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A DISCUSSION ON RANDOM MEIR-KEELER CONTRACTIONS IN FUZZY-METRIC SPACE

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Abstract. In this paper, we establish the concept of random, comparable MT_γ contraction and random, comparable Meir-Keeler contraction in the frame of complete random fuzzy metric space. We also illustrate examples to support our presented results.

Keywords: random fixed points; random comparable MT_γ contraction; probabilistic functional analysis; random fuzzy metric space; Meir-Keeler contraction.

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1. Introduction

Fuzzy sets were introduced and described using the membership functions by L.A.Zadeh 1965 [1] and has many practical applications. After the pioneering work of Zadeh, there has been a great effort to obtain fuzzy analogues of classical theories. George and Veeramani[2] updated Kramsoil and Michalek's[3] definition of the fuzzy metric space.Several researchers have defined fuzzy metric space in various ways to utilize this concept. Probabilistic functional analysis is one of the most important field as it has broad application for the probabilistic model in applied sciences. Random fixed point theory is the key of probabilistic functional analysis.

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Random fixed point is the extension of the basic fixed point theory endowed with the random analysis.

The initial results in random fixed point theory was given by Spacek[4] and Hans[5,6] It was subsequently developed by several authors[$7-17$] that also include fascinating applications of this theory in different fields. Meir-Keeler approaches have been investigated steadily by many authors [$18-20$].

In this paper, we focus on one of the great generalizations of Banach contraction principle [21]: the Meir-Keeler contraction endowed with fuzzy metric space. Meir-Keeler[22] contraction is investigated and observed by many authors; [$23-27$].

Definition 1.1. [2] The 3-tuple is called a fuzzy metric space if P is an arbitrary nonempty set, $*$ a continuous t-norm and M a fuzzy set on $P^2 \times [0,\infty)$ satisfying the following conditions, for all $p,q,r \in P$ and $t,s > 0$:

$(FMS1)$ $M(p,q,t) > 0$,

$(FMS2)$ $M(p,q,t) = 1$ if and only if$p = q$;

$(FMS3)$ $M(p,q,t) = M(q,p,t)$;

$(FMS4)$ $M(p,q,t) * M(q,r,s) \leq M(p,r,t+s)$;

$(FMS5)$ $M(p,q,t) : (0,\infty) \to [0,1]$ is continuous.

Example 1.2. Let $P = R^+$. Define for $p,q \in P, t > 0$

$$M(p,q,t) = \frac{\min\{p,q\}+t}{\max\{p,q\}+t}$$

Then $(P,.)$ is a fuzzy metric on P.

Definition 1.3. [2] Let $(P,M,*)$ is a fuzzy metric space:

(1) A sequence $\{p_n\}$ in P is said to convergent to a point $p \in P$, if

$$\lim_{n\to\infty} P(p_n,p,t) = 1$$

for all $t > 0$.

(2) A sequence $\{p_n\}$ in P is called a Cauchy sequence if

$$\lim_{n\to\infty} P(p_{n+k},p_n,t) = 1$$

for all $t > 0$ and $k > 0$.

(3) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 1.4. [28] Let ψ be a function that is defined from non-negative reals into the interval $[0,1)$ then ψ is called the MT function if the following are satisfied:

$$\lim_{s\to t^+}\sup\psi(s)=\inf_{a>0}\sup_{0<s-t<a}\psi(s)<1 \text{ for all } t\in R^+.$$

Theorem 1.5. [28] *For a mapping $\psi:R^+\to[0,1)$, the following are equivalent.*

(1) ψ is an MT function.

(2) For any non-increasing sequence $\{\delta_n\}_{n\in N}$ in R^+, we have

$$0\le\sup_{n\in N}\psi(\delta_n)<1.$$

Remark 1.6. [28] Notice that in the case that $\psi:R^+\to[0,1)$ is non-increasing or non-decreasing, then ψ is a MT function

2. Main Results

The mappings $\gamma:R_0^+\times R_0^+\times R_0^+\times R_0^+\to R_0^+$ is called a comparable function, if the following three axioms are fulfilled:

(1) γ is a non-decreasing, continuous function in each coordinate;

(2) $\gamma(r,r,r,r)\le r, \gamma(0,r,0,r)\le r$ and $\gamma(0,0,r,r)\le r$, for all $r>0$;

(3) $\gamma(r_1,r_2,r_3,r_4)=0$ if and only if $r_1=r_2=r_3=r_4=0$.

Definition 2.1. Let Υ be a nonempty subset of a random fuzzy-metric space $(P,M,*)$, ψ be a MT function and $T:\Omega\times\Upsilon\to\Upsilon$ be a random operator. Then, for $p\in\Omega, T(p,.)$ is called a random, comparable $MT_{-\gamma}$ contraction if the following condition holds:

$$M(T(\zeta(p)),Y(\xi(p)),t)\le\psi(M(\zeta(p),\xi(p),t)).\Gamma(\zeta(p),\xi(p),t),$$

where

$$\Gamma(\zeta(p),\xi(p),t)=\gamma(M(\zeta(p),\xi(p),t),M(\zeta(p),T(\zeta(p)),t),\Gamma(\xi(p),T(\xi(p)),t),$$
$$\frac{M(\zeta(p),T(\zeta(p)),t)+\Gamma(\xi(p),T(\xi(p)),t)}{2}),$$

for all $\zeta,\xi\in X$.

Theorem 2.2. *Suppose* $(P,M,*)$ *is a complete random fuzzy-metric space and* $\Upsilon \subset P$. *If* $T(p,.) : \Omega \times \Upsilon \to \Upsilon$ *is a continuous, random, comparable* $\mathsf{MT}-_{\gamma}$ *contraction, then* T *possesses a random fixed point in* P.

Proof. Given $\varphi_0(p) \in \Omega \times P$ and defining $\varphi_1(p) = T(\varphi_0(p))$ and $\varphi_{n+1}(p) = T(\varphi_n(p))$ $= T^{n+1}(\varphi_0(p))$ for each $n \in N$, since $T(p,.) : \Omega \times \Upsilon \to \Upsilon$ is a random, comparable $\mathsf{MT}-_{\gamma}$ contraction, we have

$$M(\varphi_n(p), \varphi_{n+1}(p), t) = M(T(\varphi_{n-1}(p)), T(\varphi_n(p)), t)$$

$$\leq \psi(M(\varphi_{n-1}(p), \varphi_n(p), t).\Gamma((\varphi_{n-1}(p), \varphi_n(p), t))$$

and

$$\Gamma((\varphi_{n-1}(p), \varphi_n(p), t) =$$

$$\gamma(M((\varphi_{n-1}(p), \varphi_n(p), t), M(\varphi_{n-1}(p), T(\varphi_n(p)), t), M(\varphi_n(p), T(\varphi_n(p)), t), \frac{M(\varphi_{n-1}(p), T(\varphi_n(p)), t) + M(\varphi_n(p), T(\varphi_{n-1}(p)), t)}{2})$$

$$= \gamma((\varphi_{n-1}(p), \varphi_n(p), t), M((\varphi_{n-1}(p), \varphi_n(p), t), M((\varphi_n(p), \varphi_{n+1}(p), t), \frac{M(\varphi_{n-1}(p), \varphi_{n+1}(p), t) + M(\varphi_n(p), \varphi_n(p), t)}{2})$$

If $M(\varphi_n(p), \varphi_{n+1}(p), t) > M(\varphi_{n-1}(p), \varphi_n(p), t)$ for some n, then by the conditions of the function γ we have that

$$\Gamma(\varphi_{n-1}(p), \varphi_n(p), t)\gamma((\varphi_{n-1}(p), \varphi_n(p), t), M((\varphi_{n-1}(p), \varphi_n(p), t), M((\varphi_n(p), \varphi_{n+1}(p), t), \frac{M(\varphi_{n-1}(p), \varphi_{n+1}(p), t) + M(\varphi_n(p), \varphi_n(p), t)}{2})$$

$$\leq (M(\varphi_n(p), \varphi_{n+1}(p), t), M(\varphi_n(p), \varphi_{n+1}(p), t), M(\varphi_n(p), \varphi_{n+1}(p), t), M(\varphi_n(p), \varphi_{n+1}(p), t))$$

$$\leq M(\varphi_n(p), \varphi_{n+1}(p), t).$$

In a different order pair of γ

$$M(\varphi_n(p), \varphi_{n+1}(p), t) = M(T(\varphi_n(p)), T(\varphi_{n-1}(p)), t)$$

$$\leq \psi(M(\varphi_n(p), \varphi_{n-1}(p), t)).\Gamma(\varphi_n(p), \varphi_{n-1}(p), t),$$

and

$\Gamma(\varphi_n(p),\varphi_{n-1}(p),t) = \gamma(M(\varphi_n(p),\varphi_{n-1}(p),t), M(\varphi_n(p)),T(\varphi_n(p)),t), M(\varphi_{n-1}(p)),$
$T(\varphi_{n-1}(p)),t),$

$$\frac{M(\varphi_n(p),T(\varphi_{n-1}(p)),t)+M(\varphi_{n-1}(p)),T(\varphi_n(p)),t}{2})$$

$$\leq \gamma(M(\varphi_n(p),\varphi_{n-1}(p),t), M(\varphi_n(p),\varphi_{n+1}(p),t), M(\varphi_{n-1}(p)),\varphi_n(p)),t)),$$

$$\frac{M(\varphi_n(p)),\varphi_n(p)),t)+M(\varphi_{n-1}(p),\varphi_{n+1}(p),t)}{2}).$$

If $M(\varphi_n(p),\varphi_{n+1}(p),t) > M(\varphi_{n-1}(p),\varphi_n(p),t)$ for some n, then by the conditions of the comparable function γ we have that

$$\Gamma(\varphi_n(p),\varphi_{n-1}(p),t) = \gamma(M(\varphi_n(p),\varphi_{n-1}(p),t), M(\varphi_n(p)),\varphi_{n+1}(p),t), M(\varphi_{n-1}(p),\varphi_n(p),t),$$

$$\frac{M(\varphi_n(p)),\varphi_n(p),t)+M(\varphi_{n-1}(p),\varphi_{n+1}(p),t)}{2})$$

$$\leq \gamma(M(\varphi_n(p),\varphi_{n+1}(p),t), M(\varphi_n(p),\varphi_{n+1}(p),t), M(\varphi_n(p),\varphi_{n+1}(p),t), M(\varphi_n(p),\varphi_{n+1}(p),t)$$

$$\leq M(\varphi_n(p),\varphi_{n+1}(p),t).$$

Since ψ is a MT function, we conclude that

$$M(\varphi_n(p),\varphi_{n+1}(p),t) \leq \psi(M(\varphi_{n-1}(p),\varphi_n(p),t)).M(\varphi_n(p),\varphi_{n+1}(p),t))$$

$$< M(\varphi_n(p),\varphi_{n+1}(p),t),$$

which implies a contradiction. So, we conclude that

$$M(\varphi_n(p),\varphi_{n+1}(p),t) \leq M(\varphi_{n-1}(p),\varphi_n(p),t), \text{ for each } n \in N.$$

From above argument, then sequence $\{M(\varphi_n(p),\varphi_{n+1}(p),t)\}_{n\in N\cup\{0\}}$ is non-increasing in R_0^+. Since ψ is an MT function, by Theorem 1 we conclude that

$$0 \leq \sup_{n\in N} \psi(M(\varphi_n(p),\varphi_{n+1}(p),t)) < 1.$$

Let $\lambda = sup_{n\in N}\psi(M(\varphi_n(p),\varphi_{n+1}(p),t) < 1$; then

$$0 \leq \psi(M(\varphi_n(p),\varphi_{n+1}(p),t)) \leq \lambda, \text{ for all } n \in N.$$

Following from the above arguments and by T being a random, comparable MT contraction, we conclude that for each n

$$(M(\varphi_n(p),\varphi_{n+1}(p),t))$$

$$\leq \varphi(M(\varphi_{n-1}(p),\varphi_n(p),t)).M(\varphi_{n-1}(p),\varphi_n(p),t)$$

$$\leq \lambda.M(\varphi_{n-1}(p),\varphi_n(p),t).$$

Therefore, we also conclude that

$$M(\varphi_n(p),\varphi_{n+1}(p),t)$$

$$= M(T\varphi_{n-1}(p),T\varphi_n(p),t)$$

$$\leq \lambda.M(\varphi_{n-1}(p),\varphi_n(p),t)$$

$$\leq \lambda^2.M(\varphi_{n-2}(p),\varphi_{n-1}(p),t)$$

$$\leq \dots$$

$$\leq \lambda^n.M(\varphi_0(p),\varphi_1(p),t).$$

So we have that $\lim_{n\to\infty} M(\varphi_n(p),\varphi_{n+1}(p),t) = 0$, since $\lambda < 1$, and for $n > m$,

$$M(\varphi_m(p),\varphi_n(p),t)$$

$$\leq (\lambda^m + \lambda^{m+1} + \dots + \lambda^{n-1}).M(\varphi_0(p),\varphi_1(p),t)$$

$$\leq \tfrac{\lambda^m}{1-\lambda}.M(\varphi_0(p),\varphi_1(p),t).$$

Let $0 \leq \delta$ be given. Then we can choose a natural number P such that

$$\tfrac{\lambda^m}{1-\lambda}.M(\varphi_0(p),\varphi_1(p),t) \leq \delta, \text{ for all } m \geq P,$$

and we also conclude that

$$M(\varphi_m(p),\varphi_n(p),t) < \delta, \text{ for all } m \geq P.$$

So $\{\varphi_n(p)\}$ is a Cauchy sequence in $\Omega \times P$. On account of the fact that $(P,M,*)$ is complete, there exists a $\varphi^*(p) \in \Omega \times P$ such that $\varphi_n(p)$ converges to $\varphi^*(p)$; that is,

$$\lim_{n\to\infty} \varphi_n(p) = \varphi^*(p).$$

Thus, we have

$$M(\varphi^*(p),T(\varphi^*(p)),t)$$

$$\leq M(\varphi^*(p),\varphi_{n+1}(p),t)+M(\varphi_{n+1}(p),T(\varphi^*(p)),t)$$

$$\leq M(\varphi^*(p),\varphi_{n+1}(p),t)+M(T(\varphi_n(p)),T(\varphi^*(p)),t)$$

$$\leq M(\varphi^*(p),\varphi_{n+1}(p),t)+\psi(M(\varphi_n(p),T(\varphi^*(p)),t).\Gamma(\varphi_n(p),\varphi^*(p),t)$$

$$< M(\varphi^*(p),\varphi_{n+1}(p),t)+\Gamma(\varphi_n(p),\varphi^*(p),t),$$

and

$$\Gamma(\varphi_n(p),\varphi^*(p),t)$$

$$=\gamma(M(\varphi_n(p),\varphi^*(p),t),M(\varphi_n(p),T(\varphi_n(p)),t),M(\varphi^*(p),T(\varphi^*(p)),t),$$

$$\frac{M(\varphi_n(p),T(\varphi^*(p)),t)+M(\varphi^*(p),T(\varphi^*(p)),t)}{2}$$

$$\leq \gamma(M(\varphi_n(p),\varphi^*(p),t),M(\varphi_n(p),T(\varphi_{n+1}(p)),t),M(\varphi^*(p),T(\varphi^*(p)),t),$$

$$\frac{M(\varphi_n(p),T(\varphi^*(p)),t)+M(\varphi^*(p),T(\varphi^*(p)),t)+M(\varphi^*(p),\varphi_{n+1}(p),t))}{2})$$

Taking $n\to\infty$, we have

$$\lim_{n\to\infty}\Gamma(\varphi_n(p),\varphi^*(p),t)=\gamma(0,0,M(\varphi^*(p),T(\varphi^*(p)),t),\frac{M^*(p),T(\varphi^*(p)),t)}{2})).$$

In a different order pair of γ

$$\Gamma(\varphi^*(p),\varphi_n(p),t)$$

$$=\gamma(M(\varphi^*(p),\varphi_n(p),t),M(\varphi^*(p),T(\varphi^*(p)),t),M(\varphi_n(p),T(\varphi_n(p)),t),$$

$$\frac{M(\varphi^*(p),T(\varphi_n(p)),t)+M(\varphi_n(p),T(\varphi^*(p)),t)}{2})$$

$$\leq \gamma(M(\varphi^*(p),\varphi_n(p),t),M(\varphi^*(p),T(\varphi^*(p)),t),M(\varphi_n(p),\varphi_{n+1}(p),t)$$

$$\frac{M(\varphi^*(p),\varphi_{n+1}(p),t)+M(\varphi_n(p),\varphi^*(p),t)+M(\varphi^*(p),T(\varphi^*(p)),t)}{2})$$

Taking $n\to\infty$, we have

$$\lim_{n\to\infty}\Gamma(\varphi^*(p),\varphi_n(p),t)=\gamma(0,M(\varphi^*(p),T(\varphi^*(p)),t),0,\frac{M(\varphi^*(p),T(\varphi^*(p)),t)}{2})$$

By the condition of the mapping γ, we conclude that

$$M(\varphi^*(p),T(\varphi^*(p)),t)<M(\varphi^*(p),T(\varphi^*(p)),t),$$

and this is a contradiction unless $M(\varphi^*(p), T(\varphi^*(p)), t) = 0$.

Therefore, $\varphi^*(p) = T(\varphi^*(p))$, that is $\varphi^*(p)$ is a random fixed point of T in P. $\square$

Example 2.3. Let $P = M = R_0^+, \omega = [0,1]$ and $\sum$ be the sigma algebra of Lebesgue's measurable subset of [0, 1]. We define mappings as .... by $M(p,q,t) = \frac{\min\{p,q\},t}{\max\{p,q\},t}$. Then $(M,t,*)$ is a random fuzzy metric space. Define random operator T : ......

$$T(\xi(p)) = \frac{\xi(v)+1-v^2}{5}$$

$$M(T(\xi(p)), T(\eta(p)), t) = M(\frac{\xi(p)+1-p^2}{5}, \frac{\eta(p)+1-p^2}{5}, t$$

$$= \frac{\min}{(}\frac{\xi(p)+1-p^2}{5}, \frac{\eta(p)+1-p^2}{5}) + t\max(\frac{\frac{\xi(p)+1-p^2}{5}, \frac{\eta(p)+1-p^2}{5}}{)} + t$$

$$= \frac{1}{5}[\frac{\min(\xi(p)-\eta(p)+t)}{\max(\xi(p)-\eta(p)+t)}]$$

$$= \frac{1}{5}M(\xi(p), \eta(p), t)$$

and

$$\phi(t).\gamma(M(\xi(p), \eta(p), t), M(\xi(p), T(\xi(p)), t), M(\eta(p), T(\eta(p)), t)),$$

$$\frac{M(\xi(p), T(\eta(p)), t) + M(\eta(p), T(\eta(p)), t)}{2}$$

$$\geq \frac{1}{5}.\frac{\min}{(}\frac{\xi(p)+1-p^2}{5}, \frac{\eta(p)+1-p^2}{5}) + t\max(\frac{\frac{\xi(p)+1-p^2}{5}, \frac{\eta(p)+1-p^2}{5}}{)} + t\}$$

$$\frac{1}{5}M(\xi(p), \eta(p), t)$$

and then T is continuous, random, comparable $MT-_\gamma$ contraction.

Take the measurable mapping $p : \Omega \to P$ as $\xi(p) = \{1 - v^2\}$; then, for every $p \in \Omega$,

$$T(\xi(p) = \frac{\xi(p)+1-p^2}{2}) = \frac{1-p^2+1-p^2}{2} = 1 - p^2 = \xi(p).$$

$(1 - p^2)$ is a random fixed point of T.

Theorem 2.4. *Suppose $(P, M, *)$ is a complete random fuzzy-metric space and $\Upsilon \subset P$. If $Y(l, .) : \Omega \times \Upsilon \times \Upsilon \to \Upsilon$ is a continuous, random, comparable $\mathsf{MT}-_\gamma$ contraction, then T possesses a random fixed point in P.*

Proof. Given $\varphi_0(p) \in \Omega \times P \times P$ and defining $\varphi_1(p) = T(\varphi_0(p))$ and $\varphi_{n+1}(p) = T(\varphi_n(p)) = T^{n+1}(\varphi_0(p))$ for each $n \in N$, since $T(p, .) : \Omega \times \Upsilon \times \Upsilon \to \Upsilon$ is a random, comparable $\mathsf{MT}-_\gamma$ contraction, we have

$$M(\varphi_n(p),\varphi_{n+1}(p),t) = M(T(\varphi_{n-1}(p)),T(\varphi_n(p)),t)$$

$$\leq \psi(M(\varphi_{n-1}(p),\varphi_n(p),t)).\Gamma((\varphi_{n-1}(p),\varphi_n(p),t))$$

and

$$\Gamma((\varphi_{n-1}(p),\varphi_n(p),t) =$$

$$\gamma(M((\varphi_{n-1}(p),\varphi_n(p),t)),M((\varphi_{n-1}(p),T(\varphi_n(p)),t),M(\varphi_n(p),T(\varphi_n(p)),t),$$
$$\tfrac{M(\varphi_{n-1}(p),T(\varphi_n(p)),t)+M(\varphi_n(p),t(\varphi_{n-1}(p)),t)}{2})$$

$$= \gamma((\varphi_{n-1}(p),\varphi_n(p),t),M((\varphi_{n-1}(p),\varphi_n(p),t),M((\varphi_n(p),\varphi_{n+1}(p),t),$$
$$\tfrac{M(\varphi_{n-1}(p),\varphi_{n+1}(p),t)+M(\varphi_n(p),\varphi_n(p),t)}{2})$$

If $M(\varphi_n(p),\varphi_{n+1}(p),t) > M(\varphi_{n-1}(p),\varphi_n(p),t)$ for some n, then by the conditions of the function γ we have that

$$\Gamma(\varphi_{n-1}(p),\varphi_n(p),t)\gamma((\varphi_{n-1}(p),\varphi_n(p),t),M((\varphi_{n-1}(p),\varphi_n(p),t),M((\varphi_n(p),\varphi_{n+1}(p),t),$$
$$\tfrac{M(\varphi_{n-1}(p),\varphi_{n+1}(p),t)+M(\varphi_n(p),\varphi_n(p),t)}{2})$$

$$\leq (M(\varphi_n(p),\varphi_{n+1}(p),t),M(\varphi_n(p),\varphi_{n+1}(p),t),M(\varphi_n(p),\varphi_{n+1}(p),t),M(\varphi_n(p),\varphi_{n+1}(p),t))$$

$$\leq M(\varphi_n(p),\varphi_{n+1}(p),t).$$

In a different order pair of γ

$$M(\varphi_n(p),\varphi_{n+1}(p),t) = M(T(\varphi_n(p)),T(\varphi_{n-1}(p)),t))$$

$$\leq \psi(M(\varphi_n(p),\varphi_{n-1}(p),t)).\Gamma(\varphi_n(p),\varphi_{n-1}(p),t),$$

and

$$\Gamma(\varphi_n(p),\varphi_{n-1}(p),t) = \gamma(M(\varphi_n(p),\varphi_{n-1}(p),t),M(\varphi_n(p)),T(\varphi_n(p)),t),$$
$$M(\varphi_{n-1}(p)),T(\varphi_{n-1}(p)),t),$$
$$\tfrac{M(\varphi_n(p)),T(\varphi_{n-1}(p)),t)+M(\varphi_{n-1}(p),T(\varphi_n(p)),t)}{2})$$

$$\leq \gamma(M(\varphi_n(p),\varphi_{n-1}(p),t),M(\varphi_n(p),\varphi_{n+1}(p),t),M(\varphi_{n-1}(p),\varphi_n(p),t)),$$
$$\tfrac{M(\varphi_n(p),\varphi_n(p),t)+M(\varphi_{n-1}(p),\varphi_{n+1}(p),t)}{2}).$$

If $M(\varphi_n(p),\varphi_{n+1}(p),t) > M(\varphi_{n-1}(p),\varphi_n(p),t)$ for some n, then by the conditions of the comparable function γ we have that

$$\Gamma(\varphi_n(p),\varphi_{n-1}(p),t) = \gamma(M(\varphi_n(p),\varphi_{n-1}(p),t), M(\varphi_n(p)),\varphi_{n+1}(p),t), M(\varphi_{n-1}(p)),\varphi_n(p),t), \frac{M(\varphi_n(p),\varphi_n(p),t)+M(\varphi_{n-1}(p)),\varphi_{n+1}(p),t)}{2})$$

$$\leq \gamma(M(\varphi_n(p),\varphi_{n+1}(p),t), M(\varphi_n(p),\varphi_{n+1}(p),t), M(\varphi_n(p),\varphi_{n+1}(p),t), M(\varphi_n(p),\varphi_{n+1}(p),t))$$

$$\leq M(\varphi_n(p),\varphi_{n+1}(p),t).$$

Since ψ is a MT function, we conclude that

$$M(\varphi_n(p),\varphi_{n+1}(p),t) \leq \psi(M(\varphi_{n-1}(p),\varphi_n(p),t)).M(\varphi_n(p),\varphi_{n+1}(p),t)$$

$$< M(\varphi_n(p),\varphi_{n+1}(p),t),$$

which implies a contradiction. So, we conclude that

$$M(\varphi_n(p),\varphi_{n+1}(p),t) \leq M(\varphi_{n-1}(p),\varphi_n(p),t), \text{ for each } n \in N.$$

From above argument, then sequence $\{M(\varphi_n(p),\varphi_{n+1}(p),t)\}_{n\in N\cup\{0\}}$ is non-increasing in R_0^+. Since ψ is an MT function, by Theorem 1 we conclude that

$$0 \leq \sup_{n\in N} \psi(M(\varphi_n(p),\varphi_{n+1}(p),t)) < 1.$$

Let $\lambda = sup_{n\in N}\psi(M(\varphi_n(p),\varphi_{n+1}(p),t)) < 1$; then

$$0 \leq \psi(M(\varphi_n(p),\varphi_{n+1}(p),t)) \leq \lambda, \text{ for all } n \in N.$$

Following from the above arguments and by T being a random, comparable MT contraction, we conclude that for each n

$$(M(\varphi_n(p),\varphi_{n+1}(p),t))$$

$$\leq \psi(M(\varphi_{n-1}(p),\varphi_n(p),t)).M(\varphi_{n-1}(p),\varphi_n(p),t)$$

$$\leq \lambda.M(\varphi_{n-1}(p),\varphi_n(p),t).$$

Therefore, we also conclude that

$$M(\varphi_n(p),\varphi_{n+1}(p),t)$$

$$= M(T\varphi_{n-1}(p), T\varphi_n(p), t)$$

$$\leq \lambda.M(\varphi_{n-1}(p), \varphi_n(p), t)$$

$$\leq \lambda^2.M(\varphi_{n-2}(p), \varphi_{n-1}(p), t)$$

$$\leq \ldots$$

$$\leq \lambda^n.M(\varphi_0(p), \varphi_1(l), t).$$

So we have that $\lim_{n\to\infty} M(\varphi_n(p), \varphi_{n+1}(p), t) = 0$, since $\lambda < 1$, and for $n > m$,

$$M(\varphi_m(p), \varphi_n(p), t)$$

$$\leq (\lambda^m + \lambda^{m+1} + \ldots + \lambda^{n-1}).M(\varphi_0(p), \varphi_1(l), t)$$

$$\leq \frac{\lambda^m}{1-\lambda}.M(\varphi_0(p), \varphi_1(p), t).$$

Let $0 \leq \delta$ be given. Then we can choose a natural number M such that

$$\frac{\lambda^m}{1-\lambda}.M(\varphi_0(p), \varphi_1(p), t) \leq \delta, \text{ for all } m \geq M,$$

and we also conclude that

$$M(\varphi_m(p), \varphi_n(p), t) < \delta, \text{ for all } m \geq M.$$

So $\{\varphi_n(p)\}$ is a Cauchy sequence in $\Omega \times P \times P$. On account of the fact that $(P, M, *)$ is complete, there exists a $\varphi^*(p) \in \Omega \times P \times P$ such that $\varphi_n(p)$ converges to $\varphi^*(p)$; that is,

$$\lim_{n\to\infty} \varphi_n(p) = \varphi^*(p).$$

Thus, we have

$$M(\varphi^*(p), T(\varphi^*(p)), t)$$

$$\leq M(\varphi^*(p), \varphi_{n+1}(p), t) + M(\varphi_{n+1}(p), T(\varphi^*(p)), t)$$

$$\leq M(\varphi^*(p), \varphi_{n+1}(p), t) + M(T(\varphi_n(p)), T(\varphi^*(p)), t)$$

$$\leq M(\varphi^*(p), \varphi_{n+1}(p), t) + \psi(M(\varphi_n(p), T(\varphi^*(p)), t).\Gamma(\varphi_n(p), \varphi^*(p), t)$$

$$< M(\varphi^*(p), \varphi_{n+1}(p), t) + \Gamma(\varphi_n(p), \varphi^*(p), t),$$

and

$$\Gamma(\varphi_n(p),\varphi^*(p),t)$$

$$=\gamma(M(\varphi_n(p),\varphi^*(l),t),M(\varphi_n(p),T(\varphi_n(p)),t),M(\varphi^*(p),T(\varphi^*(p)),t),$$

$$\tfrac{M(\varphi_n(p),T(\varphi^*(p)),t)+M(\varphi^*(p),T(\varphi^*(p)),t)}{2}$$

$$\leq\gamma(M(\varphi_n(p),\varphi^*(p),t),M(\varphi_n(p),T(\varphi_{n+1}(p)),t),M(\varphi^*(p),T(\varphi^*(p)),t),$$

$$\tfrac{M(\varphi_n(p),T(\varphi^*(p)),t)+M(\varphi^*(p),T(\varphi^*(p)),t)+M(\varphi^*(p),\varphi_{n+1}(p),t)}{2})$$

Taking $n\to\infty$, we have

$$\lim_{n\to\infty}\Gamma(\varphi_n(p),\varphi^*(p),t)=\gamma(0,0,M(\varphi^*(p),T(\varphi^*(p)),t),\tfrac{M^*(p),T(\varphi^*(p)),t)}{2})).$$

In a different order pair of γ

$$\Gamma(\varphi^*(p),\varphi_n(p),t)$$

$$=\gamma(M(\varphi^*(p),\varphi_n(p),t),M(\varphi^*(p),T(\varphi^*(p)),t),M(\varphi_n(p),T(\varphi_n(p)),t),$$

$$\tfrac{M(\varphi^*(p),T(\varphi_n(p)),t)+M(\varphi_n(p),T(\varphi^*(p)),t)}{2})$$

$$\leq\gamma(M(\varphi^*(p),\varphi_n(p),t),M(\varphi^*(p),T(\varphi^*(p)),t),M(\varphi_n(p),\varphi_{n+1}(p),t)$$

$$\tfrac{M(\varphi^*(p),\varphi_{n+1}(p),t)+M(\varphi_n(p),\varphi^*(p),t)+M(\varphi^*(p),T(\varphi^*(p)),t)}{2})$$

Taking $n\to\infty$, we have

$$\lim_{n\to\infty}\Gamma(\varphi^*(p),\varphi_n(p),t)=\gamma(0,M(\varphi^*(p),T(\varphi^*(p)),t)),0,\tfrac{M(\varphi^*(p),T(\varphi^*(p)),t)}{2})$$

By the condition of the mapping γ, we conclude that

$$M(\varphi^*(p),T(\varphi^*(p)),t)<M(\varphi^*(p),T(\varphi^*(p)),t),$$

and this is a contradiction unless $M(\varphi^*(p),T(\varphi^*(p)),t)=0$.

Therefore, $\varphi^*(p)=T(\varphi^*(p))$, that is $\varphi^*(p)$ is a random fixed point of T in P. □

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