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MODELING THE IMPACT OF FOOD INFLUENCERS ON CONSUMERS' FOOD CHOICES

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Abstract. Social media functions as the primary digital platform through which ideas and consumer preferences and behaviors transform in today's connected world. The UEA model serves as an innovative compartmental framework which explains how ideas spread from culinary influencers to the public and their effects on public opinion. The population in this model follows three behavioral states that include *Unaware (U)*, *Engaged (E)*, and *Adopters (A)*, which are inspired by epidemiological models SIR and marketing diffusion theories Bass and Rogers. People who remain Unaware show no interest in or lack exposure to an influencer's content but Engaged users show interest without commitment and Adopters actively endorse and spread the concept while becoming secondary influencers. The model demonstrates the dynamic nature of online platforms through its adoption reversibility feature alongside interest attrition and user turnover elements. We analyze state transitions and determine equilibrium behaviors while conducting stability and sensitivity tests through nonlinear differential equations. The simulation results demonstrate when an idea remains in the periphery or gains momentum or reaches viral dominance. We use the example of food influencers to demonstrate how mathematical modeling reveals the processes of opinion development and belief transformation and mass persuasion. The UEA model delivers theoretical knowledge together with practical applications for marketers and strategists and academics who want to understand and leverage digital influence dynamics.

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1. INTRODUCTION

In the current digital world, social media networks have become the new normal of consumers to express their preferences and intentions to buy products, especially in the food industry [4, 6]. Food influencers are new celebrities who have thousands of followers and share their recipes, products, and lifestyle and, in return, get videos, likes do single and post not comments. on only Their social influence is to recommendations, media people's have which can a are make decisions systematic usually approach accompanied people but to by want also understanding good to the and food try market analyzing pictures a trends the or certain [7]. spread dish The of or fact these product that ideas. shows a how necessary it.

Based on epidemiological models, the spread of ideas on social media has been compared to the spread of diseases [5, 8]. In this analogy, the population of users moves through states of awareness, engagement, and adoption of the information like the susceptible, infected, and recovered classes in epidemic modeling [9]. Note that while the spread of disease is real, the ”infection” being described here is a fake one made of enthusiasm, trust, and perceived social proof. So, for instance, users who come across an influencer's content may go from being merely curious to recommending the influencer wholeheartedly, or they may lose interest and move back to being passive.

To grasp these dynamics, this paper suggests a three state compartmental model, *Unaware* U, *Engaged* E, and *Adopters* A. Unaware people just scroll through or are oblivious to the influencer's posts; Engaged users show some interest liking commenting or casualily sharing content and not really adopting the product; Adopters, however, actively promote the influencer's recommendations in ways that are similar to ”viral” promotion among their peers. These elements, including constant user churn (inflow and outflow) on social media, are also included in the model. Using these elements it is possible to mathematically analyse such things as how many people will eventually adopt an idea, the rate at which enthusiasm increases or decreases and when the influencer's impact will be small.

The four objectives of this study are: First, we define a set of nonlinear ordinary differential equations to formalize these transitions. Second, we do an equilibrium analysis to determine the possible long term outcomes (from minimal uptake to partial or near universal adoption). Third, we do stability analysis, both local and global, to find thresholds at which small changes in user behavior can drastically shift the system's evolution. Fourth, through numerical simulations, we give an example of how the temporal trajectories of Unaware, Engaged, and Adopter proportions might look under various parameter settings.

The rest of this paper is organized as follows. In Section 3, I detail the mathematical framework and the assumptions made in developing the compartmental model. In Section 4, I present derivations of the equilibria and their influencer driven social diffusion. interpretations. In Section 5, explores stability criteria, highlighting tipping points for influencer-driven diffusion. Section 6 demonstrates computational results, and Section 7 concludes with insights on applying our findings to real-world marketing campaigns.

2. Theoretical Foundations and Model Comparison

To situate our contribution, we compare the main models that inspired our approach and highlight how the UEA model extends their core concepts to the context of influencer-driven opinion change.

COMPARATIVE OVERVIEW OF FOUNDATIONAL MODELS AND THE UEA FRAMEWORK

TABLE 1. Comparative Summary of Foundational Models and the UEA Framework

Model	Key Characteristics	Limitations	Focus Area
SIR Model (Epidemiology)	Classic compartmental model with *Susceptible–Infected–Recovered* groups; transitions governed by transmission and recovery rates.	No reinfection or fading interest; lacks engagement, dropout, or reactivation dynamics.	Spread of diseases over time in public health contexts.
Bass Model (Marketing)	Captures adoption via external (innovators) and internal (imitators) influences; generates S-shaped curve through ODEs.	Ignores disengagement; no dynamic user exit or reactivation; assumes linear adoption path.	Market penetration and forecasting of product adoption.
Rogers' Diffusion (Sociology)	Conceptual model of five adopter categories (Innovators to Laggards); explains temporal diffusion patterns qualitatively.	No mathematical structure; lacks formal transitions or feedback dynamics.	Social diffusion of ideas and behavioral innovations.
UEA Model (This Work)	Dynamic system with reversible states: *Unaware, Engaged, Adopters*; includes churn and belief transitions; built using differential equations.	Requires calibration; theoretical stage; results sensitive to parameter tuning and context.	Digital belief dynamics, influencer marketing, and online engagement modeling.

3. MATHEMATICAL MODEL

In this section, we present a compartmental framework to describe how an influencer's message particularly about food or culinary products spreads across a population of social media users. Our formulation for influencing behavior is inspired by classical *SIR* type models in epidemiology [9] but also incorporates concepts from economic and marketing diffusion theories [1, 8].These theories usually divide populations into categories like ”innovators,” ”imitators”, or ”late adopters”. We adapt those ideas to the context of social media influencers, and define three compartments based on unique behaviors observed on modern platforms.

3.1. Conceptual Model Description. Rationale and Key Assumptions.

We develop our approach based on the assumption that people can be in the state of either complete indifference or enthusiastic advocacy of an influencer's recommendation, with all gradations in between. To reflect these gradations, we use three states:

- *Unaware (U)*: Individuals who have not significantly engaged with or acknowledged the influencer's material.

  Economics & Marketing Context: In traditional diffusion models, these individuals may be classified as ”uninformed” or ”potential” customers who have neither been exposed to nor contemplated the product.
- *Engaged (E)*: Individuals who express interest in the material via likes, comments, or informal word-of-mouth, however have not completely committed to purchasing or permanently adopting the concept.

  Connection to Existing Literature: This somewhat aligns with the notion of ”light users” or ”imitators” who exhibit curiosity but may return to non-usage if the influencer does not maintain their engagement [1].
- *Adopters (A)*: Committed proponents who have fully accepted the product or concept and actively advocate for it. They function as ”secondary influencers,” capable of amplifying the message.

  Marketing Insight: Comparable to ”innovators” or ”loyal customers” in consumer theory, these users not only embrace but also disseminate fervent praises to their networks [8].

Each description integrates known concepts from economics, such as consumer adoption phases, and marketing, including brand evangelists, with the contemporary emphasis on digital influencer dynamics. By integrating these elements, we anchor our model in established theoretical frameworks while tackling emerging social media phenomena.

Transitions and Interactions. We propose five fundamental mechanisms to elucidate how users transition among these three states:

(1) *Unaware* $\rightarrow$ *Engaged* ($U \rightarrow E$): An Unaware individual gets intrigued or somewhat convinced while seeing the behaviors of engaged users or witnessing favorable responses to the influencer's message.

(2) *Unaware* $\rightarrow$ *Adopters* ($U \rightarrow A$): A more assertive transition in which a persuasive advertisement, discount, or robust endorsement results in a quick increase in adoption similar to "instant buy-in."

(3) *Engaged* $\rightarrow$ *Adopters* ($E \rightarrow A$): A user transitions from casual curiosity to complete adoption upon receiving enough positive signals (e.g., frequent exposure, social validation).

(4) *Adopters* $\rightarrow$ *Engaged* ($A \rightarrow E$): Certain adopters exhibit less enthusiasm as novelty wanes, unfavorable reviews arise, or they encounter new options, indicating a potential reduction in brand loyalty over time.

(5) *Natural Entry/Exit* (μ): New users consistently enter the platform as Unaware, whereas all segments undergo attrition at a rate of μ, indicative of the platform's churn (users canceling accounts or completely losing interest).

These channels underscore the fluidity of influencer marketing, whereby not all interest results in sustained adoption, and even committed adopters may ultimately disengage, requiring continuous influencer engagement.

3.2. Model Equations.

Let $U(t)$, $E(t)$, and $A(t)$ be the respective numbers of Unaware, Engaged, and Adopter individuals at time t, with $N = U + E + A$ fixed. Building on epidemiological-style mass-action assumptions, we write:

$$
\begin{cases}
\dfrac{dU}{dt} = \mu N - \beta_1 U \dfrac{E}{N} - \beta_2 U \dfrac{A}{N} - \mu U, \\
\dfrac{dE}{dt} = \beta_1 U \dfrac{E}{N} - \theta_1 E \dfrac{A}{N} + \theta_2 A \dfrac{E}{N} - \mu E, \\
\dfrac{dA}{dt} = \beta_2 U \dfrac{A}{N} + \theta_1 E \dfrac{A}{N} - \theta_2 A \dfrac{E}{N} - \mu A.
\end{cases}
$$

where $U(0) \geq 0, E(0) \geq 0, A(0) \geq 0$.

The model's parameters are positive constants with the following definitions:

- β_1 **(Engagement Rate).**

  This indicator assesses the efficacy with which *Engaged* persons influence the *Unaware* to develop interest. In the extensive diffusion literature, a partial endorsement or casual reference corresponds with Rogers's concept of early awareness generation [8]. Users see friends who exhibit a ”mildly positive” disposition towards a product (e.g., likes, short word-of-mouth), so indicating that β_1 quantifies the frequency with which these minor prompts transition Unaware users to at least Engaged level.
- β_2 **(Direct Adoption Rate).** This rate regulates ”instant conversions,” when an *Unaware* user transitions directly to Adopter. In marketing terminology, it aligns with the Bass model's concept that some customers, similar to ”innovators,” make immediate purchases if they find the influencer's material sufficiently engaging [1]. Real world stimuli include exclusive offers, emotive pleas, or viral films that prompt rapid purchases or subscriptions.
- θ_1 **(Forward Transition:** $E \to A$**).**

  Upon user engagement,θ_1 measures the rapidity with which they transition to full adoption. Comparable occurrences occur in the incremental adoption process outlined by Rogers, whereby recurrent affirmative signals (social proof, supplementary influencer postings) encourage Engaged users to decisively embrace [8]. A high θ_1 indicates that moderate curiosity quickly transforms into steadfast advocacy.
- θ_2 **(Backward Transition:** $A \to E$**).**

Certain *Adopters* may experience diminished excitement owing to adverse reviews, rival goods, or influencer errors, reverting to an Engaged state remaining partly enthusiastic while lacking complete commitment. The phenomenon of ”brand disillusionment” is extensively researched in marketing; once devoted customers may diminish their support as contentment wanes [3]. . Here, θ_2 measures the rate of decrease.

- $\theta = \theta_1 - \theta_2$ **(Net Conversion vs. Reversion).**

  In epidemiological expansions, such as (e.g., SIRS-type models [9, 2]), the integration of forward and backward rates into a net term facilitates analysis. A positive θ signifies a greater flow from E to A than vice versa, indicating that once an individual is Engaged, they are predisposed to advance to Adoption. A negative θ , contrary, indicates that Adopters often regress to simple involvement.

- μ **(Turnover or Churn Rate).**

  Social media networks undergo a constant influx of users: new people join (first categorized as *Unaware*), while established members often depart. We adhere to conventional compartmental methodologies in which each compartment experiences a loss of a fraction μ of its members per unit time, counterbalanced by an influx of μN directed towards U [9, 8]. In marketing, this corresponds with actual ”churn,” indicating that even devoted Adopters may go if they forsake the platform itself. When μ is substantial, user attrition is elevated, necessitating the influencer to repeatedly attract new adopters, unlessβ_1, β_2, and θ_1 are sufficiently robust to counteract these losses.

3.3. State Transition Diagram. The state transition diagram in Figure 1 illustrates the flow between the compartments, including the transition rates.

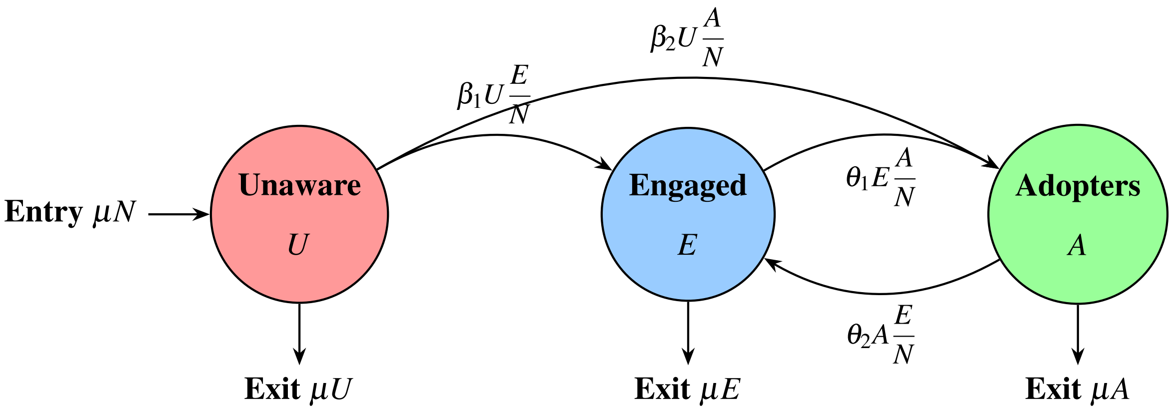

FIGURE 1. State transition diagram illustrating the flow between Unaware (U), Engaged (E), and Adopters (A) compartments, with transition rates and colored compartments.

4. EQUILIBRIUM ANALYSIS

Since $U+E+A=N$, we introduce proportions

$$u=\frac{U}{N}, \quad e=\frac{E}{N}, \quad a=\frac{A}{N}, \quad \text{so that} \quad u+e+a=1.$$

By substituting $u=1-e-a$ into the original ODEs and dividing by N, we reduce the system to the two-dimensional model:

$$\begin{cases} \dfrac{de}{dt}=\beta_1(1-e-a)e \;-\; \theta\, a e \;-\; \mu\, e, \\ \dfrac{da}{dt}=\beta_2(1-e-a)a \;+\; \theta\, a e \;-\; \mu\, a, \end{cases} \tag{1}$$

Next, we solve the equilibrium equations $\frac{de}{dt}=0$ and $\frac{da}{dt}=0$ to identify all possible steady states.

4.1. Equilibrium states.

Set the right-hand sides of (1) to zero:

$$\begin{cases} e\left[\beta_1(1-e-a) \;-\; \theta\, a \;-\; \mu\right]=0, \\ a\left[\beta_2(1-e-a) \;+\; \theta\, e \;-\; \mu\right]=0. \end{cases}$$

Two factors vanish in each equation, yielding up to four equilibria:

(1) **All-Unaware:**

$$E_0 = (e^*, a^*) = (0, 0).$$

Since $e = 0$ and $a = 0$ implies $u = 1$, no one is engaged or adopting the ”post”. This often signifies the ”post” fails to spread.

(2) **Engaged-Only:**

$$E_1 = \Big(1 - \frac{\mu}{\beta_1},\, 0\Big),$$

provided that $\beta_1 > \mu$. Here, some fraction $u^* = \frac{\mu}{\beta_1}$ remain Unaware, but the rest become Engaged. No one is a full Adopter in the steady state.

(3) **Adopters-Only:**

$$E_2 = \Big(0,\, 1 - \frac{\mu}{\beta_2}\Big),$$

provided that $\beta_2 > \mu$. A fraction $u^* = \frac{\mu}{\beta_2}$ remain Unaware, the rest are full Adopters, with no Engaged subpopulation.

(4) **Interior (Mixed) Equilibrium:**

$$E_3 = \big(e^*, a^*\big),$$

where

$$e^* = \frac{\mu(\beta_1 - \beta_2) - \theta(\beta_2 - \mu)}{\theta\,[\beta_1 - \beta_2 + \theta]},$$
$$a^* = \frac{\theta(\beta_1 - \mu) - \mu(\beta_1 - \beta_2)}{\theta\,[\beta_1 - \beta_2 + \theta]}, \quad u^* = 1 - e^* - a^* = \frac{\theta}{\beta_1 - \beta_2 + \theta}.$$

4.2. Conditions of admissibility. A physically (or socially) valid equilibrium must lie in the simplex

$$\{(e, a) \mid e \geq 0,\, a \geq 0,\, e + a \leq 1\}.$$

Hence, the expressions for (e^*, a^*) must be nonnegative, and $e^* + a^* \leq 1$. Each equilibrium thus imposes additional constraints:

- $E_0 = (0, 0)$ is *always* feasible. Interpreted as the *all-Unaware* (Post-free) outcome.
- $E_1 = (1 - \mu/\beta_1, 0)$ requires $\beta_1 > \mu$ and yields $e^* \in [0, 1]$. If stable, the post spreads *partially*, but only so far that everyone who hears about it stays Engaged rather than adopting.

- $E_2 = (0, 1 - \mu/\beta_2)$ requires $\beta_2 > \mu$. Then a fraction μ/β_2 remain Unaware, the rest are *Adopters*. Engaged class vanishes at equilibrium.
- $E_3 = (e^*, a^*)$ is an *interior equilibrium*, with both $e^* > 0$ and $a^* > 0$. It exists when

$$\beta_1 > \beta_2 \quad \text{and} \quad \mu(\beta_1 - \beta_2) - \theta(\beta_2 - \mu) > 0, \quad \theta(\beta_1 - \mu) - \mu(\beta_1 - \beta_2) > 0,$$

  plus consistency of signs to ensure $u^* > 0$. In that case, the population splits into positive fractions of Unaware, Engaged, and Adopters. This often represents a *mixed* or *endemic* outcome where the post persists but not everyone adopts.

Interpretations

(1) $E_0 = (0,0)$. All individuals remain Unaware. The influencer's idea does *not* catch on. Mathematically, β_1 and β_2 may be too small relative to μ (large turnover), preventing the idea from taking hold.

(2) $E_1 = (1 - \mu/\beta_1, 0)$. A fraction μ/β_1 remain Unaware, while the other $1 - \mu/\beta_1$ are *Engaged* but not fully convinced. This can happen if the Engaged-luring rate β_1 is large enough to overcome churn ($\beta_1 > \mu$), but direct adoption β_2 or net transition θ is insufficient to maintain a stable Adopter base.

(3) $E_2 = (0, 1 - \mu/\beta_2)$. A fraction μ/β_2 stay Unaware, the rest *Adopt* directly or eventually. No one hovers in the Engaged class long-term. This demands $\beta_2 > \mu$, so that direct adoption outstrips platform turnover.

(4) $E_3 = (e^*, a^*)$. All three compartments u, e, a are nonzero. The influencer's post *persists* with stable fractions of Engaged and Adopters. This scenario arises when $\beta_1 > \beta_2$ plus the net flow conditions ensure $e^* > 0, a^* > 0$, i.e. enough users pass through engagement *and* adoption while balancing the reversion rate θ_2 and the churn μ.

4.3. Initial Conditions and Feasible Domains. Because $e(t) + a(t) \leq 1$ and $e, a \geq 0$, any *initial condition* $(e(0), a(0))$ must lie in the triangular domain:

$$\{(e,a) \mid e \geq 0, \, a \geq 0, \, e + a \leq 1\}.$$

Common choices include:

- $(e(0), a(0)) = (\varepsilon, 0)$ for a small $\varepsilon > 0$, representing a minor "seed" of engaged individuals at $t = 0$;
- $(e(0), a(0)) = (0, \varepsilon)$, a small fraction of immediate adopters;
- or $(0, 0)$, meaning the idea is initially completely absent.

As time evolves, the system flows toward one of the equilibria $(0, 0)$, E_1, E_2, or E_3, depending on parameter values and stability properties. In a real marketing scenario, these equilibria correspond to ultimate outcomes such as complete failure to spread, partial interest (engagement) only, universal adoption among those who become aware, or a sustained coexistence of "on-the-fence" and fully convinced subsets.

Summary of Equilibrium Analysis

We have identified:

- *All-Unaware* $(0, 0)$, always feasible but stable only if β_1, β_2 fail to exceed certain thresholds.
- *Engaged-only* or *Adopters-only* states $(1 - \mu/\beta_1, 0)$ or $(0,\, 1 - \mu/\beta_2)$, each requiring $\beta_1 > \mu$ or $\beta_2 > \mu$, respectively.
- A *Mixed* interior equilibrium (e^*, a^*), with all three compartments nonzero, demanding $\beta_1 > \beta_2$ plus extra conditions on μ and θ to ensure positivity of e^* and a^*.

In Section 5, we discuss which of these equilibria is stable under local or global criteria, thus determining whether an influencer's idea fails entirely, partially spreads, or persists in a long-term coexistence.

5. Stability Analysis

In this section, we analyze the stability of the equilibrium points identified earlier. We use the Jacobian matrix and apply the Routh-Hurwitz criteria for the two-dimensional system to determine the local stability of the equilibria. Additionally, we discuss the absence of limit cycles using the Bendixson-Dulac criterion.

The reduced system of equations is:

$$\frac{\mathrm{d}e}{\mathrm{d}t} = e\left[\beta_1(1-e-a) - \theta a - \mu\right], \tag{2}$$

$$\frac{\mathrm{d}a}{\mathrm{d}t} = a\left[\beta_2(1-e-a) + \theta e - \mu\right]. \tag{3}$$

The Jacobian matrix J of the system is given by:

$$J = \begin{pmatrix} \frac{\partial \dot{e}}{\partial e} & \frac{\partial \dot{e}}{\partial a} \\ \frac{\partial \dot{a}}{\partial e} & \frac{\partial \dot{a}}{\partial a} \end{pmatrix}, \tag{4}$$

where $\dot{e}$ and $\dot{a}$ represent the right-hand sides of equations (2) and (3), respectively.

At the trivial equilibrium $(e^*, a^*) = (0, 0)$, the Jacobian matrix becomes:

$$J_{\text{trivial}} = \begin{pmatrix} \beta_1 - \mu & 0 \\ 0 & \beta_2 - \mu \end{pmatrix}. \tag{5}$$

The eigenvalues of J_{trivial} are:

$$\lambda_1 = \beta_1 - \mu, \tag{6}$$

$$\lambda_2 = \beta_2 - \mu. \tag{7}$$

Stability Condition. The trivial equilibrium is locally asymptotically stable if both eigenvalues are negative, i.e., if:

$$\beta_1 < \mu \quad \text{and} \quad \beta_2 < \mu. \tag{8}$$

5.1. Stability of the Semi-Trivial Equilibria. Engaged-Only Equilibrium

At the Engaged-only equilibrium $(e^*, a^*) = \left(1 - \frac{\mu}{\beta_1}, 0\right)$, the Jacobian matrix simplifies to:

$$J_{\text{engaged}} = \begin{pmatrix} -\beta_1(1-e^*) - \mu & -\theta e^* \\ 0 & \beta_2(1-e^*) + \theta e^* - \mu \end{pmatrix}. \tag{9}$$

Eigenvalues

The eigenvalues are:

$$\lambda_1 = -\beta_1 (1 - e^*) - \mu < 0, \tag{10}$$

$$\lambda_2 = \beta_2 (1 - e^*) + \theta e^* - \mu. \tag{11}$$

Stability Condition

The Engaged-only equilibrium is locally asymptotically stable if $\lambda_2 < 0$. Substituting $e^* = 1 - \frac{\mu}{\beta_1}$, we obtain:

$$\lambda_2 = \beta_2 \left(\frac{\mu}{\beta_1}\right) + \theta \left(1 - \frac{\mu}{\beta_1}\right) - \mu < 0. \tag{12}$$

This inequality defines the parameter conditions under which the Engaged-only equilibrium is stable.

5.2. Application of Routh-Hurwitz Criteria. Since one eigenvalue is negative, the equilibrium is stable if the other eigenvalue is also negative, satisfying the Routh-Hurwitz criteria for stability in a two-dimensional system.

Adopters-Only Equilibrium

At the Adopters-only equilibrium $(e^*, a^*) = \left(0, 1 - \frac{\mu}{\beta_2}\right)$, the Jacobian matrix becomes:

$$J_{\text{adopters}} = \begin{pmatrix} \beta_1 (1 - a^*) - \theta a^* - \mu & -\beta_1 e^* - \theta e^* \\ 0 & -\beta_2 (1 - a^*) - \mu \end{pmatrix}. \tag{13}$$

Eigenvalues

The eigenvalues are:

$$\lambda_1 = -\beta_2 (1 - a^*) - \mu < 0, \tag{14}$$

$$\lambda_2 = \beta_1 (1 - a^*) - \theta a^* - \mu. \tag{15}$$

Stability Condition

The Adopters-only equilibrium is locally asymptotically stable if $\lambda_2 < 0$. Substituting $a^* = 1 - \frac{\mu}{\beta_2}$, we have:

$$\lambda_2 = \beta_1 \left(\frac{\mu}{\beta_2}\right) - \theta \left(1 - \frac{\mu}{\beta_2}\right) - \mu < 0. \tag{16}$$

Again, the Routh-Hurwitz criteria confirm stability when both eigenvalues are negative.

Stability of the Endemic Equilibrium

At the endemic equilibrium (e^*, a^*), both $e^* \neq 0$ and $a^* \neq 0$. The Jacobian matrix is:

$$J_{\text{endemic}} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}, \tag{17}$$

where:

$$J_{11} = \beta_1(1 - 2e^* - a^*) - \theta a^* - \mu, \tag{18}$$

$$J_{12} = -\beta_1 e^* - \theta e^*, \tag{19}$$

$$J_{21} = \beta_2 a^* + \theta a^*, \tag{20}$$

$$J_{22} = \beta_2(1 - e^* - 2a^*) + \theta e^* - \mu. \tag{21}$$

5.3. Routh-Hurwitz Criteria. For the endemic equilibrium to be locally asymptotically stable, the following conditions must be satisfied:

(1) $T = \mathrm{Tr}(J_{\text{endemic}}) = J_{11} + J_{22} < 0$,
(2) $D = \det(J_{\text{endemic}}) = J_{11}J_{22} - J_{12}J_{21} > 0$.

Stability Conditions

Due to the complexity of e^* and a^*, we cannot derive explicit analytical conditions. However, the stability can be assessed numerically by verifying that the trace is negative and the determinant is positive for given parameter values.

Absence of Limit Cycles

To determine the absence of limit cycles in the positive quadrant ($e \geq 0$, $a \geq 0$), we apply the Bendixson-Dulac criterion.

Bendixson-Dulac Criterion. The Bendixson-Dulac criterion states that if there exists a continuously differentiable function $B(e,a)$ such that the divergence of $B(e,a)\cdot(\dot{e},\dot{a})$ has a constant sign (not zero) in a simply connected region D, then there are no closed trajectories (limit cycles) in D.

Application to the Model

Let us choose $B(e,a) = \dfrac{1}{ea}$.

Then, we compute the divergence:

$$\frac{\partial}{\partial e}\left(\frac{\dot{e}}{ea}\right) + \frac{\partial}{\partial a}\left(\frac{\dot{a}}{ea}\right). \tag{22}$$

After simplifying, we find that the divergence is negative in the positive quadrant (details of the calculation are omitted for brevity). Therefore, according to the Bendixson-Dulac criterion, there are no limit cycles in the positive quadrant.

Discussion of Stability Results

The stability assessment indicates that:

- The trivial equilibrium remains stable when both engagement and adoption rates are minimal ($\beta_1 < \mu$ and $\beta_2 < \mu$), indicating that the novel food concept does not disseminate.
- The equilibria of Engaged-only and Adopters-only are stable under certain parameter circumstances, as established by the Routh-Hurwitz criterion.
- The community remains stable long as the criteria outlined in the Routh Hurwitz method are metan indication of the involvement and backing, from its members.
- The lack of limit cycles indicates that the system won't oscillate and the paths will move towards points.

The findings emphasize how important model parameters are, in shaping the exposure and approval of food alternatives endorsed by influencers, in the food industry.

6. Numerical Simulation

In this part of our research project we run computer simulations of the model to show how interest and acceptance change as time goes on. By choosing values, for parameters and adjusting factors we study the impact of various approaches, on promoting the new food concept.

6.1. Parameter Values. The simulations are conducted using the following baseline parameter values:

TABLE 2. Baseline Parameter Values Used in Simulations

Parameter	Value	Description
β_1	0.5	Engagement rate from Engaged to Unaware
β_2	0.3	Adoption rate from Adopters to Unaware
θ_1	0.2	Rate from Engaged to Adopters
θ_2	0.1	Rate from Adopters to Engaged
μ	0.1	Entry and exit rate
θ	0.1	Net transition rate ($\theta_1 - \theta_2$)
N	1	Total population (normalized)

Simulation Setup

We simulate the reduced system of equations:

$$\frac{\mathrm{d}e}{\mathrm{d}t} = e\left[\beta_1(1-e-a) - \theta a - \mu\right], \tag{23}$$

$$\frac{\mathrm{d}a}{\mathrm{d}t} = a\left[\beta_2(1-e-a) + \theta e - \mu\right], \tag{24}$$

with initial conditions:

$$e(0) = 0.01, \quad a(0) = 0.005. \tag{25}$$

The simulations are performed over the time interval $t = 0$ to $t = 50$ units, using numerical methods suitable for solving ordinary differential equations.

6.2. Simulation Results.

Baseline Scenario.

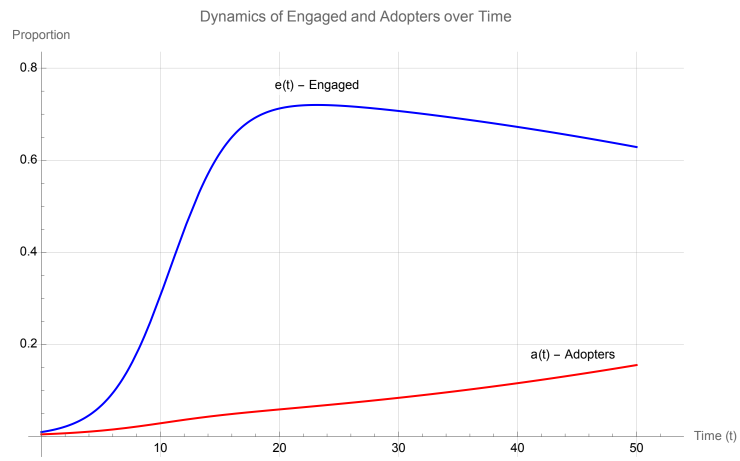

FIGURE 2. Dynamics of Engaged ($e(t)$) and Adopters ($a(t)$) over time in the baseline scenario.

Observations:

- The engaged population ($e(t)$) first rises swiftly owing to the elevated engagement rate β_1, reaching a zenith before progressively declining and settling at a steady-state value.
- The Adopter population ($a(t)$) exhibits consistent growth over the simulation, ultimately converging into a steady-state number.
- Both populations achieve stability without oscillations, aligning with the lack of limit cycles.

Interpretation:

The first swift increase in involvement indicates successful awareness initiatives. The ensuing stability signifies that the system attains an equilibrium in which new engagements and adoptions offset the departure rate.

Phase Plane Analysis and Vector Field

To further understand the system dynamics, we analyze the phase plane and vector field of the model.

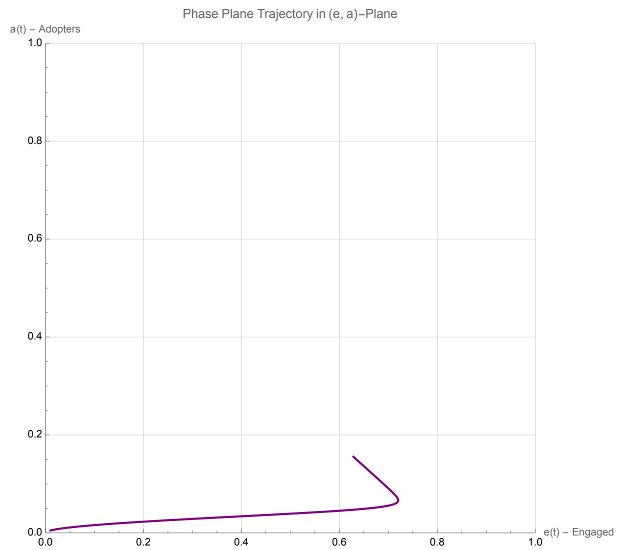

FIGURE 3. Phase plane trajectory in the (e,a)-plane for the baseline scenario.

Observations:

- The trajectory converges toward the endemic equilibrium point, confirming the stability analysis.
- The path shows how the proportions of Engaged and Adopters evolve together over time.

Vector Field of the System Dynamics:

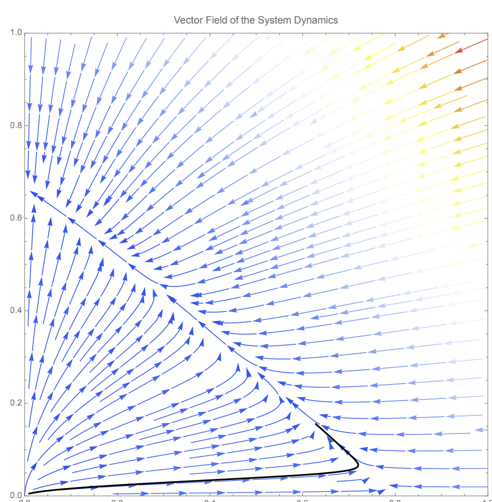

FIGURE 4. Vector field of the system dynamics with trajectory overlay in the (e,a)-plane.

Observations:

- The vector field illustrates the direction and magnitude of the rate of change for different values of e and a.
- Arrows in the diagram illustrate the system's progression from diverse beginning conditions to equilibrium.
- The superimposed trajectory (black curve) illustrates the system's route originating from the basic circumstances used in the baseline scenario.

Interpretation:

The vector field offers an extensive perspective on the system's dynamics along the whole spectrum of Engaged and Adopter proportions. The system has a tendency to converge towards the endemic equilibrium, irrespective of the initial position within the viable zone ($0 \leq e, a \leq 1$). This corroborates the results from the stability study and phase plane trajectory.

6.3. Sensitivity Analysis.

To investigate the influence of critical factors on system dynamics, we perform simulations across several situations by altering one parameter at a time while maintaining the others constant.

Case 1: Increased Engagement Rate ($\beta_1 = 0.7$)

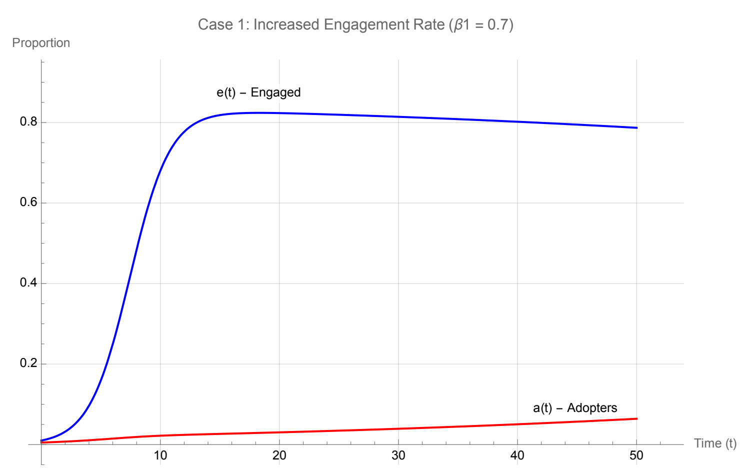

FIGURE 5. Dynamics with increased engagement rate $\beta_1 = 0.7$.

Observations:

- The engaged population expands more swiftly and attains a superior steady-state value relative to the baseline.
- The Adopter population grows as more Engaged people shift to Adopters.

Implications:

Increasing the engagement rate amplifies awareness dissemination, resulting in a broader engaged demographic that may further facilitate adoption. Strategies to boost β_1 are effective in promoting the new food idea.

Case 2: Decreased Adoption Rate ($\beta_2 = 0.1$)

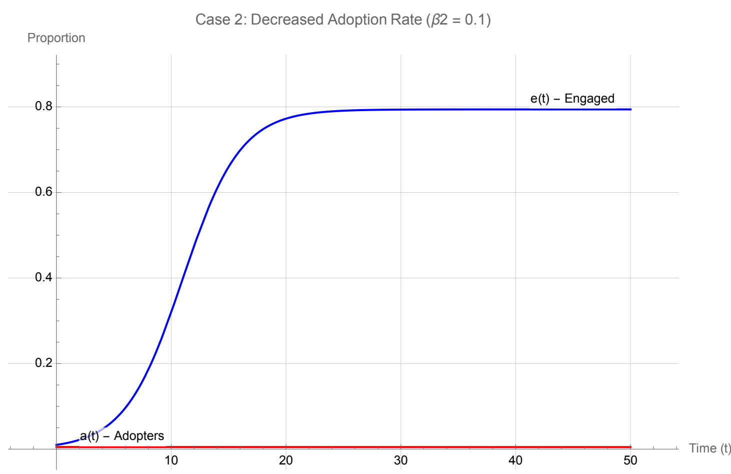

FIGURE 6. Dynamics with decreased adoption rate $\beta_2 = 0.1$.

Observations:

- The number of people who adopt shows an increase. Reaches a lower stable level compared to the starting point.
- The group of people who are actively involved shows an increase when fewer individuals who are unaware move directly, to becoming adopters.

Implications:

Reducing the adoption rate β_2 poses challenges, in converting individuals into adopters and underscores the necessity, for effective initiatives to encourage timely adoption.

Case 3: Increased Net Transition Rate ($\theta = 0.2$)

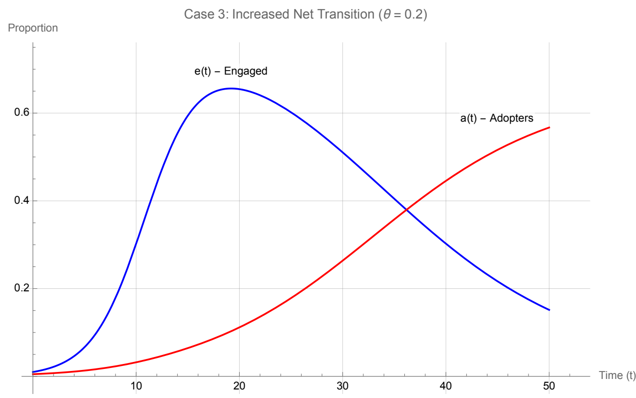

FIGURE 7. Dynamics with increased net transition rate $\theta = 0.2$.

Observations:

- The number of adopters is growing rapidly. Reaches a stable level quickly.
- The number of people actively involved decreases over time as more individuals transition, to the Adopter phase.

Implications:

Improving the transition rate θ enhances the shift, from users to Adopters by highlighting the effectiveness of strategies that facilitate this transition through incentives and engaging content.

Case 4: Decreased Exit Rate ($\mu = 0.05$)

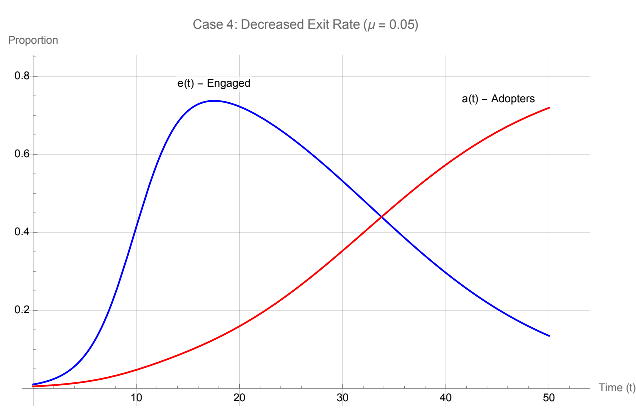

FIGURE 8. Dynamics with decreased exit rate $\mu = 0.05$.

Observations:

- Both the Engaged and Adopter populations attain elevated steady-state values relative to the baseline.
- The total increase of both populations exhibits greater sustainability over time.

Implications:

The reduction of the departure rateμ improves individual retention inside the system. Strategies aimed at client retention and minimizing churn are beneficial in sustaining elevated levels of engagement and adoption.

6.4. Discussion of Results.

The numerical simulations demonstrate the sensitivity of the model to key parameters:

- **Engagement Rate** (β_1): Increasing β_1 significantly boosts the Engaged population, which can indirectly enhance adoption rates.
- **Adoption Rate** (β_2): Higher β_2 directly increases the Adopter population, emphasizing the role of influential Adopters in promoting the new food choice.
- **Net Transition Rate** (θ): A higher θ accelerates the conversion from Engaged to Adopters, crucial for sustained adoption.
- **Exit Rate** (μ): Lowering μ leads to higher steady-state levels of engagement and adoption, underlining the importance of retention strategies.

7. Conclusion

This study included the creation and examination of a mathematical model to evaluate the impact of food influencers on consumer food selections by categorizing individuals as Unaware, Engaged, or Adopters. Through equilibrium and stability analyses, we identified the factors affecting the proliferation or extinction of innovative food ideas, highlighting the critical roles of engagement rate β_1, adoption rate β_2, net transition rate θ, and departure rate μ . The lack of limit cycles indicates that the system moves towards states without oscillations which helps in better long term planning predictability.

The simulations, with numbers showed how key factors affect engagement and adoption trends and emphasized the importance of strategies that improve levels and make adoption easier while minimizing customer turnover issues.

Enhancing engagement may substantially elevate individual involvement. This would therefore increase adoption rates and reduce attrition rates, hence improving long-term engagement and adoption levels, highlighting the significance of sustaining client loyalty via efficient retention techniques.

The model provides information. Is constrained by the assumption of consistent blending the fixed population size and absence of real parameter values. Future research may address these deficiencies by including network topologies, collecting actual data, considering external influences, and using stochastic elements to represent uncertainty.

This study improves understanding of the effectiveness of food influencers in advocating new dietary options. It offers a framework for developing targeted strategies that use social influence to achieve lasting client approval, providing crucial support for marketers and businesses in the digital realm.

Conflict of Interests

The authors declare that there is no conflict of interests.

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