Available online at http://scik.org
J. Math. Comput. Sci. 11 (2021), No. 2, 2031-2046
https://doi.org/10.28919/jmcs/5367
ISSN: 1927-5307

SEMI P-CORRESPONDENT TOPOLOGIES AND NOWHERE P DENSE SETS

T. VINITHA[1,*], T.P. JOHNSON[2]

[1]Department of Mathematics, Al-Ameen College, Edathala, Kerala India

[2]School of Engineering and Applied Sciences Division, Cochin Univrsity of Science and Technology, Kerala, India

Abstract. The objective of this paper is to introduce the concept of semi p-open sets. Also we introduce the idea of nowhere p-dense sets and obtained an equivalent condition for a set to be nowhere p-dense in terms of p-open sets. We proved that any semi p-open set can be written as disjoint union of a p-open set and a nowhere p-dense set. We studied various mappings involving semi p-open sets and p-open sets and analysed the behavior of p-open sets, semi p-open sets and nowhere p-dense sets under such mappings.

Keywords: semi p-open sets; semi-irresolute; semi-continuous; nowhere p-dense sets.

2010 AMS Subject Classification: 54C05, 54C10.

1. Introduction

Norman Levine in [7] introduced the concept of semi-open sets and semi-continuity in topological spaces. Later a lot of research work has been done in topology using semi-open sets. In any continuous lattice L ; Gierz et. al. [4] introduced prime element and later Ales Pultr [5], P. T. Johnstone [8] etc initiated the study of irreducible open set in the lattice of open sets of any arbitrary topological space inspired by the definition of prime element in any lattice by

*Corresponding author
E-mail address: vinitha01@gmail.com
Received December 30, 2020

Gierz. Motivated by those definition of irreducible open set we in [13] introduced prime open set shortly p-open set in the lattice of open sets of any arbitrary topological space. In [13] we studied the concept of generalised closed set introduced by Levine using p-open sets and thus introduce generalised p-closed sets. Also we studied some new weaker separation axioms using the concepts of p-open sets and generalised p-closed sets.

In this paper we try to apply the concept of p-open sets to semi open sets and thereby we introduce the notion of semi p-open sets. Mean while we introduce nowhere p-dense sets and obtained that any semi p-open set can be written as the disjoint union of p-open and nowhere p-dense sets. We studied semi p-continuous, semi-irresolute and semi p-open mappings using p-open and semi p-open sets. Examined the implications amongst each of the mappings and analysed the behavior of semi p-open sets, p-open sets and nowhere p-dense sets under such mappings. Also studied about semi p-homeomorphism and proved that nowhere p-dense sets are preserved under semi p-homeomorphisms. Also obtained that any semi p-homeomorphic image of a topological space of first category can be written as the union of nowhere p-dense sets in it.

2. Preliminaries

Definition 2.1. [13] *Let (X,T) be any arbitrary topological space. The open sets in T forms a complete lattice with smallest element 0 and largest element 1 ; where $0=\phi$ and $1=X$. We define an open set G$\neq$ 1 in T to be prime open set if $H\cap K\subseteq G\Rightarrow H\subseteq G$ or $K\subseteq G$; where H,K are open sets in T such that $H\cap K\neq\phi$. Clearly 0 and 1 are prime in T. Prime open sets are denoted by p-open sets. Complements of p-open sets are called p-closed sets.*

Theorem 2.2. [13] *Let (X,T) be a hausdorff space and $x\in X$ then the only p-open sets are* $X-\{x\}$.

Definition 2.3. [13] *Let (X,T) be a topological space and let $A\subseteq X$, then the p-closure of A with respect to T is defined as the minimal p-closed super set of A in X and is denoted as $p\text{-}cl(A)$.*

Proposition 2.4. [13] *Let (X,T) be a topological space, then for every p-open set $A\subseteq X$ there always exists a unique p-closed set containing A.*

Definition 2.5. [13] *Let (X,T) be a topological space and let $A \subseteq X$, then the p-interior of A with respect to T is defined as the maximal p-open subset of A in X and is denoted as p-int(A).*

Proposition 2.6. [13] *Let (X,T) be a topological space, then for every p-closed set $A \subseteq X$ there always exists a unique p-open set contained in A.*

Theorem 2.7. [13] *Let (X,T) be a topological space and $Y \subseteq X$. U p-open in X implies $U \cap Y$ p-open in Y.*

Proposition 2.8. [13] *Let (X,T) be a topological space, $A \subseteq X$ and $x \in X$. Then $x \in$p-$cl(A)$ if and only if every p-open set containing 'x' intersects A.*

Definition 2.9. [13] *Let (X,T) be a topological space and $A \subseteq X$; an element $x \in X$ is called a p-limit point/p-cluster point of $A \subseteq X$ if every p-open set containing 'x' intersects A.*

Definition 2.10. [7] *A set A in a topological space X will be termed semi-open if there exists an open set O such that $O \subseteq A \subseteq \overline{O}$; where $\overline{O}$ is the closure of O in X.*

Definition 2.11. [12] *Let $(X,T),(Y,T^{'})$ be two topological spaces and let $f:(X,T) \rightarrow (Y,T^{'})$ be a mapping between this two topological spaces. f is called p-continuous if the inverse image of p-open sets in $T^{'}$ are p-open in T and is said to be p-open if p-open sets are mapped to p-open sets only.*

Definition 2.12. [12] *Let $(X,T),(Y,T^{'})$ be two topological spaces and $f:(X,T) \rightarrow (Y,T^{'})$ be a mapping. f is said to be a p-homeomorphism if f is one-one, onto and both f, f^{-1} are p-continuous.*

Definition 2.13. [1] *A function f is said to be semi-continuous if inverse image of open sets are semi -open.*

Definition 2.14. [1] *A function $f: X \rightarrow Y$ is said to be irresolute if and only if for every semi-open set S of Y $f^{-1}(S)$ is semi-open in X.*

Definition 2.15. [1] *Let X and Y be topological spaces , a function $f: X \rightarrow Y$ is pre semi-open if every semi-open set in X is mapped to semi-open set in Y only.*

3. Semi p-Open Sets and Nowhere p-Dense Sets

Definition 3.1. *Let (X,T) be a topological space and $A \subseteq X$. A is said to be semi p-open if there exists a p-open set 'O' such that $O \subseteq A \subseteq p\text{-}cl(A)$ and A is said to be semi p-closed if its complement is semi p-open.*

Remark 3.2. *Trivially p-open implies semi p-open but converse is not true ; for example Let $X = \{a,b,c,d\}$ and $\tau = \{X,\phi,\{a\},\{b\},\{a,b\},\{a,b,c\},\{a,b,d\}\}$ be a topology on X. In (X,τ) , $\{a,c\}$ is not p-open but it is semi p-open.*

Remark 3.3. *For a haursdorff space, p-open sets and semi p-open sets coincides.*

Remark 3.4. *For any arbitrary topological space , semi open sets are not always semi p-open. For example consider real line with usual topology, then $(0,1]$ is semi-open but not semi p-open.*

Theorem 3.5. *Let (X,T) be a topological space and $A \subseteq X$. A is semi p-open iff $A \subseteq p\text{-}cl(p\text{-}int(A))$.*

Proof. For necessary part we assume A as a semi p-open set which implies there exists a p-open set G such that

$$G \subseteq A \subseteq p\text{-}cl(G) \tag{1}$$

Now $G \subseteq A$ and G is p-open, which implies $G \subseteq p\text{-}int(A)$ which again implies

$$p\text{-}cl(G) \subseteq p\text{-}cl(p\text{-}int(A)) \tag{2}$$

Now (1) and (2) implies $A \subseteq p\text{-}cl(p\text{-}int(A))$.

Conversely assume that $A \subseteq p\text{-}cl(p\text{-}int(A))$. Take p-$int(A) = G$, then G is a p-open set such that $G \subseteq A \subseteq p\text{-}cl(G)$. That is A is semi p-open. □

Corollary 3.6. *Let (X,T) be a topological space and let $A \subseteq X$ be a semi p-open set in X then A is semi-open if $p\text{-}cl(p\text{-}int(A)) \subseteq cl(int(A))$.*

Remark 3.7. *Generally semi p-open sets are not always semi-open. Let $X = \{1,2,3,4\}$ and $\tau = \{X,\phi,\{1\},\{2,3\},\{1,4\},\{1,2,3\},\{2,3,4\}\}$. In the topological space (X,τ) , $\{1,2,4\}$ is semi p-open but not semi-open.*

Remark 3.8. *Union of semi p-open sets need not be semi p-open. For example let* $X = \{a,b,c\}$ *and the topology on it be* $\tau = \{X, \phi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}, \{a,b,d\}\}$. *Here* $\{a\}$ *and* $\{b\}$ *are semi p-open but* $\{a,b\}$ *is not semi p-open.*

Remark 3.9. *Intersection of two semi p-open sets need not be semi p-open. Consider any arbitrary set with cardinality greater than three and with discrete topology, clearly* $X-\{x_1\}, X-\{x_2\}$ *are semi p-open but their intersection is not semi p-open.*

Proposition 3.10. *Let* (X,T) *be a topological space and let A be a semi p-open set in* (X,T). *Also let* $A \subseteq B \subseteq p\text{-}cl(A)$, *then B is also semi p-open.*

Proof. Given A as a semi p-open set then by definition of semi p-open set there exist a p-open set 'O' such that

$$O \subseteq A \subseteq p\text{-}cl(O) \tag{3}$$

$O \subseteq A$ and $A \subseteq B$ which implies

$$O \subseteq B \tag{4}$$

From (3) we obtain $A \subseteq p\text{-}cl(O)$
$\Rightarrow p\text{-}cl(A) \subseteq p\text{-}cl(O)$
$\Rightarrow B \subseteq p\text{-}cl(A) \subseteq p\text{-}cl(O)$. Hence by (4) $O \subseteq B \subseteq p\text{-}cl(O)$. Thus B is also semi p-open. □

Theorem 3.11. *Let* (X,T) *be any topological space and* $\mathscr{G} = \{G_\alpha\}$ *be a collection of sets in X such that*

(1) $T \subset \mathscr{G}$.
(2) $G_\alpha \in \mathscr{G}$ *and* $G_\alpha \subseteq H \subseteq p\text{-}cl(G_\alpha)$ *implies* $H \in \mathscr{G}$.

Then the collection of all semi p-open sets in X belongs to $\mathscr{G}$ *and it is the smallest collection of sets in X satisfying 1 and 2.*

Theorem 3.12. *Let* (X,T) *be a topological space with a subspace* (Y,T_Y) *where* $Y \subseteq X$. *If* $A \subseteq Y$ *is semi p-open in* (X,T), *then A is semi p-open in* (Y,T_Y).

Proof. Given A is semi p-open in (X,T) then by definition of semi p-open set there exists a p-open set 'O' such that

$$O \subseteq A \subseteq p\text{-}cl_X(O) \tag{5}$$

where p-$cl_X(O)$ is the p-closure of 'O' with respect to (X,T).

We have $O \subset A \subset Y$ which implies $O \subset Y$.

Now (5) $\Rightarrow O \cap Y \subset A \cap Y \subset p\text{-}cl_X(O) \cap Y$

$\Rightarrow O \cap Y \subset A \cap Y \subseteq p\text{-}cl_Y(O)$

Since $O \subset Y$, $O \cap Y = Y$. Hence we obtain $O \subseteq A \subseteq p\text{-}cl_Y(O)$. Thus A is semi p-open in (Y,T_Y). □

Remark 3.13. *Converse of above result need not be true. For example consider the discrete topological space $X = \{x_1,x_2,x_3\}$ and let $Y = \{x_1,x_2\}$. Then $Y - \{x_2\}$ is semi p-open in Y but not semi p-open in X.*

Definition 3.14. *Let (X,T) be a topological space and let $A \subset X$ then we define $D_p(A)$ as the set of all p-limit points of A with respect to T.*

Remark 3.15. *Clearly $D(A) \subseteq D_p(A)$ but not conversely where $D(A)$ denotes the set of all limit points of A.*

Theorem 3.16. *Let (X,T) be a topological space and let $A \subseteq X$ then $p\text{-}cl(A) = A \cup D_p(A)$.*

Proof. To prove that $A \cup D_p(A) \subseteq p\text{-}cl(A)$. Let $x \in A \cup D_p(A)$, then $x \in A$ or $x \in D_p(A)$. If $x \in A$, then trivially $x \in p\text{-}cl(A)$. If $x \in D_p(A)$, then 'x' is a p-limit point of A and by Proposition : 2.8 ; $x \in p\text{-}cl(A)$. Thus $A \cup D_p(A) \subseteq p\text{-}cl(A)$. Conversely assume that $x \in p\text{-}cl(A)$. To prove that p-$cl(A) \subseteq A \cup D_p(A)$. If $x \in A$ the result trivially follows. If $x \notin A$ then since $x \in p\text{-}cl(A)$ every p-open set 'U' containing 'x' intersects A which implies 'x' is a p-limit point of A. Hence $x \in A \cup D_p(A)$. Thus p-$cl(A) \subseteq A \cup D_p(A)$ always and hence p-$cl(A) = A \cup D_p(A)$ □

Definition 3.17. *A subset A of a topological space (X,T) is said to be nowhere p-dense if $p\text{-}int(\overline{A}) = \phi$.*

Remark 3.18. *Nowhere p-dense does not implies nowhere dense. Consider (R,U) and let A be the set of all rationals between 0 and 1 then A is nowhere p-dense but not nowhere dense.*

Remark 3.19. *Trivially if $int(\overline{A}) = \phi$ then p-$int(\overline{A}) = \phi$. Hence nowhere dense implies nowhere p-dense.*

Proposition 3.20. *Let (X,T) be a topological space and $A \subseteq X$ then A is nowhere p-dense if and only if every non-empty p-open set in X contains a non-empty open set which is disjoint from A.*

Proof. For necessity assume that A is nowhere p-dense, that is p-$int(\overline{A}) = \phi$. We have to prove that every non-empty p-open set in X contains a non-empty open set which is disjoint from A. Let U be the given non-empty p-open set. Clearly U is not a subset of $\overline{A}$, if $U \subseteq \overline{A}$ then p-$int(\overline{A}) \neq \phi$ which is not possible. Hence $U \cap (X - \overline{A})$ is a non-empty open set disjoint from $\overline{A}$ and thus disjoint from A and such that $U \cap (X - \overline{A}) \subset U$. Thus proved the necessary part.
Conversely assume the sufficiency part in order to prove that p-$int(\overline{A}) = \phi$. On contradiction let p-$int(\overline{A}) \neq \phi$ then there exists a p-open set G such that $G \subset \overline{A}$. Thus any point of G happens to be a limit point of A ; that is all open sets containing points of G must intersects A which implies there does not exists an open set in G disjoint from A contradicting our assumption. Hence p-$int(\overline{A}) = \phi$. □

Proposition 3.21. *Let O be p-open in X; then p-$cl(O) - O$ is nowhere p-dense in X.*

Proof. By above result we have p-$cl(O) = O \cup D_p(O)$ which implies p-$cl(O) - O \subseteq D_p(O)$. That is p-$cl(O) - O$ contains all p-limit points of O. We have to prove that p-$cl(O) - O$ is nowhere p-dense. Let U be a non-empty p-open set in X. If $U \subseteq O$; then $U \cap (p\text{-}cl(O) - O) = \phi$ that is U itself is a p-open set disjoint from p-$cl(O) - O$. Hence p-$cl(O) - O$ is nowhere p-dense. If $O \cap U = \phi$ then U contains no points of O which implies it does not contains any p-limit points of O which implies $U \cap (p\text{-}cl(O) - O) = \phi$. Now if both of the above cases fails that is if $O \cap U \neq \phi$ and U not a subset of O then $U \cap O$ is a non-empty open subset of U as well as O and $(p\text{-}cl(O) - O) \cap (U \cap O) = \phi$. Thus in this case also U contains an open set disjoint from p-$cl(O) - O$ which implies p-$cl(O) - O$ is nowhere p-dense. □

Theorem 3.22. *Let A be a semi p-open set in a topological space (X,T). Then A will be of the form $A = O \cup B$ where O is a p-open set in X such that $O \cap B = \phi$ and B is nowhere p-dense.*

Proof. Given A as a semi p-open set which implies there exists a p-open set O such that $O \subseteq A \subseteq p\text{-}cl(O)$. Clearly any arbitrary set A can be written as $A = O \cup (A - O)$. Now let $B = A - O$, since $A \subseteq p\text{-}cl(O)$ we have $B \subseteq p\text{-}cl(O) - O$. By above lemma p-$cl(O) - O$ is nowhere p-dense and that implies B is also nowhere p-dense. Thus $A = O \cup B$ and it satisfies all conditions of the theorem. □

Theorem 3.23. *Let (X,T) be a topological space and let $\mathscr{P}$ denote the collection of p-open sets in T. If $\mathscr{G}$ denote the collection of p-interior of all semi p-open sets in X then $\mathscr{G} = \mathscr{P}$.*

Proof. Let $P \in \mathscr{P}$, then p-$int(P) = P$ itself which implies $P \in \mathscr{G}$. Hence $\mathscr{P} \subset \mathscr{G}$. Now let $G \in \mathscr{G} \Rightarrow G = p\text{-}int(G_1)$ for some G_1 semi p-open in X which is a maximal p-open subset of G_1 which implies $G \in \mathscr{P}$. Hence $\mathscr{G} \subset \mathscr{P}$ and thus $\mathscr{G} = \mathscr{P}$. □

Lemma 3.24. *Let A be a semi p-open set in a topological space (X,T). Then there exists a p-open set O such that $(A - O) \subseteq D_p(A)$.*

Proof. Given A as a semi p-open set then by definition of semi p-open set there exists a p-open set O such that $O \subseteq A \subseteq p\text{-}cl(O)$. Now $A - O \subseteq p\text{-}cl(O) - O \subseteq D_p(O)$. Hence for any semi p-open set A there exists a p-open set O such that $A - O \subseteq D_p(O)$. □

Theorem 3.25. *Let A be a semi p-open set in a topological space (X,T). Then there exists a p-open set O such that $D_p(A - O) \subseteq D_p(O)$.*

Proof. Let $y \in D_p(A - O)$ then any p-open set containing 'y' must contain points of $A - O$ but by above lemma $A - O \subseteq D_p(O)$ which implies any p-open set containing points of $A - O$ must contain points of O. Hence any p-open set containing 'y' must contain points of 'O' that is $y \in D_p(O)$. Thus $D_p(A - O) \subseteq D_p(O)$. □

4. Mappings Involving Semi p-Open Sets and p-Open Sets

Lemma 4.1. *A function $f : X \to Y$ is p-continuous if and only if for every $A \subset X$ $f(p\text{-}cl(A)) \subseteq p\text{-}cl(f(A))$.*

Proof. Assume that $f : X \to Y$ is p-continuous. Now consider $f(A) \subseteq p\text{-}cl(f(A))$
$\Rightarrow A \subseteq f^{-1}(f(A)) \subseteq f^{-1}[p\text{-}cl(f(A))]$.
Since f is p-continuous and p-$cl(f(A))$ is p-closed, $f^{-1}[p\text{-}cl(f(A))]$ is a p-closed set containing A
$\Rightarrow p\text{-}cl(A) \subseteq f^{-1}[p\text{-}cl(f(A))]$
$\Rightarrow f(p\text{-}cl(A)) \subseteq f(f^{-1}[p\text{-}cl(f(A))]) = p\text{-}cl(f(A))$. Hence $f(p\text{-}cl(A)) \subseteq p\text{-}cl(f(A))$. Conversely assume that $f(p\text{-}cl(A)) \subseteq p\text{-}cl(f(A))$ to prove that f is p-continuous. Let B be a p-closed set in Y it is enough to prove that $f^{-1}(B)$ is p-closed in X. That is to prove that p-$cl(f^{-1}(B)) = f^{-1}(B)$.
Consider $f(p\text{-}cl(f^{-1}(B)) \subseteq p\text{-}cl(f(f^{-1}(B))) = p\text{-}cl(B) = B$
$\Rightarrow p\text{-}cl[f^{-1}(B)] \subseteq f^{-1}(B)$
$\Rightarrow f^{-1}(B)$ is p-closed and hence f is p-continuous. □

Theorem 4.2. *Let $f : (X,T) \to (Y,T')$ be a p-continuous , p-open mapping between the topological spaces (X,T) and (Y,T'). If A is semi p-open in (X,T), then $f(A)$ is semi p-open in (Y,T').*

Proof. Given A is semi p-open in (X,T) , then by theorem : 3.22 $A = O \cup B$ where O is p-open and $B \subseteq p\text{-}cl(O) - O$ which implies
$O \subseteq A \Rightarrow f(O) \subseteq f(A)$
$= f(O) \cup f(B)$
$\subseteq f(O) \cup f(p\text{-}cl(O))$
$\subseteq f(O) \cup p\text{-}cl(f(O))$
$\subseteq p\text{-}cl(f(O))$.
Thus $f(O) \subseteq f(A) \subseteq p\text{-}cl(f(O))$. Since $f(O)$ is p-open in Y, $f(A)$ is semi p-open in Y. □

Definition 4.3. *Let $f : (X,T) \to (Y,T')$ be a mapping between two topological spaces (X,T) and (Y,T') , then f is said to be semi p-continuous if inverse image of p-open set in Y is semi p-open in X.*

Example 4.1. *Let $X = Y = \{a,b,c,d\}$. Also let $T = \{X, \phi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}, \{a,b,d\}\}$ and $T' = \{X, \phi, \{b\}, \{c\}, \{b,c\}\}$ be two topologies on X . Define $f : (X,T) \to (Y,T')$ by $f(a) = f(c) = c$, $f(b) = b, f(d) = d$. Then f is semi p-continuous.*

Remark 4.4. *The above example indicates that semi p-continuity does not implies p-continuity and it does not implies even continuity.*

Remark 4.5. *Trivially p-continuity implies semi p-continuity.*

Remark 4.6. *Semi-continuity neither implies nor implied by semi p-continuity. For example Let $X = Y = \{a,b,c\}$, T be the discrete topology and $T' = \{X, \phi, \{c\}\}$. Now define $f : (X,T) \to (Y,T')$ as the identity mapping . Then f is semi continuous but not semi p-continuous. Now consider the function $g : (R,U) \to (R,D)$ where R is the real line with Discrete topology D and usual topology U ; g is the identity mapping. Clearly g is semi p-continuous but not semi-continuous.*

Remark 4.7. *Let X be a T_2 space and f is a function such that $f : X \to Y$ is semi p-continuous then it is p-continuous.*

Theorem 4.8. *Let (X,T), (Y,T') be two topological spaces and $f : (X,T) \to (Y,T')$ be a mapping such that f is a single valued function. If f is semi p-continuous then for any $f(x) \in G'$, G' p-open in Y there exists G semi p-open in X such that $x \in G$ and $f(G) \subset G'$.*

Proof. Let $f(x) \in G'$. Clearly $f^{-1}(G')$ is semi p-open in X and contains 'x'. Now let $G = f^{-1}(G')$ then $x \in G$ and $f(G) \subset G'$. □

Definition 4.9. *Let (X,T) , (Y,T') be two topological spaces ; then $f : (X,T) \to (Y,T')$ is said to be semi - irresolute if and only if inverse image of semi p-open set in Y is semi p-open in X.*

Example 4.2. *Let $X = Y = \{a,b,c,d\}$ and τ, τ' be two topologies on X such that $\tau = \{X, \phi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}, \{a,b,d\}\}$ and $\tau' = \{X, \phi, \{c\}\}$. Define a function $f : (X,\tau) \to (Y,\tau')$ as $f(a) = f(b) = c$, $f(c) = d$ and $f(d) = a$. Then f is semi- irresolute but not p-continuous.*

Remark 4.10. *Both p-continuity and semi p-continuity does not implies semi -irresoluteness. For example Let* $X = Y = \{a,b,c,d\}$ *and let* $T = \{X,\phi,\{a\},\{c\},\{a,c\},\{a,c,d\},\{a,b,c\}\}$ *,* $T' = \{X,\phi,\{a\}\}$ *be two topologies on* X*. Consider* $f : (X,T) \to (Y,T')$ *as* $f(a) = c, f(b) = b, f(c) = a, f(d) = d$ *.Then* f *is p-continuous and semi p-continuous but not semi -irresolute.*

Lemma 4.11. *If* $f : X \to Y$ *is p-continuous and p-open , then* $f^{-1}(p\text{-}cl(A)) = p\text{-}cl(f^{-1}(A))$*.*

Proof. Since f is p-open , f^{-1} is p-continuous and hence $f^{-1}(p\text{-}cl(A)) \subseteq p\text{-}cl(f^{-1}(A))$. For the other part we have $A \subseteq p\text{-}cl(A)$ which implies

$$f^{-1}(A) \subseteq f^{-1}(p\text{-}cl(A)) \tag{6}$$

Since f is p-continuous and p-$cl(A)$ is p-closed always, $f^{-1}(p\text{-}cl(A))$ is p-closed and thus (6) implies p-$cl(f^{-1}(A)) \subseteq f^{-1}(p\text{-}cl(A))$. Thus $f^{-1}(p\text{-}cl(A)) = p\text{-}cl(f^{-1}(A))$. □

Theorem 4.12. *Let* $f : X \to Y$ *be p-continuous and p-open , then* f *is semi irresolute.*

Proof. To prove that every semi p-open set in Y is mapped on to semi p-open set in X. Let G be a semi p-open set in Y then by definition of semi p-open set there exists a set O such that O is p-open and $O \subseteq G \subseteq p\text{-}cl(O)$

$$\Rightarrow f^{-1}(O) \subseteq f^{-1}(G) \subseteq f^{-1}(p\text{-}cl(O)) = p\text{-}cl(f^{-1}(O)) \tag{7}$$

Since f is p-continuous, $f^{-1}(O)$ is p-open in X and hence (7) implies $f^{-1}(G)$ is semi p-open in X. □

Theorem 4.13. *Let* (X,T)*,* (Y,T') *be two topological spaces then* $f : (X,T) \to (Y,T')$ *is a semi - irresolute function if and only if for every semi p-closed subset G of* T'*,* $f^{-1}(G)$ *is semi p-closed in* T*.*

Proof. Proof is trivial by taking complements. □

Theorem 4.14. *Composition of semi-irresolute functions are semi -irresolute.*

Definition 4.15. *Let* (X,T)*,* (Y,T') *be two topological spaces , then a function* $f : X \to Y$ *is semi p-open if for every semi p-open set A in X ;* $f(A)$ *is semi p-open in* Y*.*

Example 4.3. *Let f be the identity function from (R,D) to (R,U) where R is the real line , D is the discrete topology and U is the usual topology . Then f is semi p-open but not pre semi -open.*

Theorem 4.16. *If $f : X \to Y$ is p-continuous and p-open then f is semi-irresolute and semi p-open.*

Proof. If f is given to be p-continuous and p-open, then f should be semi-irresolute by *Theorem : 4.14*. Also the proof of semi p-openness analogously follows from the proof of *Theorem : 4,14* and *Lemma : 4.13.* □

5. SEMI P-HOMEOMORPHISM AND NO WHERE P-DENSE SETS

Definition 5.1. *A function $f : X \to Y$ is said to be a semi p-homeomorphism if f is one-one, onto , semi p-open and semi -irresolute.*

Remark 5.2. *Homeomorphism implies p-homeomorphism implies semi p-homeomorhism and none of the converse implications holds. For example , let $X = \{a,b,c,d\}$ and let $T = \{X,\phi,\{a,b\},\{a\},\{a,b,c\},\{a,b,d\}\}$, $T' = \{X,\phi.\{a\}\}$ be two topologies on X. Consider the function $f : (X,T) \to (X,T')$ defined by $f(a) = b, f(b) = c, f(c) = d, f(d) = a$; then f is a semi p-homeomorphism but not a p-homeomorphism.*

Definition 5.3. *Let (X,T) be a topological space and let $A \subseteq X$ then semi p-closure of A denoted by semi p-cl(A) is defined as the minimal semi p-closed super set of A.*

Example 5.1. *$X = \{a,b,c\}, T = \{X,\phi,\{a\},\{a,b\},\{a,b,c\},\{a,b,d\}\}$. Let $A = \{c,d\}$, then semi p-cl$(\{c,d\}) = \{c,d\} = \overline{\{c,d\}}$ and p-cl$(\{c,d\}) = X$. Now let $B = \{b\}$, semi p-cl$(\{b\}) = \{b\}$ and p-cl$(\{b\}) = \{b,c,d\} = \overline{\{b\}}$.*

Proposition 5.4. *Let (X,T) be a topological space and $A \subseteq X$, then semi p-cl$(A) \subseteq$ p-cl(A).*

Proof. Since p-closed implies semi p-closed, the above result is trivial. □

Theorem 5.5. *A function $f : X \to Y$ is semi -irresolute if and only if $f($semip-cl$(A)) \subseteq$ semip-cl$(f(A))$.*

Proof. Let $A \subseteq X$ and consider semi p-$cl(f(A))$ which is semi p-closed in Y. Hence f^{-1}(semi p-$cl(f(A)))$ is semi p-closed in X.

But $f(A) \subseteq$semi p-$cl(f(A))$

$f^{-1}(f(A)) \subseteq f^{-1}$(semi p-$cl(f(A)))$. That is f^{-1}(semi p-$cl(f(A)))$ is a semi p-closed super set of A and by definition of semi p-closure

semi p-$cl(A) \subseteq f^{-1}$(semi p-$cl(f(A)))$.

Now taking f on both sides f(semi p-$cl(A)) \subseteq f(f^{-1}$(semi p-$cl(f(A)))) \subseteq$semi p-$cl(f(A))$.

Hence f(semi p-$cl(A)) \subseteq$semi p-$cl(f(A))$ and thus necessary part is proved.

Conversely let G be a semi p-closed set in Y to prove that $f^{-1}(G)$ is semi p-closed in X. Consider $f^{-1}(G)$ and applying our assumption on $f^{-1}(G)$ we have

f(semi p-$cl(f^{-1}(G))) \subseteq$semi p-$cl(f(f^{-1}(G)))$

$\subseteq$semi p-$cl(G) = G$.

$\Rightarrow$semi p-$cl(f^{-1}(G)) \subseteq f^{-1}(G)$ and then only possibility is $f^{-1}(G) =$semi p-$cl(f^{-1}(G)$. Thus $f^{-1}(G)$ is semi p-closed and hence f is semi -irresolute. □

Theorem 5.6. *A function $f : X \to Y$ is semi - irresolute if and only if for every $H \subseteq Y$; semi p-$cl(f^{-1}(H)) \subseteq f^{-1}$(semi p-$cl(H))$.*

Proof. Necessarily we assume that f is semi-irresolute and consider semi p-$cl(H)$ for $H \subseteq Y$. Since f is semi-irresolute, f^{-1}(semi p-$cl(H))$ is semi p-closed in X.

But $H \subseteq$semi p-$cl(H)$

$\Rightarrow f^{-1}(H) \subseteq f^{-1}$(semi p-$cl(H))$

$\Rightarrow$semi p-$cl(f^{-1}(H)) \subseteq f^{-1}$(semi p-$cl(H))$. Conversely let H be a semi p-closed set in Y to prove that $f^{-1}(H)$ is semi p-closed in X. Clearly $f^{-1}(H) \subseteq$semi p-$cl(f^{-1}(H)) \subseteq f^{-1}$(semi p-$cl(H)) = f^{-1}(H)$. Hence $f^{-1}(H)$ is semi p-closed in X and thus f is semi-irresolute. □

Corollary 5.7. *If $f : X \to Y$ is a semi p-homeomorphism then semi p-$cl(f^{-1}(B)) = f^{-1}$(semi p-$cl(B))$ for every $B \subseteq Y$.*

Corollary 5.8. *If $f : X \to Y$ is a semi p-homeomorphism then semi p-$cl(f(B)) = f$(semi p-$cl(B))$ for every $B \subseteq Y$.*

Definition 5.9. *Semi p-interior of $A \subseteq X$ in a topological space X is defined as maximal semi p-open subset of A and is denoted as semi p-int (A).*

Theorem 5.10. *If $f : X \to Y$ is a semi p-homeomorphism then*

(1) semi p-int$(f^{-1}(B)) = f^{-1}($semi p-int(B)).

(2) semi p-int$(f(B)) = f($semi p-int(B))

Proof. Proof is trivial using theorem 5.6 and theorem 5.7. □

Theorem 5.11. *Let (X,T) be a topological space and $A \subseteq X$. Then A is nowhere p-dense if and only if semi p-int(semi p-cl$(A)) = \phi$.*

Proof. Clearly semi p-$cl(A) \subseteq p\text{-}cl(A)$ and p-int$(A) \subseteq$semi p-$int(A)$
which implies p-int(p-cl$(A)) \subseteq$semi p-int(semi p-$cl(A)) = \phi$. Thus if semi p-int(semi p-$cl(A)) = \phi$ then p-int(p-$cl(A)) = \phi$. But $\overline{A} \subseteq p\text{-}cl(A)$. If p-$cl(A)$ contains no non empty p-open set then $\overline{A}$ also contains no non-empty p-open set which implies p-$int(\overline{A}) = \phi$ implies A is no where p-dense. Thus sufficiency part holds. For necessity assume that A is no where p-dense
$\Rightarrow$p-int$(\overline{A}) = \phi$ implies semi p-int$(\overline{A}) = \phi$. But semi p-$cl(A) \subseteq \overline{A}$. Hence if semi p-$int(\overline{A}) = \phi$, then semi p-int(semi p-$cl(A)) = \phi$ and hence the result. □

Theorem 5.12. *If $f : X \to Y$ is a semi p-homeomorphism and $A \subseteq X$ is nowhere p-dense in X then $f(A)$ is no where p-dense in Y.*

Proof. Assume that A is no where p-dense in X ; that is p-int$(\overline{A}) = \phi$ which implies semi p-int(semi p-$cl(A)) = \phi$. We have to prove that semi p-int(semi p-$cl(f(A))) = \phi$.
But semi p-int(semi p-$cl(f(A))) = f($semi p-int(semi p-$cl(A))) = f(\phi) = \phi$. Hence $f(A)$ is nowhere p-dense in Y. □

Theorem 5.13. *Let (X,T) be a topological space of first category and $f : (X,T) \to (Y,T')$ be a semi p-homeomorphism from (X,T) to another topological space (Y,T'). Then (Y,T') can be written as union of no where p-dense sets in it.*

Proof. Given X is of first category ; that is $X = \cup G_i : i = 1,2.....\infty$ where each G_i is nowhere dense in X. Now consider $Y = f(X) = f(\cup G_i) = \cup f(G_i) : i = 1,2,3....\infty$. Then by above theorem each $f(G_i)$ is no where p-dense and hence the result. □

Definition 5.14. *Let X be any arbitrary set and τ, τ' be topologies on X, then τ and τ' are said to be semi p-correspondent topologies on X if (X, τ) and (X, τ') has the same collection of sets.*

Example 5.2. *Any two hausdorff topologies on X is semi p-correspondent.*

Corollary 5.15. *Any two semi p-correspondent topologies on any arbitrary set X determines precisely the same nowhere p-dense subsets.*

Proof. Proof is trivial by definition of semi p-correspondent topologies and by theorem 5.14. □

ACKNOWLEDGEMENT

The first author wishes to thank Dr.T.P. Johnson, Professor, Cochin University of Science and Technology for the valuable suggestions.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

[1] S.G. Crossley, S.K. Hildebrand, Semi-topological properties. Fundam. Math. 74(3) (1972), 233-254.

[2] G. Birkhoff, On combination of topologies, Fundam. Math. 26 (1936), 156-166.

[3] G. Gratzer, Lattice Theory. First Concepts and Distributive Lattices, W. H. Freeman and Co., San Fransisco, 1971.

[4] G. Gierz, K.H. Hofmann, K. Keimel, et al. A compendium of continuous lattices, Springer-Verlag, Berlin, 1980.

[5] J. Picado, A. Pultr, Frames and Locales: Topology without Points, Frontiers in Mathematics, vol. 28. Springer, Basel (2012).

[6] J.P. Lee, On Semi-Homeomorphisms, In. J. Math. Math. Sci. 13 (1990), 129-134.

[7] N. Levine, Semi-open sets ande Semi-continuity in Topological spaces, Amer. Math. Mon. 70 (1963), 36-41.

[8] P.T. Johnstone, Stone Spaces, Cambridge University Press, Cambridge, 1986.

[9] R.E. Larson, S.J. Andima, The lattice of topologies: A survey, Rocky Mt. J. Math. 5 (1975), 177-198.

[10] S. Willard, General Topology, Academic Press, New York, 1970.

[11] T.R. Hamlett, Semi-Continuous Functions, Math. Chronicle, 4 (1976), 101-107.

[12] T. Vinitha, T.P. Johnson, p-compactness and C-p.compactness, Glob. J. Pure Appl. Math. 13 (2017), 5539-5550.

[13] T. Vinitha, T.P. Johnson, On Generalised p-closed sets, Int. J. Pure Appl. Math. 117 (2017), 609-619.