2\sqrt{3}} \right) \pi \right\} \right)^{-(3\alpha+1)} + \left(1 - \frac{S}{2} \left\{ 1 - \cos \left(\frac{\sqrt{3}+1}{2\sqrt{3}} \right) \pi \right\} \right)^{-(3\alpha+1)} \right] \right]}{\left[l_u + l_d + \frac{l_s}{R_0^{3\alpha+1}} \right]} \quad (22)$$

$$\Lambda_T = \frac{\lambda_T}{\lambda_N} = \left[l_u + l_d + \frac{SR_0 \tan \alpha_d}{2} \left[\left[R_0 - \frac{SR_0(\sqrt{3}-1)}{2\sqrt{3}} \right]^{-(3\alpha+1)} + \left[R_0 - \frac{SR_0(\sqrt{3}+1)}{2\sqrt{3}} \right]^{-(3\alpha+1)} \right] \right. \\ \left. + \frac{l_s}{[R_0(1-S)]^{3\alpha+1}} - \frac{SR_0 \tan \alpha_d}{2} \left[\left[1 - \frac{S(\sqrt{3}-1)}{2\sqrt{3}} \right]^{-(3\alpha+1)} + \left[1 - \frac{S(\sqrt{3}+1)}{2\sqrt{3}} \right]^{-(3\alpha+1)} \right] \right] \bigg/ \left[l_u + l_d + \frac{l_s}{R_0^{3\alpha+1}} \right] \quad (23)$$

Now, the non-dimensional blood flow resistance with the impact of Non-Newtonian behaviour of blood is considered as the ratio of the flow resistance of Non-Newtonian blood in a stenosed artery

to the flow resistance of Newtonian blood in a normal artery. Hence, it can be stated as

$$\xi_R = \left[\frac{(3\alpha + 1)}{\pi\alpha} \right]^\alpha \cdot \frac{Q^{\alpha-1}\pi}{4} \frac{\left[l_u + l_d + \frac{l_s}{[R_0(1-S)]^{3\alpha+1}} \right]}{\left[l_u + l_d + \frac{l_s}{R_0^{3\alpha+1}} \right]} \quad (24)$$

$$\begin{aligned} \xi_C = & \left[\frac{(3\alpha + 1)}{\pi\alpha} \right]^\alpha \cdot \frac{Q^{\alpha-1}\pi}{4} \left[l_u + l_d + \frac{l_s}{2R_0^{3\alpha+1}} \left\{ \left(1 - \frac{S}{2} \left(1 - \cos \left(\frac{\sqrt{3}-1}{2\sqrt{3}} \right) \pi \right) \right)^{-(3\alpha+1)} \right. \right. \\ & \left. \left. + \left(1 - \frac{S}{2} \left(1 - \cos \left(\frac{\sqrt{3}+1}{2\sqrt{3}} \right) \pi \right) \right)^{-(3\alpha+1)} \right\} \right] \bigg/ \left[l_u + l_d + \frac{l_s}{R_0^{3\alpha+1}} \right] \end{aligned} \quad (25)$$

$$\begin{aligned} \xi_T = & \left[\frac{(3\alpha + 1)}{\pi\alpha} \right]^\alpha \cdot \frac{Q^{\alpha-1}\pi}{4} \\ & \left[l_u + l_d + \frac{SR_0 \tan \alpha_u}{2} \left[\left[R_0 - \frac{SR_0(\sqrt{3}-1)}{2\sqrt{3}} \right]^{-(3\alpha+1)} + \left[R_0 - \frac{SR_0(\sqrt{3}-1)}{2\sqrt{3}} \right]^{-(3\alpha+1)} \right] \right. \\ & \left. + \frac{l_s}{[R_0(1-S)]^{3\alpha+1}} - \frac{SR_0 \tan \alpha_d}{2} \left[\left[1 - \frac{S(\sqrt{3}-1)}{2\sqrt{3}} \right]^{-(3\alpha+1)} + \left[1 - \frac{S(\sqrt{3}+1)}{2\sqrt{3}} \right]^{-(3\alpha+1)} \right] \right] \bigg/ \left(l_u + l_d + \frac{l_s}{R_0^{3\alpha+1}} \right) \end{aligned} \quad (26)$$

When stenosis is not present in artery ($R(z) = R_0$), the skin-friction becomes,

$$\tau_N = \mu \left[\frac{Q(3\alpha+1)}{\pi R_0^3 \alpha} \right]^\alpha \quad (27)$$

The non-dimensional skin friction τ is shown as
$$\tau = \frac{\tau_R}{\tau_N} = \left(\frac{R_0}{R} \right)^{3\alpha} \quad (28)$$

The dimensionless skin-friction due to stenosed artery for different shapes are expressed as

$$\tau_{R_1} = \left(\frac{R_0}{R_0(1-S)} \right)^{3\alpha} = \left(\frac{1}{1-S} \right)^{3\alpha} = (1-S)^{-3\alpha} \quad (29)$$

$$\tau_c = \frac{1}{\left[1 - \frac{S}{2} \left\{1 - \cos \frac{\pi(z-l_u)}{l_s}\right\}\right]^{3\alpha}} \quad (30)$$

$$\tau_T = \begin{cases} \left(\frac{R_0}{R_0 - (z-l_u) \cdot \tan \alpha_u}\right)^{3\alpha} & \text{for } l_u \leq z \leq l_u + SR_0 / \tan \alpha_u \\ \left(\frac{1}{1-S}\right)^{3\alpha} & \text{for } l_u + SR_0 / \tan \alpha_u \leq z \leq l_u + l_s - SR_0 / \tan \alpha_d \\ \left(\frac{1}{R_0 - (l_u + l_s - z) \cdot \tan \alpha_d}\right)^{3\alpha} & \text{for } l_u + l_s - SR_0 / \tan \alpha_d \leq z \leq l_u + l_s \end{cases} \quad (31)$$

The non-dimensional blood skin friction (η) with the impact of Non-Newtonian behaviour of blood is considered as the ratio of the skin friction of Non-Newtonian blood in a stenosed artery to the skin friction of Newtonian blood in a normal artery. Hence, it can be stated as

$$\eta = \frac{\tau_R}{\tau_{Ne}} = \frac{R_0^3 \pi}{4Q} \left(\frac{Q(3\alpha+1)}{\pi R^3 \alpha} \right)^\alpha, \text{ where, } \tau_{Ne} \text{ represent the skin-friction of Newtonian blood and it is}$$

$$\text{given as } \tau_{Ne} = \mu \left(\frac{4Q}{\pi R_0^3} \right).$$

For different geometry of stenosis, it can be expressed as

$$\eta_R = \frac{\pi R_0^3}{4Q} \cdot \left[\frac{Q(3\alpha+1)}{\pi \alpha} \right]^\alpha \cdot [R_0(1-S)]^{-3\alpha} \quad (32)$$

$$\eta_c = \frac{R_0^3 \pi}{4Q} \left[\frac{Q(3\alpha+1)}{\pi \alpha} \right]^\alpha \cdot \left[R_0 \left\{ 1 - \frac{S}{2} \left(1 - \cos \frac{\pi(z-l_u)}{l_s} \right) \right\} \right]^{-3\alpha} \quad (33)$$

$$\eta_T = \begin{cases} \frac{R_0^3 \pi}{4Q} \left[\frac{Q(3\alpha+1)}{\pi \alpha} \right]^\alpha \cdot [R_0 - (z-l_u) \cdot \tan \alpha_u]^{-3\alpha} & \text{for } l_u \leq z \leq l_u + SR_0 / \tan \alpha_u \\ [R_0(1-S)]^{-3\alpha} \cdot \frac{R_0^3 \pi}{4Q} \left[\frac{Q(3\alpha+1)}{\pi \alpha} \right]^\alpha & \text{for } l_u + SR_0 / \tan \alpha_u \leq z \leq l_u + l_s - SR_0 / \tan \alpha_d \\ \frac{R_0^3 \pi}{4Q} \left[\frac{Q(3\alpha+1)}{\pi \alpha} \right]^\alpha \cdot \left[\frac{R_0 - (l_u + l_s - z) \tan \alpha_d}{R_0} \right]^{-3\alpha} & \text{for } l_u + l_s - SR_0 / \tan \alpha_d \leq z \leq l_u + l_s \end{cases} \quad (34)$$

5. NUMERICAL RESULT AND DISCUSSION

In this mathematical model, we studied the impact of different shape of stenosis and non – Newtonian behavior of blood on physiologically indispensable flow quantities, like flow rate, skin friction, and flow resistance. For graphical representation, we executed the dimensionless flow quantities from equations (21) to (26) and equations (29) to (34) by using MATLAB.

TABLE 1. Parameter value

Parameters	Values
Flow rate (Q)	0-5 cm^3/s
Length of artery	3cm
Radius of normal artery	1mm
Viscosity(μ)	2-5 mpas
A	0.3-1

5.1. EFFECT OF DIFFERENT SHAPES OF STENOSIS ON FLOW RESISTANCE

The effect of dimensionless flow resistance ($\Lambda_R, \Lambda_C, \Lambda_T$) along R_{min}/R_0 with different shape of stenosis is shown in fig. 4 to 6 for different value of α . Since every shape is depend on degree of stenosis S. As R_{min}/R_0 decreases, the degree of stenosis S increases. From fig.4 to 6, it is observed that when α is increasing, the dimensionless resistance of flow for rectangular, cosine and trapezoidal shape is decreasing and it is also decreasing with R_{min}/R_0 . It has been noticed that as value of R_{min}/R_0 reaches towards 0, the resistance of flow is attained maximum. Therefore, retardation in flow is maximum, which represents the location of total blockage of artery and causes heart failure. Moreover, it is also pointed out that non-dimensional flow resistance in rectangular, cosine and trapezoidal shape is decreasing slightly when the blood takes Newtonian property i.e. $\alpha = 1$. A comparison of results are done with the result of karimi and it has been observed that results agree with the result of karimi.

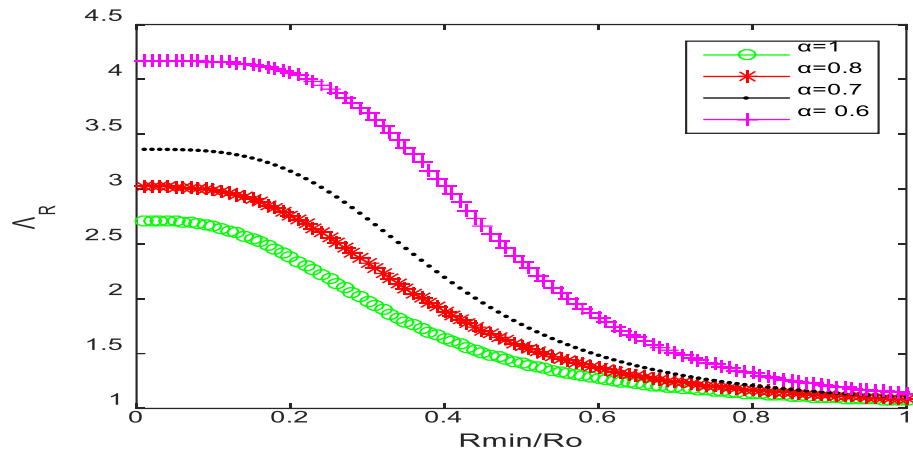


Fig.4. Variation in non-dimensional flow resistance with R_{min}/R_0 for rectangular shape having different value of α

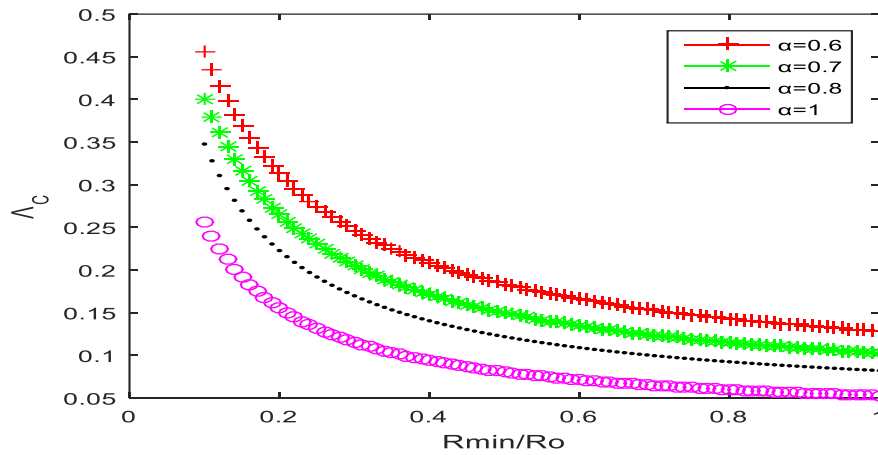


Fig.5. Variation in non-dimensional flow resistance with R_{min}/R_0 for cosine shape having different value of α

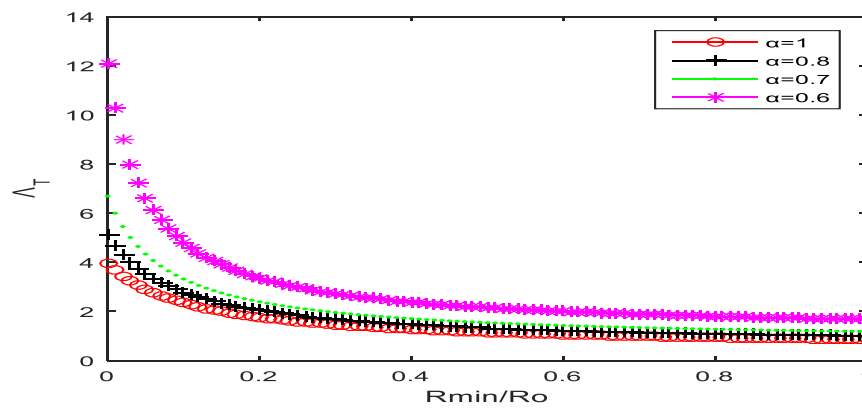


Fig.6. Variation in non-dimensional flow resistance with R_{min}/R_0 for trapezoidal shape having different value of α

5.2. IMPACT OF NON-NEWTONIAN CHARACTERISTICS OF BLOOD ON FLOW RESISTANCE

The variation in non-dimensional flow resistance (ξ_R, ξ_C, ξ_T) with non-Newtonian behavior of blood for various shapes of stenosis is investigated in fig.7 to 9. The impact of power-law index on flow resistance along R_{min}/R_0 is shown in these figures. It is found from fig.7 to 9 that as the value of α is increasing, the value of resistance to flow is decreasing. Moreover, it is also pointed out that non-dimensional flow resistance in rectangular, cosine and trapezoidal shape is decreasing slightly when the blood takes Newtonian property i.e. $\alpha = 1$. It is also noticed from these figures that rectangular shape has higher resistance to flow than cosine and trapezoidal shapes. It is observed from fig.4 to 9, that impact of stenosis and impact of non-Newtonian property of blood are exhibited the same behavior.

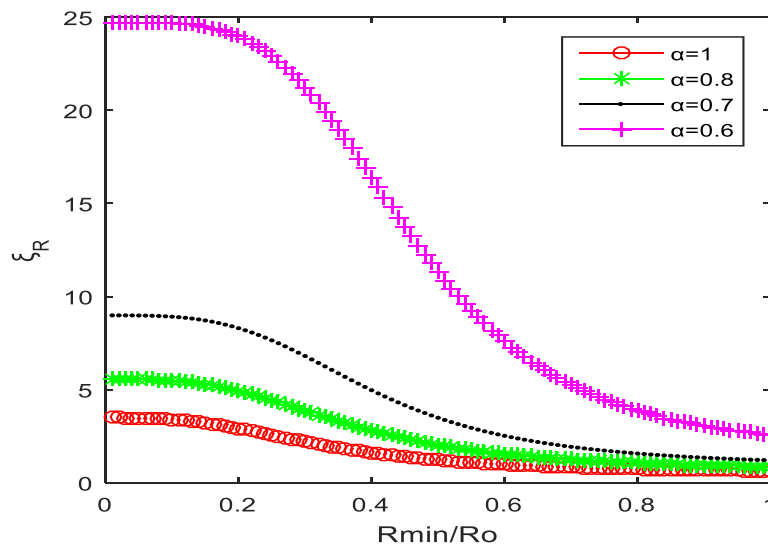


Fig.7. Variation in non-dimensional flow resistance with R_{min}/R_0 for rectangular shape having different value of α

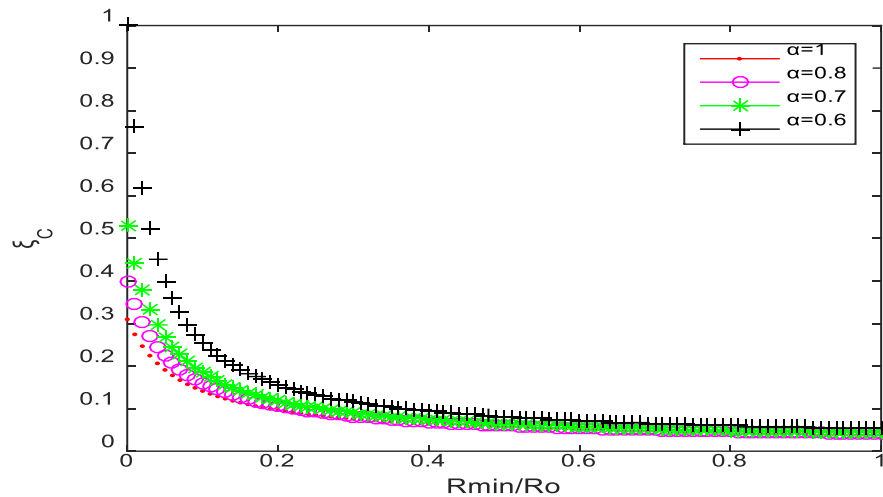


Fig.8. Variation in non-dimensional flow resistance with R_{min}/R_0 for cosine shape having different value of α

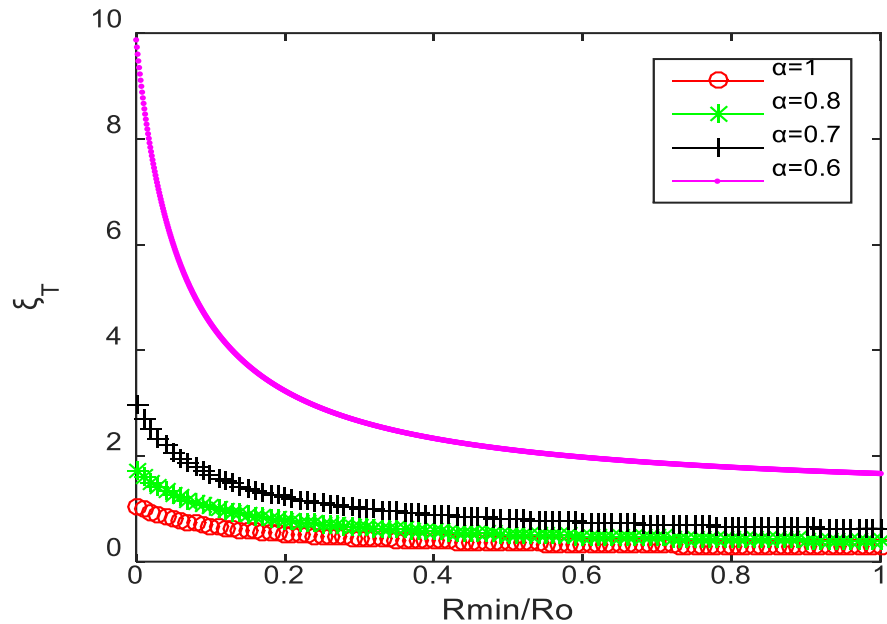


Fig.9. Variation in non-dimensional flow resistance with R_{min}/R_0 for trapezoidal shape having different value of α

6. SKIN FRICTION

6.1. IMPACT OF DIFFERENT SHAPE OF STENOSIS ON SKIN FRICTION

The variation on dimensionless skin friction ($\tau_{R1}, \tau_C, \tau_T$) for various geometry of stenosis is represented in fig.10 to 12. The effect of power-law index for rectangular shape with degree of stenosis is shown in fig.10. It is also pointed out that when the degree of stenosis arrives towards 1 i.e. R_{min}/R_0 , reaches 0, the dimensionless shear stress is attained maximum. Furthermore, the

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dimensionless skin friction is increasing as the value of power-law index is increasing. The impact of power-law index on non-dimensional skin friction for cosine shape along axial distance is displayed by fig.11. It is detected that skin friction is increasing with the axial distance. It is also noticed that dimensionless skin friction for cosine shape is decreasing when the value of α is increasing. The impact of power-law index on non-dimensional skin friction for trapezoidal shape along axial distance is displayed by fig.12. It is noted that dimensionless skin friction for trapezoidal shape is decreasing when the value of α is increasing and it is increasing along axial distance. Furthermore, it is investigated that dimensionless skin friction increases slightly when $\alpha=1$ and it is also examined from these figures that dimensionless skin-friction for cosine shape has higher value than trapezoidal shape along axial distance. A comparison of dimensionless skin friction results are done with the results of karimi and it has been observed that results agree with the results of karimi.

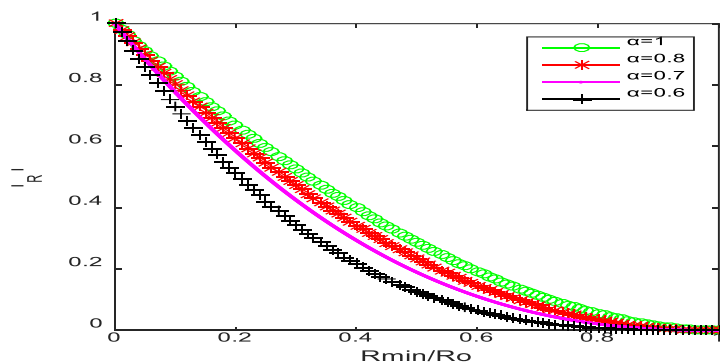


Fig.10 Variation in dimensionless skin friction with R_{min}/R_0 for rectangular shape having different values of α

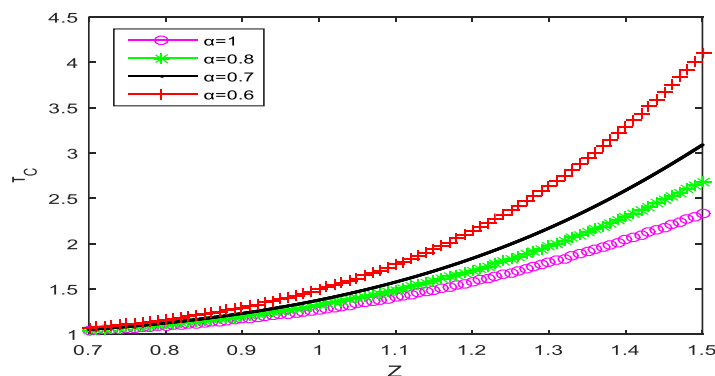


Fig.11 Variation in dimensionless skin friction along axial direction for cosine shape having different value of α

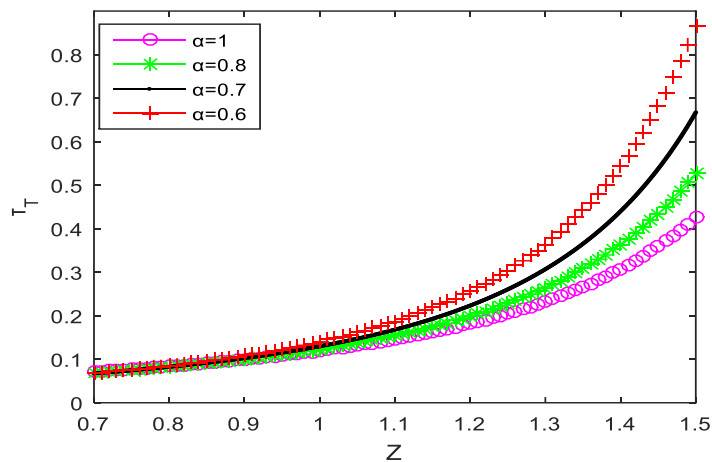


Fig.12. Variation in dimensionless skin friction for trapezoidal shape along axial direction having different values of α

6.2. EFFECT OF DIFFERENT SHAPE OF STENOSIS WITH NON-NEWTONIAN BEHAVIOR OF BLOOD ON SKIN FRICTION

The variation on dimensionless skin friction (η_R, η_C, η_T) with non-Newtonian behavior of blood for various geometry of stenosis is represented in fig.13 to 15. The effect of power law index for rectangular shape with degree of stenosis is depicted in fig.13. It is pointed out that dimensionless skin friction is increasing when the value of α is increasing but it is decreasing along R_{min}/R_0 . It is also pointed out that when the degree of stenosis arrives towards 1 i.e. R_{min}/R_0 reaches 0, the dimensionless shear stress is attained maximum. The impact of power-law index on non-dimensional skin friction for cosine shape along axial distance is represented by fig.14. It is found that skin friction is increasing with the axial distance. It is also noticed that dimensionless skin friction for cosine shape is decreasing when the value of α is increasing. The impact of power-law index on non-dimensional skin friction for trapezoidal shape along axial distance is depicted in fig.15. It is also noted that dimensionless skin friction for trapezoidal shape is decreasing when the value of α is increasing and it is increasing along axial distance. Furthermore, it is investigated that dimensionless skin friction increases slightly when $\alpha=1$. It is noted that dimensionless skin friction for cosine shape with non-Newtonian behavior along axial direction has attained higher value than trapezoidal shape. It is observed from fig. 10 to 15 that the impact of different geometry of stenosis on skin friction is same as impact of non-Newtonian behavior on skin friction.

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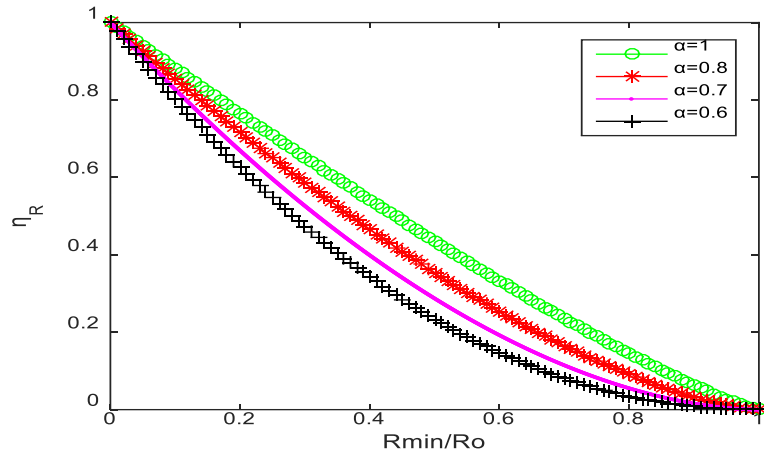


Fig.13. Variation in dimensionless skin friction with R_{min}/R_0 for rectangular shape having different values of α

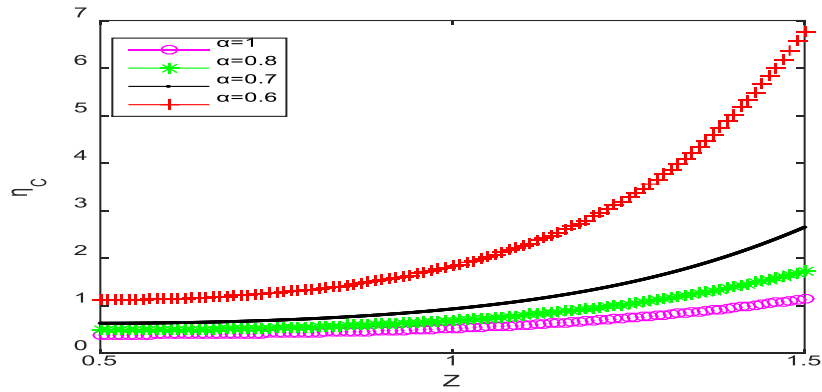


Fig.14. Variation in dimensionless skin friction for cosine shape having different values of α

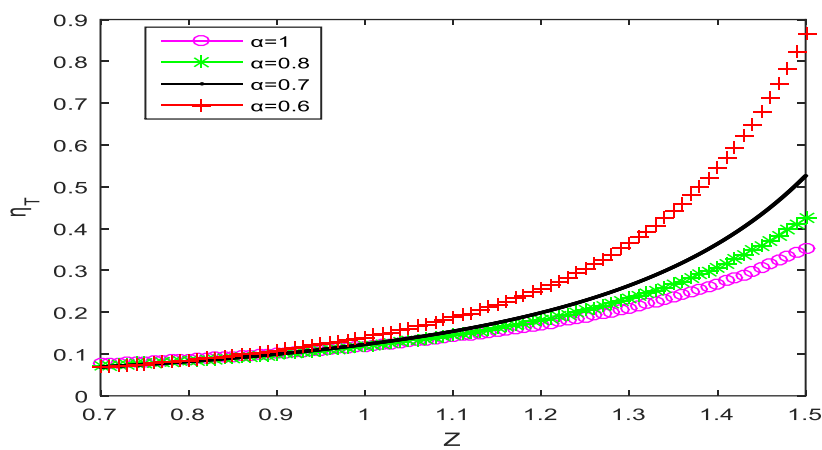


Fig.15. Variation in dimensionless skin friction for trapezoidal shape having different values of α

7. EFFECT OF DIFFERENT SHAPES OF STENOSIS ON FLOW RATE

The variation on flow rate (Q_R, Q_C, Q_T) of blood for various geometry of stenosis is represented by fig.16 to 18. The impact of power-law index (α) on flow rate with R_{min}/R_0 for rectangular, cosine and trapezoidal shape is shown in these figures. It is found from fig.16 to 18 that flow rate is increasing when the value of α is increasing. Furthermore, it is also pointed out that when the degree of stenosis arrives towards 1 i.e. R_{min}/R_0 reaches 0, the flow rate is attained minimum and it can cause heart failure. It is also observed that flow rate for both trapezoidal and cosine shape are attained the same maximum value and flow rate for both trapezoidal and cosine shape have higher value than rectangular shape.

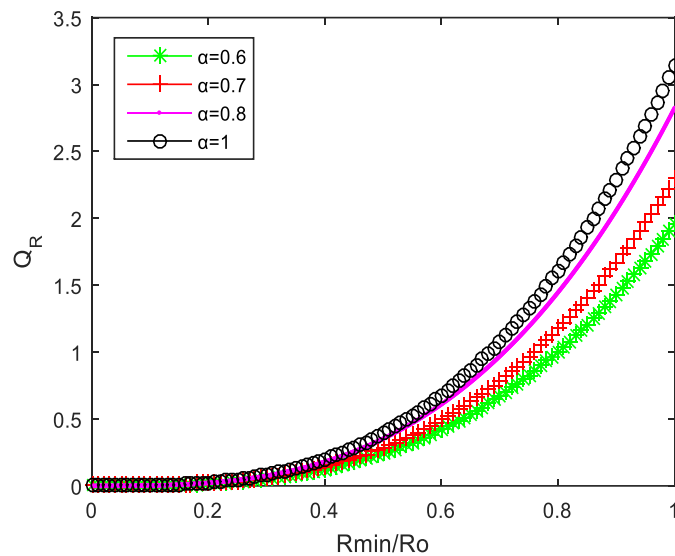


Fig.16. Variation on flow rate with R_{min}/R_0 for rectangular shape having different values of α

NON-NEWTONIAN BLOOD FLOW MODEL

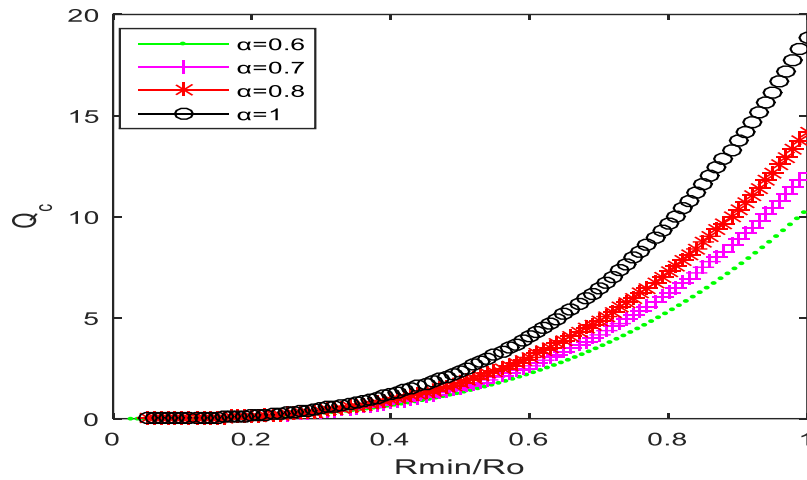


Fig.17. Variation on flow rate with R_{min}/R_0 for cosine shape having different values of α

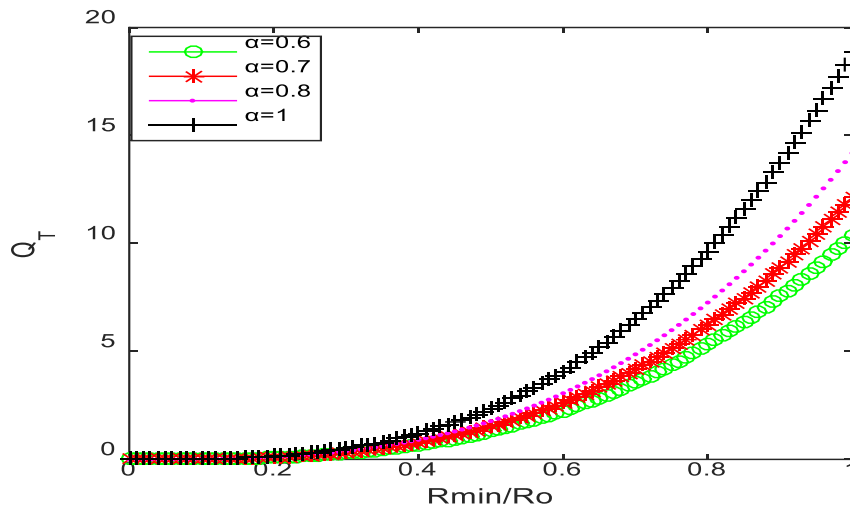


Fig.18. Variation on flow rate for trapezoidal shape with R_{min}/R_0 having different values of α

8. CONCLUSION

In this present work, we have studied the effect of different geometry of stenosis with Characteristics of non-Newtonian blood on various flow quantities like flow resistance, skin friction and flow rate. The main findings are given below:

1. The impact of flow-behaviour index on flow resistance with non-Newtonian behaviour of blood shows that rectangular shape has higher resistance to flow than cosine and trapezoidal shape.

2. The cosine shape has higher skin-friction along axial distance than trapezoidal shape.
3. The impact of various geometry of stenosis on non-dimensional skin friction in stenosed artery is same as impact of characteristics of non-Newtonian blood on non-dimensional skin friction.
4. Both trapezoidal and cosine shape are attained the same maximum value of flow rate and trapezoidal and cosine shape has higher flow rate of blood than rectangular shape.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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