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SOME NEW 4-CORDIAL GRAPHS

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Abstract. We discuss here 4-cordial labeling of some standard graph families. We prove that wheels, fans and friendship graphs are 4-cordial. We also prove that gear graph, double fan and helm admit 4-cordial labeling.

Keywords: Abelian group; 4-cordial labeling; gear graph; double fan.

2010 AMS Subject Classification: 05C78.

1. Introduction

Throughout this work, by a graph we mean finite, connected, undirected, simple graph $G = (V(G), E(G))$ of order $|V(G)|$ and size $|E(G)|$.

Definition 1.1. A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices(edges) then the labeling is called a *vertex labeling(an edge labeling).*

According to Beineke and Hegde[1], labeling of discrete structure is a frontier between graph theory and theory of numbers. A latest survey on various graph labeling problems can be found in Gallian[2].

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Definition 1.2. Let $< A, * >$ be any Abelian group. A mapping $f : V(G) \rightarrow A$ is *A-cordial labeling* of graph G (For an edge $e = uv$, $f(e) = f(u) * f(v)$) if for all $a, b \varepsilon A$ the following two conditions are satisfied

(i) $|v_f(a) - v_f(b)| \leq 1$,

(ii) $|e_f(a) - e_f(b)| \leq 1$,

where $v_f(a)$=the number of vertices with label a, $v_f(b)$=the number of vertices with label b, $e_f(a)$=the number of edges with label a and $e_f(b)$=the number of edges with label b.

The concept of A-cordial labeling was introduced by Hovey[4] and proved the following results.

- All the connected graphs are 3-cordial.
- All the trees are 3-cordial.
- All the trees are 4-cordial.
- Cycles are k-cordial for all odd k.

Youssef [5] proved the following results.

- The *complete graph* K_n is 4-cordial $\Longleftrightarrow n \leq 6$.
- The *complete bipartite graph* $K_{m,n}$ is 4-cordial $\Longleftrightarrow m \ or \ n \not\equiv 2(mod 4)$.
- The graph C_n^2 is 4-cordial $\Longleftrightarrow n \not\equiv 2(mod 4)$.

Here we consider the following definitions of standard graphs.

- The *wheel* W_n is $C_n + K_1$.
- The *fan* f_n is $P_n + K_1$.
- The *friendship graph* F_n is one point union of n copies of cycle C_3.
- The *gear graph* G_n is obtained by subdividing each rim edge of the wheel W_n.
- The *double fan* Df_n is obtained by $P_n + 2K_1$.
- The *helm* H_n is obtained by adding a pendant edge at each rim vertex of the wheel W_n.

For any undefined term in graph theory we rely upon Gross and Yellen[3].

2. Main results

Theorem 2.1. *All the wheels* W_n *are* $4-$*cordial.*

Proof. Let $G = W_n$ be the wheel. Let $v_1, v_2, ..., v_n$ be the rim vertices of W_n and v_0 be the apex vertex . We note that $|V(G)| = n+1$ and $|E(G)| = 2n$. To define $4-$ cordial labeling $f : V(G) \rightarrow < Z_4, +_4 >$ we consider the following cases.

Case 1: $n \equiv 0, 1, 3, 5, 6 (mod 8)$

$f(v_0) = 0;$

$f(v_i) = 0; \quad i \equiv 0, 4(\text{mod } 8);$

$f(v_i) = 1; \quad i \equiv 1, 6(\text{mod } 8);$

$f(v_i) = 2; \quad i \equiv 3, 7(\text{mod } 8);$

$f(v_i) = 3; \quad i \equiv 2, 5(\text{mod } 8); \quad 1 \leq i \leq n.$

Case 2: $n \equiv 2, 4 (mod 8)$

$f(v_0) = 0;$

$f(v_1) = 1;$

$f(v_2) = 3;$

$f(v_3) = 2;$

$f(v_4) = 2;$

$f(v_i) = 0; \quad i \equiv 0, 4(\text{mod } 8);$

$f(v_i) = 1; \quad i \equiv 2, 5(\text{mod } 8);$

$f(v_i) = 2; \quad i \equiv 3, 7(\text{mod } 8);$

$f(v_i) = 3; \quad i \equiv 1, 6(\text{mod } 8); \quad 5 \leq i \leq n.$

Case 3: $n \equiv 7 (mod 8)$

$f(v_0) = 0;$

$f(v_1) = 1;$

$f(v_2) = 3;$

$f(v_3) = 2;$

$f(v_4) = 0;$

$f(v_5) = 2;$

$f(v_i) = 0; \quad i \equiv 1, 5(\text{mod } 8);$

$f(v_i) = 1; \quad i \equiv 3, 6(\text{mod } 8);$

$f(v_i) = 2; \quad i \equiv 0, 4(\text{mod } 8);$

$f(v_i) = 3; \quad i \equiv 2,7 (\text{mod } 8); \quad 6 \leq i \leq n.$

The labeling pattern defined above covers all possible arrangement of vertices. In each possibilities the graph under consideration satisfies the vertex conditions and edge conditions for 4−cordial labeling as shown in *Table 2.1*. That is G admits 4−cordial labeling.

Let $n = 8a + b$, $a, b \in N \cup \{0\}$.

b	Vertex conditions	Edge conditions
0	$v_f(0) = v_f(1)+1 = v_f(2)+1 = v_f(3)+1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$
1	$v_f(0) = v_f(1) = v_f(2)+1 = v_f(3)+1$	$e_f(0)+1 = e_f(1) = e_f(2) = e_f(3)+1$
2	$v_f(0)+1 = v_f(1) = v_f(2) = v_f(3)$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$
3	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$	$e_f(0)+1 = e_f(1) = e_f(2)+1 = e_f(3)$
4	$v_f(0)+1 = v_f(1)+1 = v_f(2) = v_f(3)+1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$
5	$v_f(0) = v_f(1)+1 = v_f(2)+1 = v_f(3)$	$e_f(0) = e_f(1)+1 = e_f(2)+1 = e_f(3)$
6	$v_f(0) = v_f(1) = v_f(2)+1 = v_f(3)$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$
7	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$	$e_f(0) = e_f(1)+1 = e_f(2) = e_f(3)+1$

Table 2.1

Illustration 2.2. The wheel W_{10} and its 4−cordial labeling is shown in *Figure 2.1*.

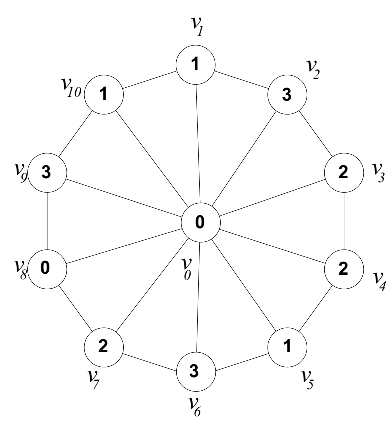

Figure 2.1 4-cordial labeling of wheel W_{10}.

Theorem 2.3. *All the fans f_n are* 4−*cordial.*

Proof. Let $G = f_n$ be the fan. Let $v_1, v_2, ..., v_n$ be the path vertices of f_n and v_0 be the apex vertex. We note that $|V(G)| = n+1$ and $|E(G)| = 2n-1$. We define 4− cordial labeling

$f : V(G) \to < Z_4, +_4 >$ as follows.

$f(v_0) = 0;$

$f(v_i) = 0; \quad i \equiv 0, 4(\text{mod } 8);$

$f(v_i) = 1; \quad i \equiv 2, 5(\text{mod } 8);$

$f(v_i) = 2; \quad i \equiv 3, 7(\text{mod } 8);$

$f(v_i) = 3; \quad i \equiv 1, 6(\text{mod } 8); \quad 1 \leq i \leq n.$

The labeling pattern defined above covers all possible arrangement of vertices. In each possibilities the graph under consideration satisfies the vertex conditions and edge conditions for $4-$cordial labeling as shown in *Table 2.2*. That is G admits $4-$cordial labeling.

Let $n = 8a + b$, $a, b \in N \cup \{0\}$.

b	Vertex conditions	Edge conditions
0	$v_f(0) = v_f(1) + 1 = v_f(2) + 1 = v_f(3) + 1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3) + 1$
1	$v_f(0) = v_f(1) + 1 = v_f(2) + 1 = v_f(3)$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2) + 1 = e_f(3)$
2	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3)$	$e_f(0) = e_f(1) = e_f(2) + 1 = e_f(3)$
3	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2) + 1 = e_f(3)$
4	$v_f(0) = v_f(1) + 1 = v_f(2) + 1 = v_f(3) + 1$	$e_f(0) = e_f(1) + 1 = e_f(2) = e_f(3)$
5	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1 = e_f(3) + 1$
6	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3)$	$e_f(0) = e_f(1) = e_f(2) + 1 = e_f(3)$
7	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1 = e_f(3) + 1$

Table 2.2

Illustration 2.4. The fan f_8 and its $5-$cordial labeling is shown in *Figure 2.2*.

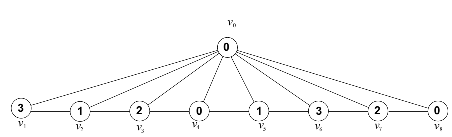

Figure 2.2 4-cordial labeling of fan f_8.

Theorem 2.5. *All the friendship graphs F_n are $4-$cordial.*

Proof. Let $G = F_n$ be the friendship graph. Let $v_1, v_2, ..., v_{2n}$ be partition vertices of n triangles consecutively of f_n and v_0 be the central vertex. We note that $|V(G)| = 2n+1$ and $|E(G)| = 3n$.

To define $4-$ cordial labeling $f : V(G) \to < Z_4, +_4 >$ we consider the following cases.

Case 1: $n \equiv 0,1,2,3,4(mod 8)$

$f(v_0) = 0;$

$f(v_i) = 0; \quad i \equiv 5,9,13,15(\text{mod } 16);$

$f(v_i) = 1; \quad i \equiv 1,4,8,14(\text{mod } 16);$

$f(v_i) = 2; \quad i \equiv 2,6,10,11(\text{mod } 16);$

$f(v_i) = 3; \quad i \equiv 0,3,7,12(\text{mod } 16); \quad 1 \leq i \leq 2n.$

Case 2: $n \equiv 5(mod 8)$

$f(v_0) = 0;$

$f(v_i) = 0; \quad i \equiv 5,9,13,15(\text{mod } 16);$

$f(v_i) = 1; \quad i \equiv 1,4,8,14(\text{mod } 16);$

$f(v_i) = 2; \quad i \equiv 2,6,10,11(\text{mod } 16);$

$f(v_i) = 3; \quad i \equiv 0,3,7,12(\text{mod } 16); \quad 1 \leq i \leq 2n-2$

$f(v_{2n-1}) = 2;$

$f(v_{2n}) = 3.$

Case 3: $n \equiv 6(mod 8)$

$f(v_0) = 0;$

$f(v_i) = 0; \quad i \equiv 5,9,13,15(\text{mod } 16);$

$f(v_i) = 1; \quad i \equiv 1,4,8,14(\text{mod } 16);$

$f(v_i) = 2; \quad i \equiv 2,6,10,11(\text{mod } 16);$

$f(v_i) = 3; \quad i \equiv 0,3,7,12(\text{mod } 16); \quad 1 \leq i \leq 2n-2$

$f(v_{2n-1}) = 3;$

$f(v_{2n}) = 1.$

Case 4: $n \equiv 7(mod 8)$

$f(v_0) = 0;$

$f(v_i) = 0; \quad i \equiv 5,9,13,15(\text{mod } 16);$

$f(v_i) = 1; \quad i \equiv 1,4,8,14(\text{mod } 16);$

$f(v_i) = 2; \quad i \equiv 2, 6, 10, 11 (\text{mod } 16);$

$f(v_i) = 3; \quad i \equiv 0, 3, 7, 12 (\text{mod } 16); \quad 1 \le i \le 2n-4$

$f(v_{2n-3}) = 3;$

$f(v_{2n-2}) = 1;$

$f(v_{2n-1}) = 2;$

$f(v_{2n}) = 3.$

The labeling pattern defined above covers all possible arrangement of vertices. In each possibilities the graph under consideration satisfies the vertex conditions and edge conditions for $4-$cordial labeling as shown in *Table 2.3*. That is G admits $4-$cordial labeling.

Let $n = 8a + b$, $a, b \in N \cup \{0\}$.

b	Vertex conditions	Edge conditions
0	$v_f(0) = v_f(1)+1 = v_f(2)+1 = v_f(3)+1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$
1	$v_f(0) = v_f(1) = v_f(2) = v_f(3)+1$	$e_f(0)+1 = e_f(1) = e_f(2) = e_f(3)$
2	$v_f(0)+1 = v_f(1) = v_f(2)+1 = v_f(3)+1$	$e_f(0)+1 = e_f(1) = e_f(2)+1 = e_f(3)$
3	$v_f(0) = v_f(1) = v_f(2) = v_f(3)+1$	$e_f(0)+1 = e_f(1)+1 = e_f(2) = e_f(3)+1$
4	$v_f(0)+1 = v_f(1) = v_f(2)+1 = v_f(3)+1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$
5	$v_f(0)+1 = v_f(1) = v_f(2) = v_f(3)$	$e_f(0)+1 = e_f(1) = e_f(2) = e_f(3)$
6	$v_f(0)+1 = v_f(1) = v_f(2)+1 = v_f(3)+1$	$e_f(0) = e_f(1)+1 = e_f(2) = e_f(3)+1$
7	$v_f(0)+1 = v_f(1) = v_f(2) = v_f(3)$	$e_f(0)+1 = e_f(1)+1 = e_f(2) = e_f(3)+1$

Table 2.3

Illustration 2.6. The friendship graph F_5 and its $4-$cordial labeling is shown in *Figure 2.3*.

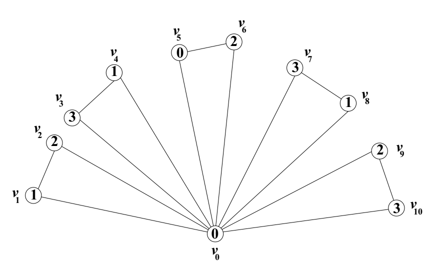

Figure 2.3 4-cordial labeling of friendship graph F_5.

Theorem 2.7. *All gear graphs G_n are $4-$cordial.*

Proof. Let $G = G_n$ be the gear graph. Let $v_1, v_2, ..., v_{2n}$ be the rim vertices of G_n and v_0 be the apex vertex . We note that $|V(G)| = 2n+1$ and $|E(G)| = 3n$. To define $4-$ cordial labeling $f : V(G) \rightarrow < Z_4, +_4 >$ we consider the following cases.

Case 1: $n \equiv 0,3,4,7(mod 8)$

$f(v_0) = 0;$

$f(v_i) = 0; \quad i \equiv 4,7(\text{mod } 8);$

$f(v_i) = 1; \quad i \equiv 2,5(\text{mod } 8);$

$f(v_i) = 2; \quad i \equiv 0,3(\text{mod } 8);$

$f(v_i) = 3; \quad i \equiv 1,6(\text{mod } 8). \quad 1 \leq i \leq 2n.$

Case 2: $n \equiv 1,5(mod 8)$

$f(v_0) = 0;$

$f(v_i) = 0; \quad i \equiv 4,7(\text{mod } 8);$

$f(v_i) = 1; \quad i \equiv 2,5(\text{mod } 8);$

$f(v_i) = 2; \quad i \equiv 0,3(\text{mod } 8);$

$f(v_i) = 3; \quad i \equiv 1,6(\text{mod } 8); \quad 1 \leq i \leq 2n-2$

$f(v_{2n-1}) = 1;$

$f(v_{2n}) = 3.$

Case 3: $n \equiv 2,6(mod 8)$

$f(v_0) = 0;$

$f(v_i) = 0; \quad i \equiv 4,7(\text{mod } 8);$

$f(v_i) = 1; \quad i \equiv 2,5(\text{mod } 8);$

$f(v_i) = 2; \quad i \equiv 0,3(\text{mod } 8);$

$f(v_i) = 3; \quad i \equiv 1,6(\text{mod } 8); \quad 1 \leq i \leq 2n-4$

$f(v_{2n-3}) = 1;$

$f(v_{2n-2}) = 3;$

$f(v_{2n-1}) = 2;$

$f(v_{2n}) = 0.$

The labeling pattern defined above covers all possible arrangement of vertices. In each possibilities the graph under consideration satisfies the vertex conditions and edge conditions for 4−cordial labeling as shown in *Table 2.4*. That is G admits 4−cordial labeling.

Let $n = 8a + b$, $a, b \in N \cup \{0\}$.

b	**Vertex conditions**	**Edge conditions**
0,4	$v_f(0) = v_f(1)+1 = v_f(2)+1 = v_f(3)+1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$
1,5	$v_f(0) = v_f(1) = v_f(2)+1 = v_f(3)$	$e_f(0) = e_f(1)+1 = e_f(2) = e_f(3)$
2,6	$v_f(0) = v_f(1)+1 = v_f(2)+1 = v_f(3)+1$	$e_f(0)+1 = e_f(1)+1 = e_f(2) = e_f(3)$
3,7	$v_f(0) = v_f(1) = v_f(2)+1 = v_f(3)$	$e_f(0)+1 = e_f(1)+1 = e_f(2) = e_f(3)+1$

Table 2.4

Illustration 2.8. The gear graph G_7 and its 4− cordial labeling is shown in *Figure 2.4*.

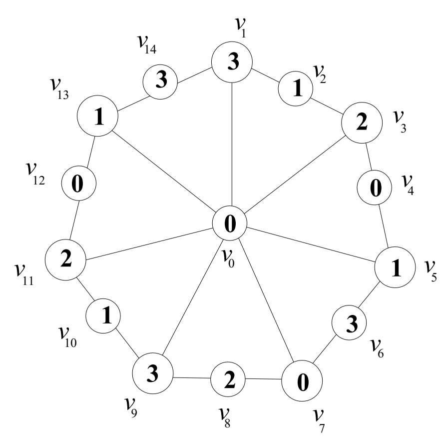

Figure 2.4 4-cordial labeling of gear graph G_7.

Theorem 2.9. *All the Double fans DF_n are* 4−*cordial.*

Proof. Let $G = DF_n$ be the doublefan graph. Let $v_1, v_2, ..., v_n$ are vertices of common path and u_1, u_2 be the apex vertex . We note that $|V(G)| = n+2$ and $|E(G)| = 3n-1$. To define 4−cordial labeling $f : V(G) \rightarrow < Z_4, +_4 >$ we consider the following cases.

<u>Case 1</u>: $n \equiv 0,3,4,5(mod8)$

$f(u_1) = 0;$

$f(u_2) = 1;$

$f(v_i) = 0; \quad i \equiv 0,4(\text{mod } 8);$

$f(v_i) = 1; \quad i \equiv 1,6(\text{mod } 8);$

$f(v_i) = 2; \quad i \equiv 3,7(\text{mod } 8);$

$f(v_i) = 3; \quad i \equiv 2,5(\text{mod } 8); \quad 1 \leq i \leq n.$

<u>Case 2</u>: $n \equiv 1,7(mod 8)$

$f(u_1) = 0;$

$f(u_2) = 3;$

$f(v_i) = 0; \quad i \equiv 0,4(\text{mod } 8);$

$f(v_i) = 1; \quad i \equiv 1,6(\text{mod } 8);$

$f(v_i) = 2; \quad i \equiv 3,7(\text{mod } 8);$

$f(v_i) = 3; \quad i \equiv 2,5(\text{mod } 8); \quad 1 \leq i \leq n.$

<u>Case 3</u>: $n \equiv 2(mod 8)$

$f(u_1) = 0;$

$f(u_2) = 3;$

$f(v_i) = 0; \quad i \equiv 0,4(\text{mod } 8);$

$f(v_i) = 1; \quad i \equiv 1,6(\text{mod } 8);$

$f(v_i) = 2; \quad i \equiv 3,7(\text{mod } 8);$

$f(v_i) = 3; \quad i \equiv 2,5(\text{mod } 8); \quad 1 \leq i \leq n-1$

$f(v_n) = 2.$

<u>Case 4</u>: $n \equiv 6(mod 8)$

$f(u_1) = 0;$

$f(u_2) = 1;$

$f(v_i) = 0; \quad i \equiv 0,4(\text{mod } 8);$

$f(v_i) = 1; \quad i \equiv 1,6(\text{mod } 8);$

$f(v_i) = 2; \quad i \equiv 3,7(\text{mod } 8);$

$f(v_i) = 3; \quad i \equiv 2,5(\text{mod } 8); \quad 1 \leq i \leq n-1$

$f(v_n) = 2.$

The labeling pattern defined above covers all possible arrangement of vertices. In each possibilities the graph under consideration satisfies the vertex conditions and edge conditions for $4-$cordial labeling as shown in *Table 2.5*. That is G admits $4-$cordial labeling.

Let $n = 8a + b$, $a, b \in N \cup \{0\}$.

b	**Vertex conditions**	**Edge conditions**
0	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3) + 1$	$e_f(0) = e_f(1) + 1 = e_f(2) = e_f(3)$
1	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3)$	$e_f(0) = e_f(1) = e_f(2) + 1 = e_f(3) + 1$
2	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1 = e_f(3) + 1$
3	$v_f(0) + 1 = v_f(1) = v_f(2) + 1 = v_f(3) + 1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$
4	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3) + 1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3) + 1$
5	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3)$	$e_f(0) = e_f(1) + 1 = e_f(2) + 1 = e_f(3)$
6	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2) + 1 = e_f(3)$
7	$v_f(0) + 1 = v_f(1) + 1 = v_f(2) + 1 = v_f(3)$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$

Table 2.5

Illustration 2.10. The double fan DF_9 and its 4-cordial labeling is shown in *Figure 2.5*.

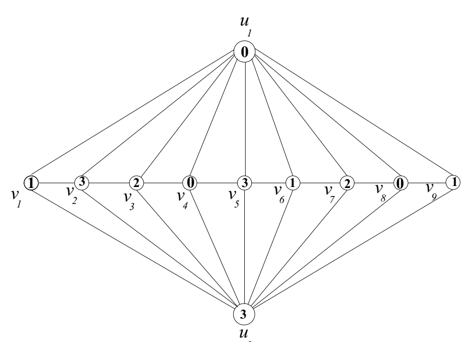

Figure 2.5 4-cordial labeling of gear graph DF_9.

Theorem 2.11. *All Helms the H_n are $4-$cordial.*

Proof. Let $G = H_n$ be the helm. Let $v_1, v_2, ..., v_n$ be the vertices of degree 3 of H_n. Let $v_1^{'}, v_2^{'}, ..., v_n^{'}$ be the pendant vertices and v_0 be the apex vertex of H_n. We note that $|V(G)| = 2n + 1$ and $|E(G)| = 3n$. To define $4-$ cordial labeling $f : V(G) \rightarrow < Z_4, +_4 >$ we consider the following cases.

Case 1: $n \equiv 0,5(mod 8)$

$f(v_0) = 0;$

$f(v_i) = 0; \quad i \equiv 0,4(\text{mod } 8);$

$f(v_i) = 1; \quad i \equiv 1,6(\text{mod } 8);$

$f(v_i) = 2; \quad i \equiv 3,7(\text{mod } 8);$

$f(v_i) = 3; \quad i \equiv 2,5(\text{mod } 8). \quad 1 \leq i \leq n$

$f(v_i^{'}) = 0; \quad i \equiv 2,6(\text{mod } 8);$

$f(v_i^{'}) = 1; \quad i \equiv 1,5(\text{mod } 8);$

$f(v_i^{'}) = 2; \quad i \equiv 4,7(\text{mod } 8);$

$f(v_i^{'}) = 3; \quad i \equiv 0,3(\text{mod } 8); \quad 1 \leq i \leq n.$

Case 2: $n \equiv 1,3,6(mod 8)$

$f(v_0) = 0;$

$f(v_i) = 0; \quad i \equiv 0,4(\text{mod } 8);$

$f(v_i) = 1; \quad i \equiv 1,6(\text{mod } 8);$

$f(v_i) = 2; \quad i \equiv 3,7(\text{mod } 8);$

$f(v_i) = 3; \quad i \equiv 2,5(\text{mod } 8); \quad 1 \leq i \leq n.$

$f(v_i^{'}) = 0; \quad i \equiv 2,6(\text{mod } 8);$

$f(v_i^{'}) = 1; \quad i \equiv 1,5(\text{mod } 8);$

$f(v_i^{'}) = 2; \quad i \equiv 4,7(\text{mod } 8);$

$f(v_i^{'}) = 3; \quad i \equiv 0,3(\text{mod } 8); \quad 1 \leq i \leq n-1$

$f(v_{n'}) = 2.$

Case 3: $n \equiv 2(mod 8)$

$f(v_0) = 0;$

$f(v_1) = 1;$

$f(v_2) = 3;$

$f(v_3) = 2;$

$f(v_4) = 2;$

$f(v_i) = 0; \quad i \equiv 0,4(\text{mod } 8);$

$f(v_i) = 1; \quad i \equiv 1,6(\text{mod } 8);$

$f(v_i) = 2; \quad i \equiv 3,7(\text{mod } 8);$

$f(v_i) = 3; \quad i \equiv 2,5(\text{mod } 8). \quad 5 \leq i \leq n$

$f(v_i^{'}) = 0; \quad i \equiv 2,6(\text{mod } 8);$

$f(v_i^{'}) = 1; \quad i \equiv 1,5(\text{mod } 8);$

$f(v_i^{'}) = 2; \quad i \equiv 4,7(\text{mod } 8);$

$f(v_i^{'}) = 3; \quad i \equiv 0,3(\text{mod } 8); \quad 1 \leq i \leq n-2$

$f(v_{n-1'}) = 2;$

$f(v_{n'}) = 0.$

Case 4: $n \equiv 4(mod 8)$

$f(v_0) = 0;$

$f(v_1) = 1;$

$f(v_2) = 3;$

$f(v_3) = 2;$

$f(v_4) = 2;$

$f(v_i) = 0; \quad i \equiv 0,4(\text{mod } 8);$

$f(v_i) = 1; \quad i \equiv 1,6(\text{mod } 8);$

$f(v_i) = 2; \quad i \equiv 3,7(\text{mod } 8);$

$f(v_i) = 3; \quad i \equiv 2,5(\text{mod } 8); \quad 5 \leq i \leq n.$

$f(v_i^{'}) = 0; \quad i \equiv 2,6(\text{mod } 8);$

$f(v_i^{'}) = 1; \quad i \equiv 1,5(\text{mod } 8);$

$f(v_i^{'}) = 2; \quad i \equiv 4,7(\text{mod } 8);$

$f(v_i^{'}) = 3; \quad i \equiv 0,3(\text{mod } 8). \quad 1 \leq i \leq n.$

Case 5: $n \equiv 7(mod 8)$

$f(v_0) = 0;$

$f(v_1) = 1;$

$f(v_2) = 3;$

$f(v_3) = 2;$

$f(v_4) = 0;$

$f(v_5) = 2;$

$f(v_i) = 0; \quad i \equiv 1,5 (\text{mod } 8);$

$f(v_i) = 1; \quad i \equiv 3,6 (\text{mod } 8);$

$f(v_i) = 2; \quad i \equiv 0,4 (\text{mod } 8);$

$f(v_i) = 3; \quad i \equiv 2,7 (\text{mod } 8); \quad 6 \leq i \leq n.$

$f(v_1^{'}) = 1;$

$f(v_2^{'}) = 2;$

$f(v_3^{'}) = 3;$

$f(v_4^{'}) = 3;$

$f(v_i^{'}) = 0; \quad i \equiv 3,7 (\text{mod } 8);$

$f(v_i^{'}) = 1; \quad i \equiv 2,6 (\text{mod } 8);$

$f(v_i^{'}) = 2; \quad i \equiv 1,4 (\text{mod } 8);$

$f(v_i^{'}) = 3; \quad i \equiv 0,5 (\text{mod } 8); \quad 6 \leq i \leq n.$

The labeling pattern defined above covers all possible arrangement of vertices. In each possibilities the graph under consideration satisfies the vertex conditions and edge conditions for $4-$cordial labeling as shown in *Table 2.6*. That is G admits $4-$cordial labeling.
Let $n = 8a + b$, $a, b \in N \cup \{0\}$.

b	Vertex conditions	Edge conditions
0	$v_f(0) = v_f(1) + 1 = v_f(2) + 1 = v_f(3) + 1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$
1	$v_f(0) = v_f(1) = v_f(2) = v_f(3) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3)$
2	$v_f(0) + 1 = v_f(1) + 1 = v_f(2) = v_f(3) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1 = e_f(3)$
3	$v_f(0) = v_f(1) = v_f(2) = v_f(3) + 1$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2) + 1 = e_f(3)$
4	$v_f(0) + 1 = v_f(1) + 1 = v_f(2) = v_f(3) + 1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$
5	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3)$	$e_f(0) = e_f(1) + 1 = e_f(2) = e_f(3)$
6	$v_f(0) + 1 = v_f(1) = v_f(2) + 1 = v_f(3) + 1$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2) = e_f(3)$
7	$v_f(0) + 1 = v_f(1) = v_f(2) = v_f(3)$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2) = e_f(3) + 1$

Table 2.6

Illustration 2.12. The Helm H_{13} and its 4-cordial labeling is shown in *Figure 2.6*.

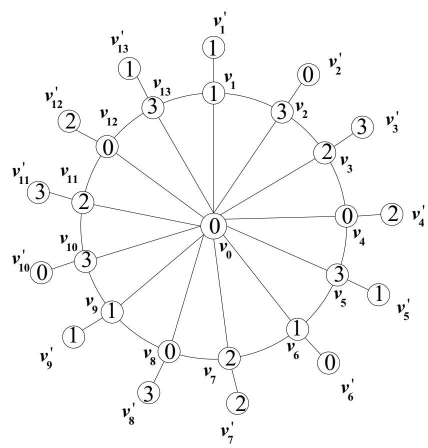

Figure 2.6 4-cordial labeling of helm H_{13}.

Concluding Remarks

It is always interesting to find out graph or graph families which admits a particular labeling. Here we investigate some standard graph families which admit 4-cordial labeling. To investigate similar results for other graph families is an open area of research.

Conflict of Interests

The authors declare that there is no conflict of interests.

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